• Modern Information Retrieval (1999)  
  Ricardo-Baeza Yates and Berthier Ribeiro-Neto

• Flexible Pattern Matching in Strings (2002)  
  Gonzalo Navarro and Mathieu Raffinot

• Algorithms on strings (2001)  
  M. Crochemore, C. Hancart and T. Lecroq

• http://www-igm.univ-mlv.fr/~lecroq/string/index.html
String Matching

String matching: definition of the problem (text,pattern)

- **Exact matching:** depends on what we have: text or patterns
  - **The patterns** ---&gt; Data structures for the patterns
    - 1 pattern ---&gt; The algorithm depends on $|p|$ and $|\Sigma|$
    - k patterns ---&gt; The algorithm depends on k, $|p|$ and $|\Sigma|$
  - Extensions
  - Regular Expressions
  - **The text** ----&gt; Data structure for the text (suffix tree, ...)

- **Approximate matching:**
  - Dynamic programming
  - Sequence alignment (pairwise and multiple)
  - Sequence assembly: hash algorithm

- **Probabilistic search:** Hidden Markov Models
Exact string matching: one pattern

How does the string algorithms made the search?

For instance, given the sequence

CTACTACTACGTCTATACTGATCGTAGCTACTACATGC

search for the pattern ACTGA.

and for the pattern TACTACGGTATGACTAA
Exact string matching: Brute force algorithm

Example:

Given the pattern ATGTA, the search is
Exact string matching: Brute force algorithm

- How the comparison is made?

Text:

Pattern:

From left to right: prefix

- Which is the next position of the window?

Text:

Pattern:

The window is shifted only one cell
Exact string matching: one pattern

How does the matching algorithms made the search?

There is a sliding window along the text against which the pattern is compared:

Text:

Pattern:

At each step the comparison is made and the window is shifted to the right.

Which are the facts that differentiate the algorithms?

1. How the comparison is made.
2. The length of the shift.
Exact string matching: one pattern (text on-line)

Experimental efficiency (Navarro & Raffinot)

BNDM : Backward Nondeterministic Dawg Matching
BOM : Backward Oracle Matching
Horspool algorithm

• How the comparison is made?

Text:

Pattern:

• Which is the next position of the window?

Text:

Pattern:

Shift until the next occurrence of “a” in the pattern:

We need a preprocessing phase to construct the shift table.
Horspool algorithm: example

Given the pattern ATGTA

• The shift table is:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>C</th>
<th>G</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Given the pattern ATGTA

- The shift table is:

<table>
<thead>
<tr>
<th></th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
</tr>
<tr>
<td>G</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td></td>
</tr>
</tbody>
</table>
Horspool algorithm: example

Given the pattern ATGTA

- The shift table is:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
</tr>
<tr>
<td>C</td>
<td>5</td>
</tr>
<tr>
<td>G</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td></td>
</tr>
</tbody>
</table>
Given the pattern ATGTA

- The shift table is:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Horspool algorithm : example

Given the pattern ATGTA

- The shift table is:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
</tr>
<tr>
<td>C</td>
<td>5</td>
</tr>
<tr>
<td>G</td>
<td>2</td>
</tr>
<tr>
<td>T</td>
<td>1</td>
</tr>
</tbody>
</table>
Horspool algorithm: example

Given the pattern ATGTA

- The shift table is:

<table>
<thead>
<tr>
<th>Nucleotide</th>
<th>Shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
</tr>
<tr>
<td>C</td>
<td>5</td>
</tr>
<tr>
<td>G</td>
<td>2</td>
</tr>
<tr>
<td>T</td>
<td>1</td>
</tr>
</tbody>
</table>

- The searching phase: GTA CTA GAG GAC GT A TGT A C T G ...

ATGTA
ATGTA
ATGTA
ATGTA
ATGTA
ATGTA
ATGTA
Exemple algorisme de Horspool

Given the pattern ATGTA

• The shift table is:
  
<table>
<thead>
<tr>
<th>Nucleotide</th>
<th>Shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
</tr>
<tr>
<td>C</td>
<td>5</td>
</tr>
<tr>
<td>G</td>
<td>2</td>
</tr>
<tr>
<td>T</td>
<td>1</td>
</tr>
</tbody>
</table>

• The searching phase: GTA CTA GAG GAC GTATGTA C TG ...
Given the pattern ATGTA, the shift table is:

<table>
<thead>
<tr>
<th>Letter</th>
<th>Shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
</tr>
<tr>
<td>C</td>
<td>5</td>
</tr>
<tr>
<td>G</td>
<td>2</td>
</tr>
<tr>
<td>T</td>
<td>1</td>
</tr>
</tbody>
</table>

Given a random text over an equally likely probability distribution (EPD):

1. Determine the expected shift of the window. And, if the PD is not equally likely?

2. Determine the expected number of shifts assuming a text of length n.

3. Determine the expected number of comparisons in the suffix search phase.
Experimental efficiency (Navarro & Raffinot)

BNDM: Backward Nondeterministic Dawg Matching
BOM: Backward Oracle Matching

| $|\Sigma|$ |
|-----|-----|-----|-----|-----|-----|-----|
| 2   | 4   | 8   | 16  | 32  | 64  | 128 | 256 |

Long. pattern
Search for suffixes of T that are factors of

Which is the next position of the window ?

How the comparison is made?

That is denoted as

Once the next character x is read

B(x): mask of x in the pattern P.
For instance, if B(x) = \((0 \ 0 \ 1 \ 1 \ 0 \ 0 \ 0)\)

\[ D = (0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0) \ \& \ (0 \ 0 \ 1 \ 1 \ 0 \ 0 \ 0) = (0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0) \]

Which is the next position of the window ?

Depends on the value of the leftmost bit of \( D \)
Given the pattern ATGTA

- The mask of characters is:
  
  \[
  B(A) = (10001) \\
  B(C) = (00000) \\
  B(G) = (00100) \\
  B(T) = (01010)
  \]

- The searching phase:

\[
\begin{align*}
D_1 &= (01010) \\
D_2 &= (10100) & (00000) = (00000) \\
D_1 &= (00100) \\
D_2 &= (01000) & (00100) = (00000) \\
D_1 &= (10001) \\
D_2 &= (00010) & (01010) = (00010) \\
D_3 &= (00100) & (00100) = (00100) \\
D_4 &= (01000) & (00000) = (00000)
\end{align*}
\]

Exemple algorisme BNDM

- Given the pattern ATGTA
- The mask of characters is:
  - B(A) = (1 0 0 0 1)
  - B(C) = (0 0 0 0 0)
  - B(G) = (0 0 1 0 0)
  - B(T) = (0 1 0 1 0)

- The searching phase:
  - GTACTAGAGGACGTATGTAGCTG...
  - D1 = (1 0 0 0 1)
  - D2 = (0 0 0 1 0) & (0 1 0 1 0) = (0 0 0 1 0)
  - D3 = (0 0 1 0 0) & (0 0 1 0 0) = (0 0 1 0 0)
  - D4 = (0 1 0 0 0) & (0 1 0 1 0) = (0 1 0 0 0)
  - D5 = (1 0 0 0 0) & (1 0 0 1) = (1 0 0 0 0)
  - D6 = (0 0 0 0 0) & (*****) = (0 0 0 0 0) Trobat!

ATGTA
Given the pattern ATGTA

• The mask of characters is:

B(A) = (10001)
B(C) = (00000)
B(G) = (00100)
B(T) = (01010)

How is the shift determined?

• The searching phase:

\( D_1 = (01010) \)
\( D_2 = (10100) \& (00000) = (00000) \)
\( D_1 = (01010) \)
\( D_2 = (10100) \& (10001) = (10000) \)
\( D_3 = (00000) \& (10001) = (00000) \)