DPLL(T):Fast Decision Procedures

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Computer Aided-Verification (CAV)

Boston, July 2004

In Memoriam

Harald Ganzinger (1950-2004)

Overview of this talk

1. Introduction
2. Lazy vs eager approach
   - Lazy approach: advantages and disadvantages
   - Eager approach: advantages and disadvantages
3. DPLL(T): Our framework for SMT
   - The DPLL algorithm
   - Branching heuristics, unit propagation and conflict analysis
   - Comparison with existing approaches
4. A concrete case: EUF with offsets
   - A solver for EUF
   - Experimental results
5. Conclusions and future work

SMT: Satisfiability modulo theories

$$g(a)=c \;\wedge\; (\, f(g(a))\neq f(c) \,\vee\, g(a)=d \,) \;\wedge\; c\neq d$$

- Theories of interest: EUF [Burch and Dill '94], CLU [Bryant, Lahiri and Seshia '02], separation logic [BLS '03], arrays, ...
- Applications: circuit design, compiler optimization, planning, scheduling, software/hardware verification, ...

Lazy vs eager approach

Lazy approach

The following three steps are iterated

- SAT solver looks for a propositional model
- Specialized procedure for conjunctions of literals checks its consistency
- If model consistent then formula is SAT, otherwise a lemma is added precluding the model

– constraints imposed by the theory introduced on demand

– Lazy/eager notification, online/offline SAT solver, extraction of inconsistency proofs [Armando et al ’00; deMoura and Ruess ’02; Barret, Dill and Stump ’02; Flanagan et al ’03, etc]

Lazy vs eager approach

Lazy approach

- Advantages:
  - Use of off-the-shelf theory solvers
  - Can use of almost off-the-shelf SAT solvers
- Disadvantages:
  - Information from the theory only used to validate propositional models
  - Too many iterations may be required
- Tools: SVC, CVC (Lite), ICS, VeriFun, MathSAT

Lazy vs eager approach

Eager approach

- formula converted into an equisatisfiable propositional one to be checked by a SAT solver
- Two steps (for CLU)
  - Functional symbols are removed, only constants left
  - (in)Equality is removed
- Small-domain encoding (SD) [Pnuelli et al '99, BLS '02], Per-constraint encoding (EIJ) [Bryant, German and Velev '02; Bryant and Velev '02], Hybrid methods [BLS '02, '03]

Lazy vs eager approach

Eager approach: different encodings

Given the equality formula:

$$(k_1 = k_2 \lor k_3 = k_4) \land (k_2 = k_3 \lor k_1 = k_4 \lor k_2 = k_4)$$

Small Domain encoding (SD): propositional formula small but suffers from loss of structure

$$(x_{11} \lor ((x_{31} \land x_{41}) \lor (\neg x_{31} \land x_{32} \land \neg x_{41}))) \land$$
$$(x_{31} \lor ((x_{11} \land x_{41}) \lor (\neg x_{11} \land x_{12} \land \neg x_{41})) \lor x_{41})$$

Per-constraing encoding (EIJ): structure preserved but size may be exponential if pred/succ allowed

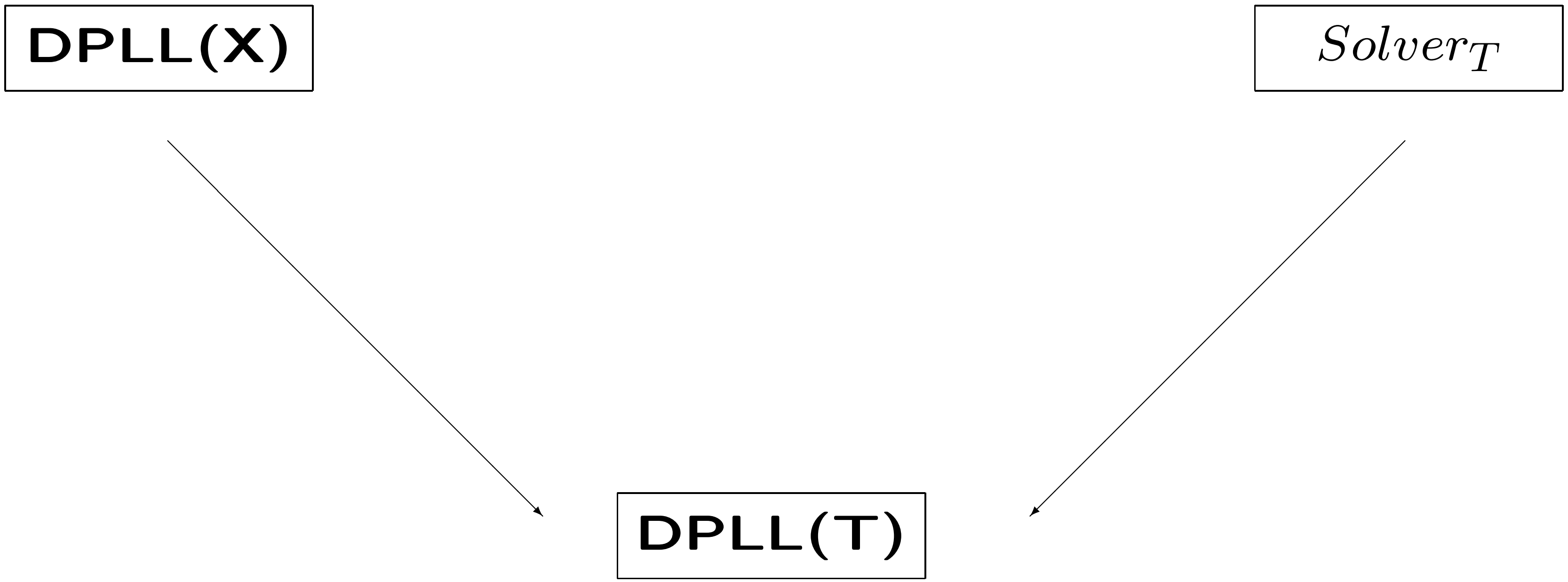
$$(e_{12} \lor e_{34}) \land (e_{23} \lor e_{14} \lor e_{24})$$
$$e_{12} \land e_{24} \Rightarrow e_{14}$$
$$e_{12} \land e_{14} \Rightarrow e_{24}$$
$$\ldots$$

Lazy vs eager approach

Eager approach

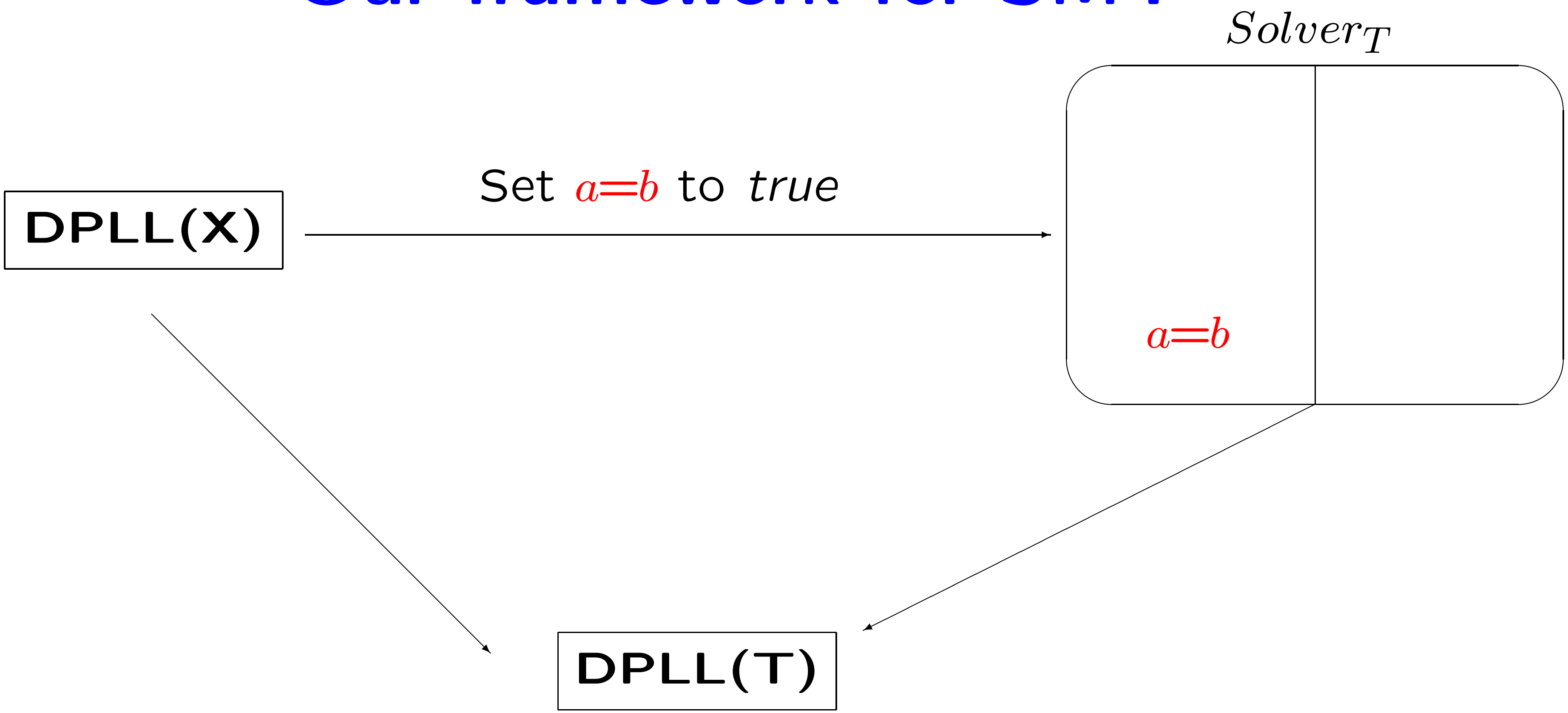
- Advantages:
  - Best SAT solver may be used as is
  - Theory information compiled into the translated formula
- Disadvantages:
  - Loss of formula structure, exponential blowup in size
  - Limited range of application
- Tools: UCLID

Our framework for SMT



Based on theoretical calculus in [Tinelli'02]

Our framework for SMT



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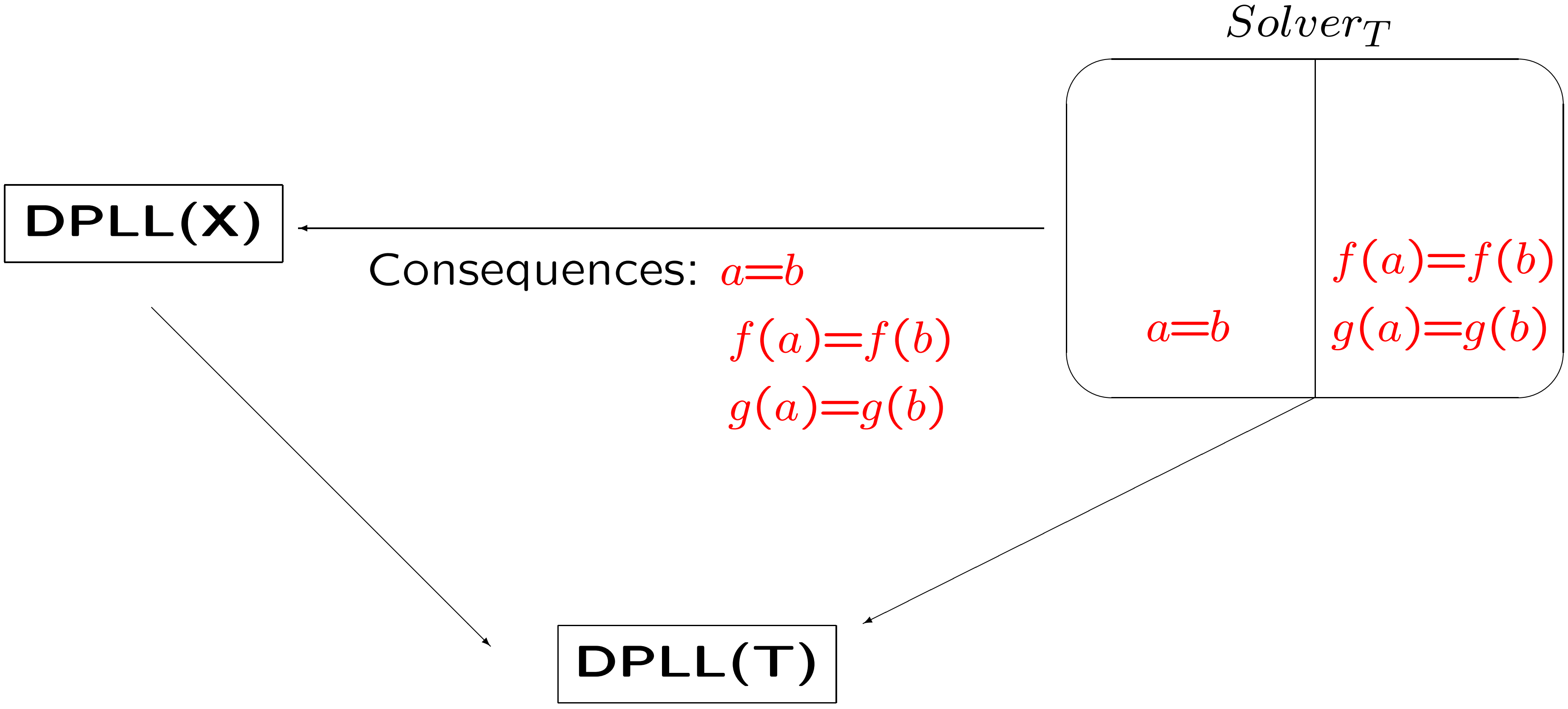

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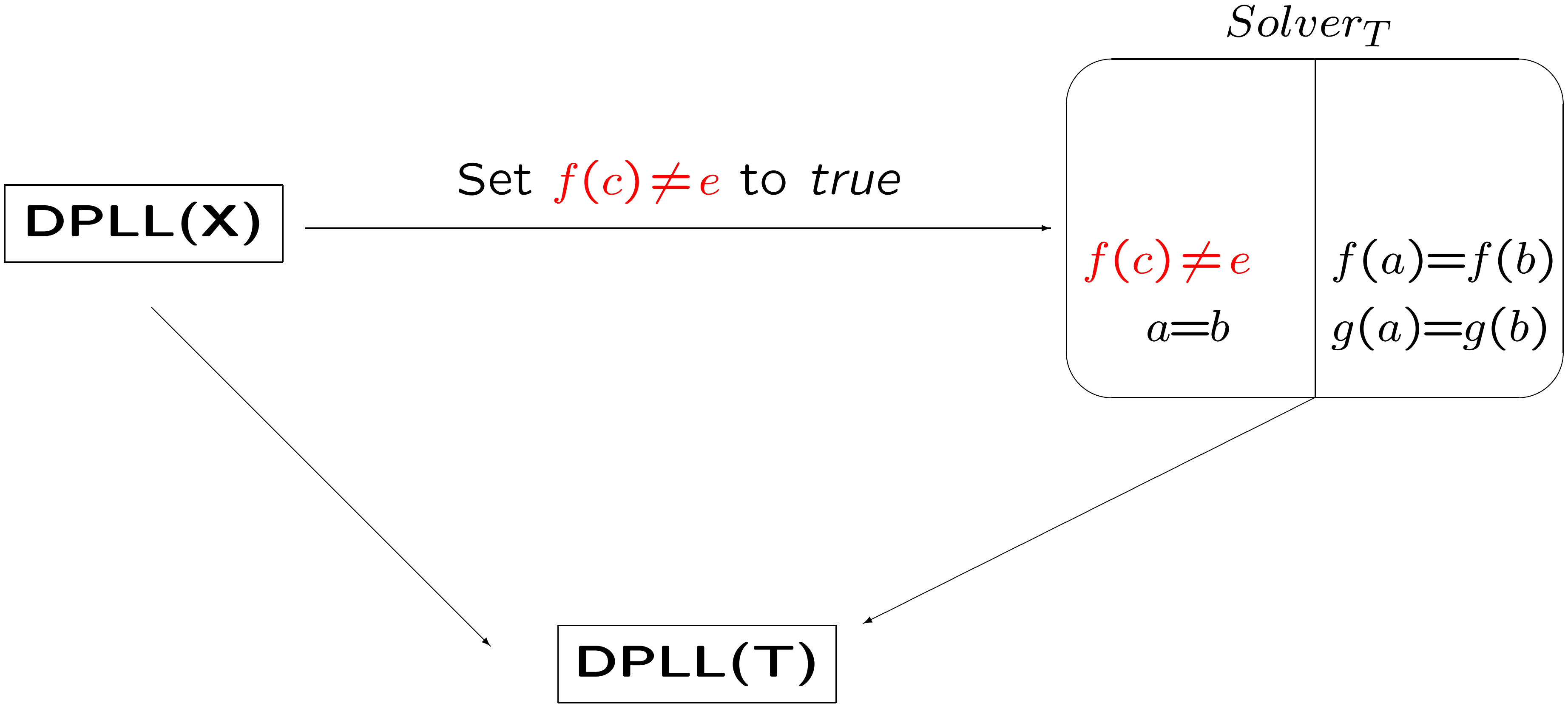

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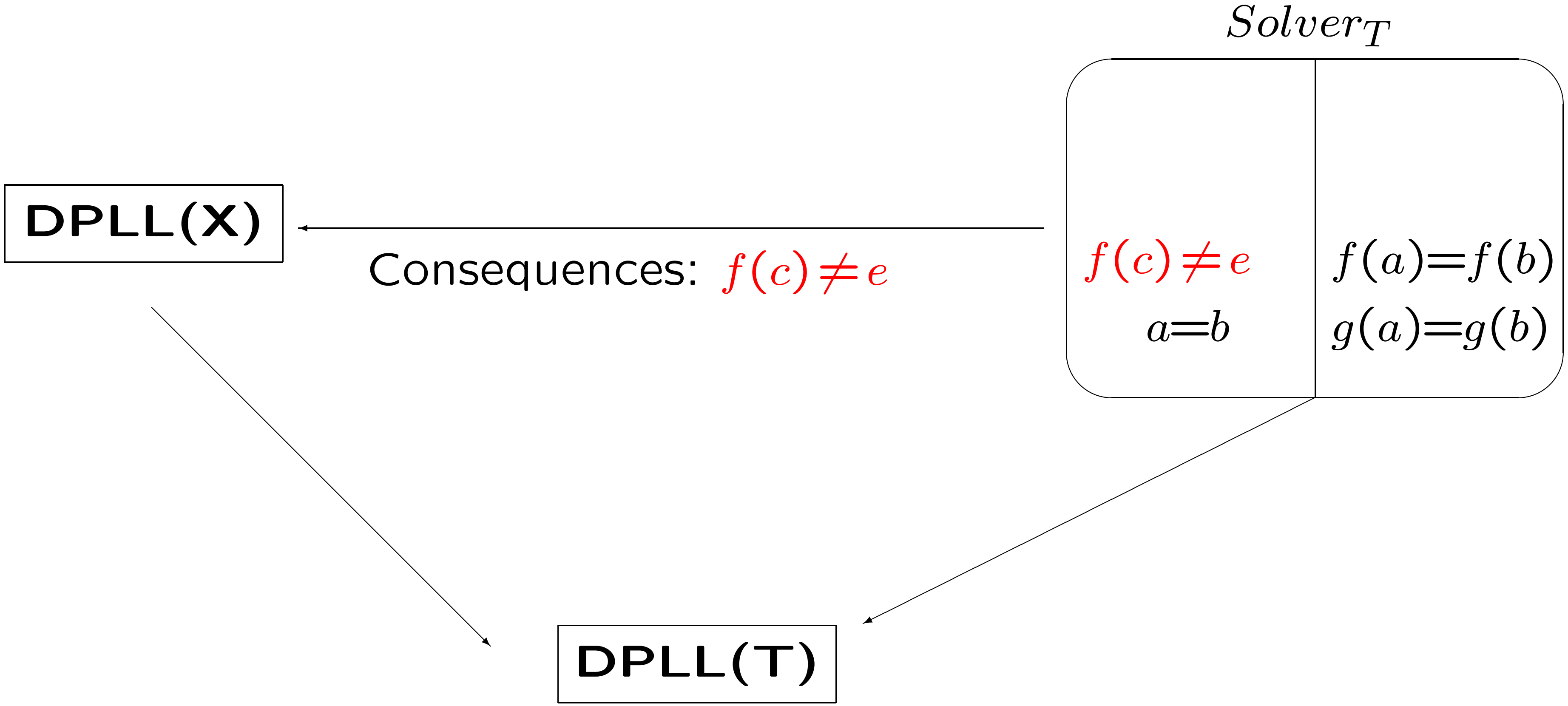

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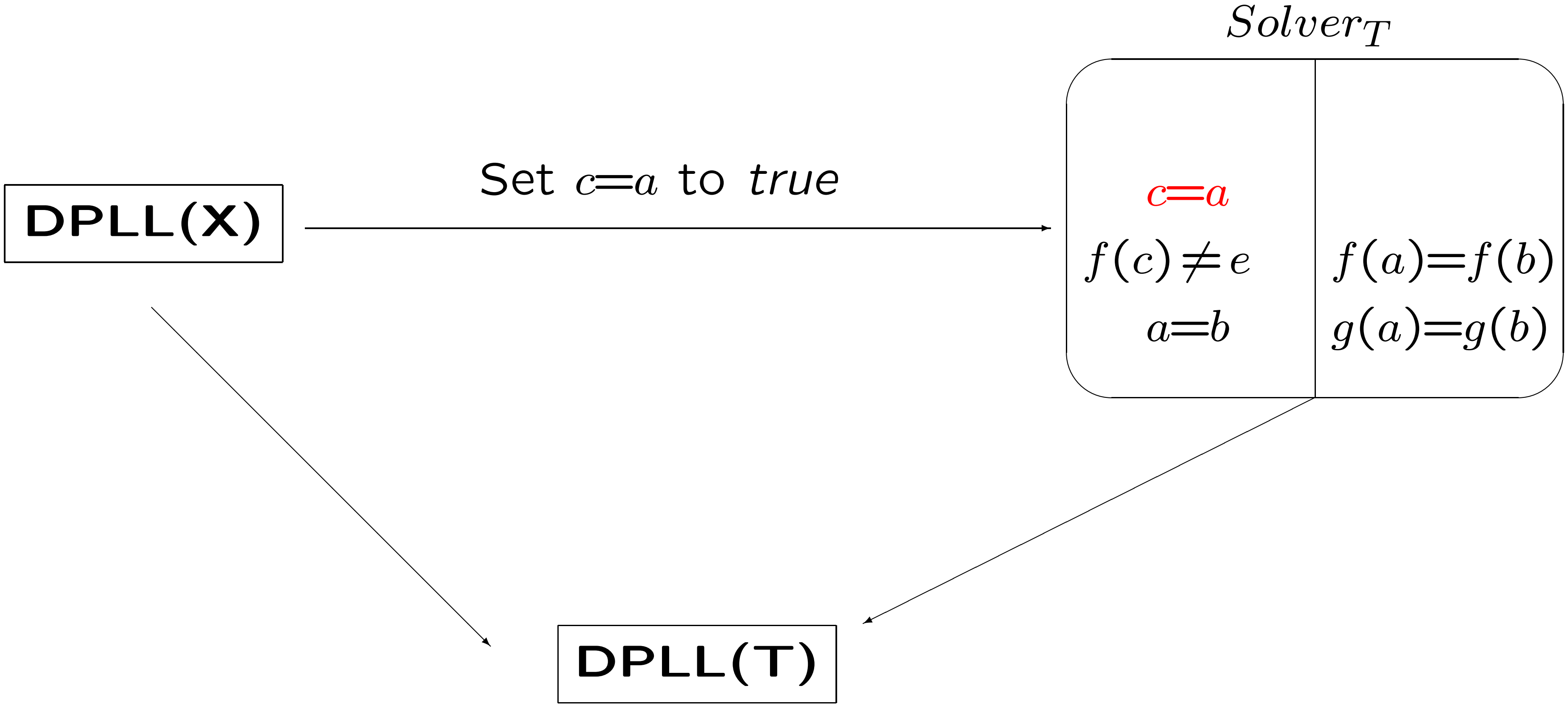

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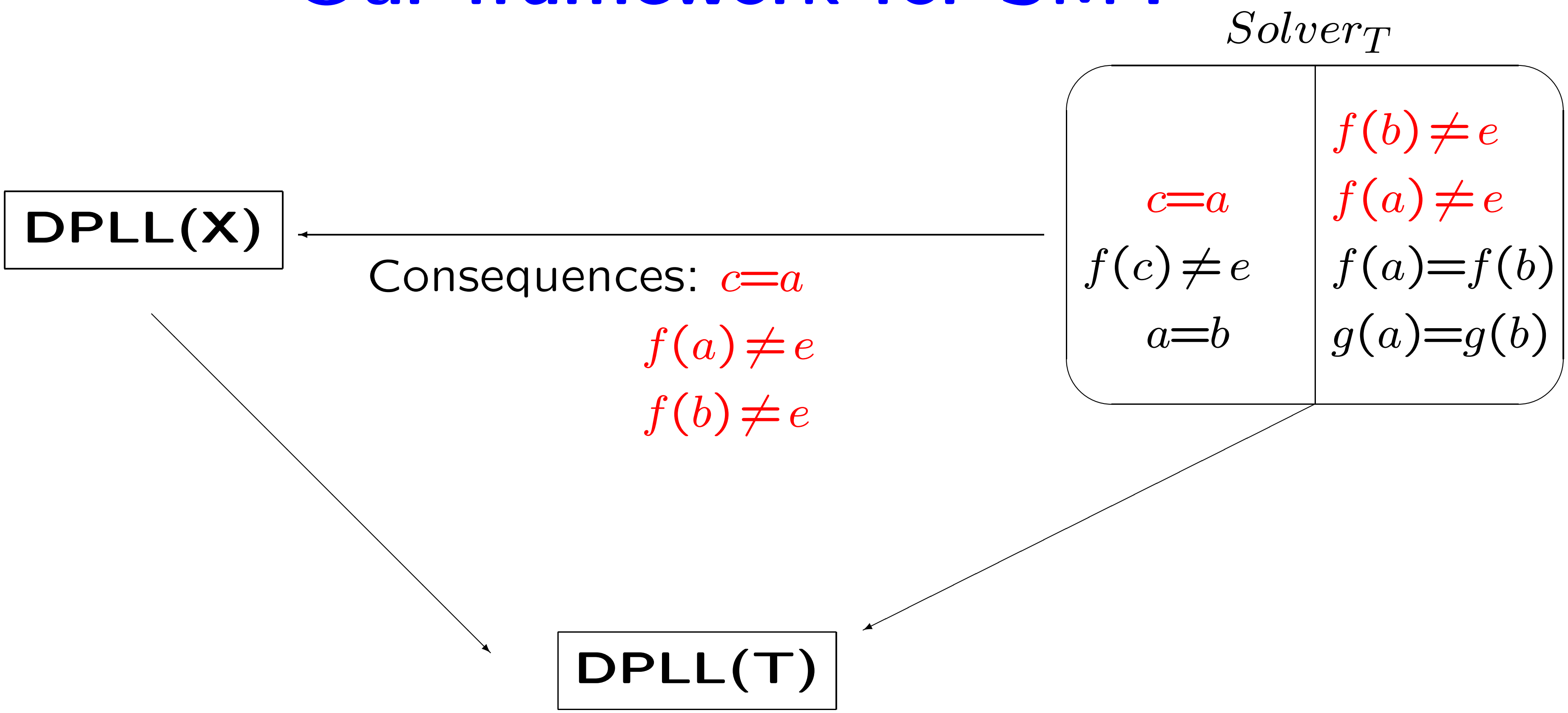

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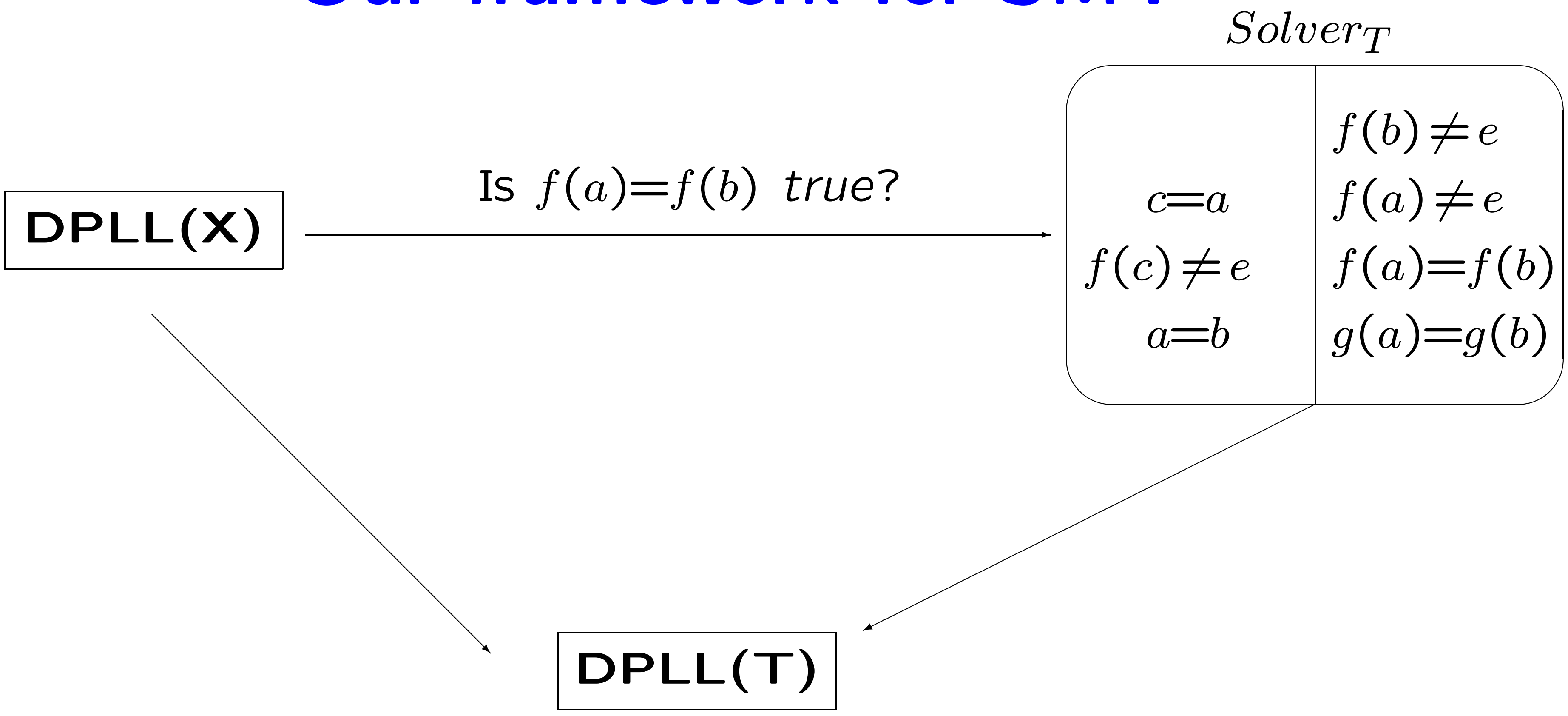

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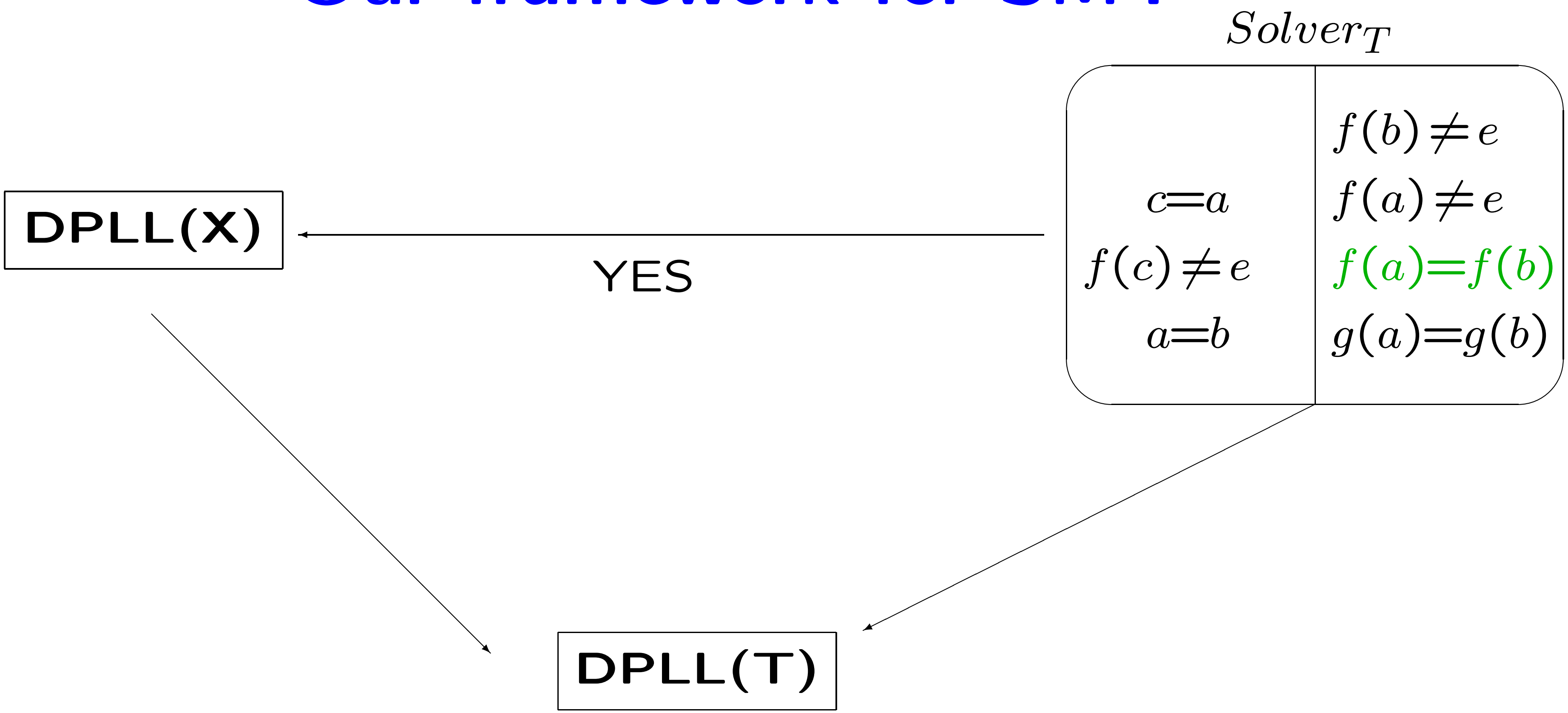

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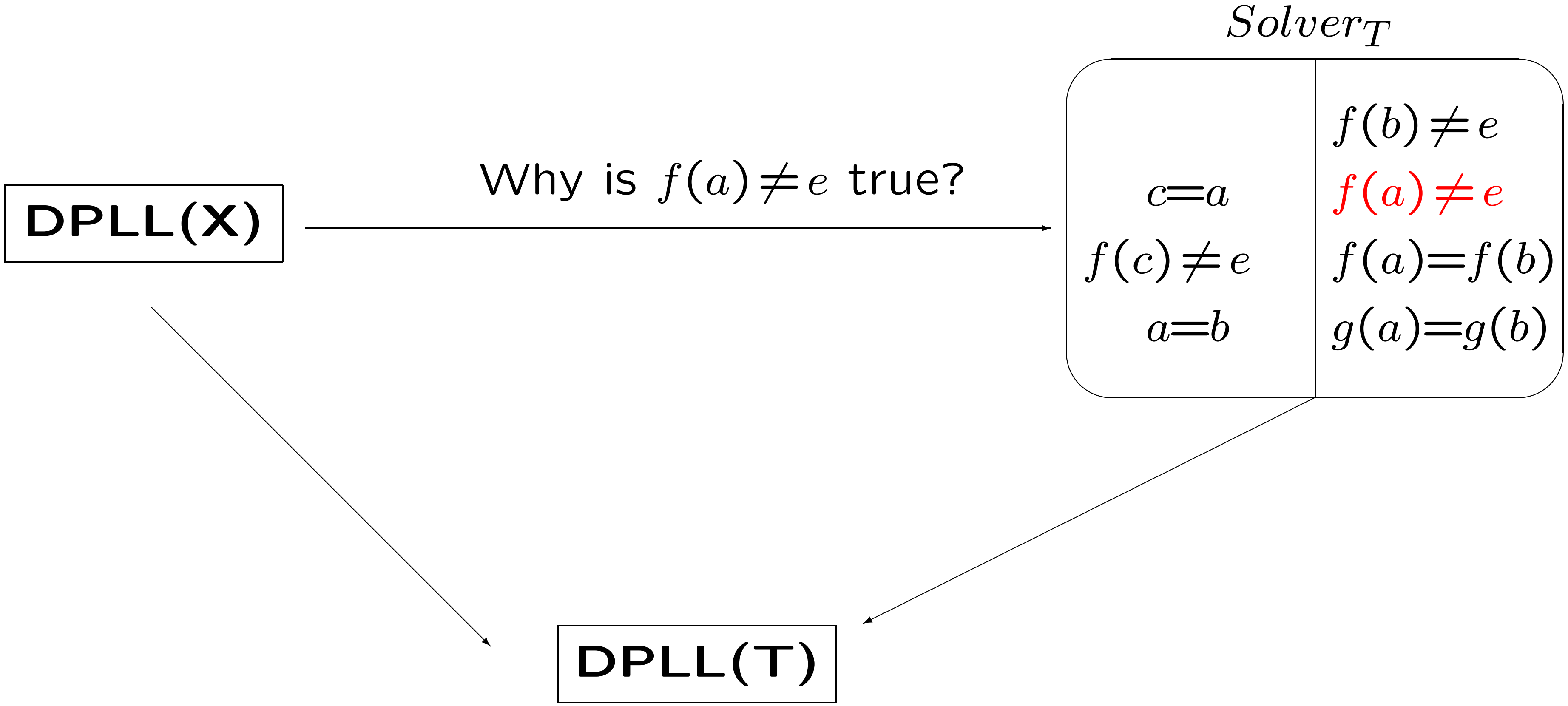

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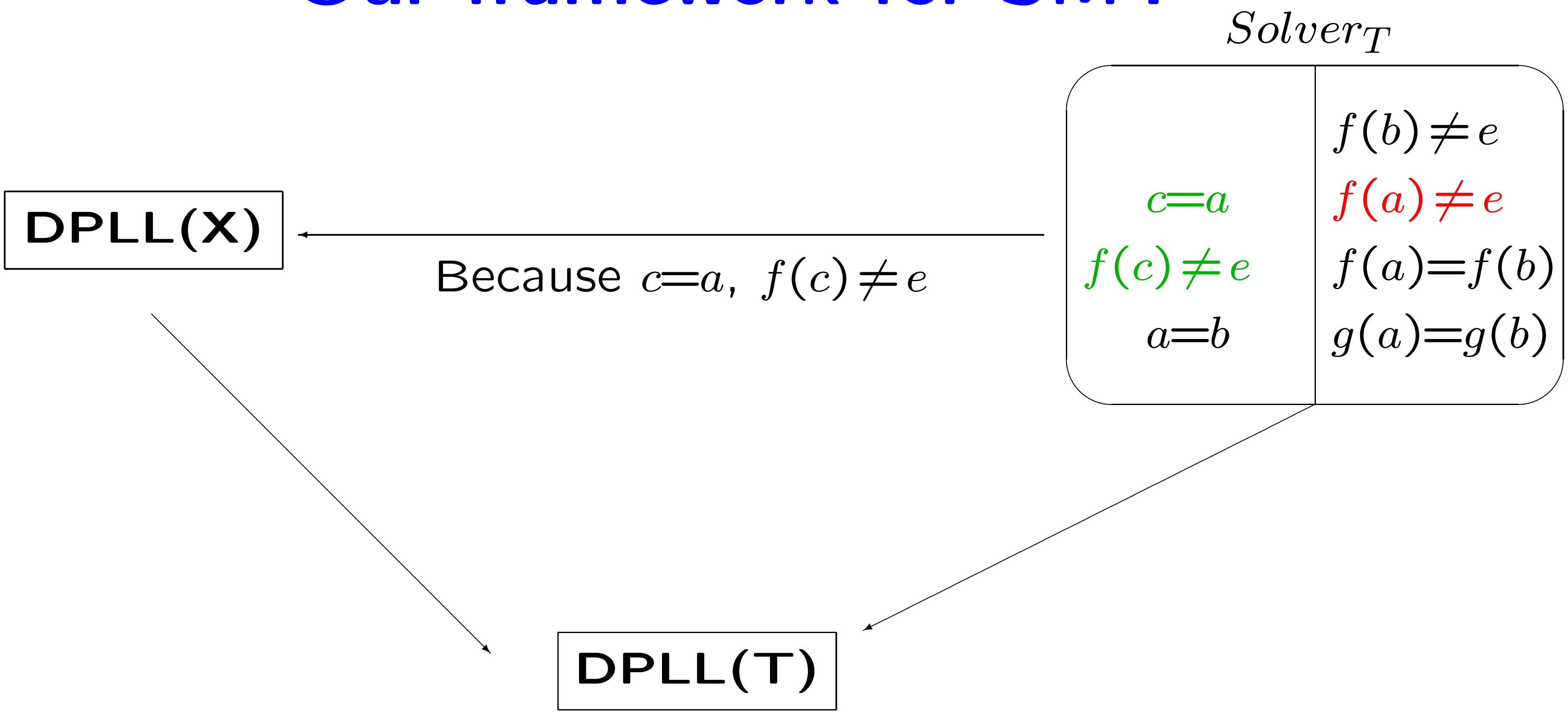

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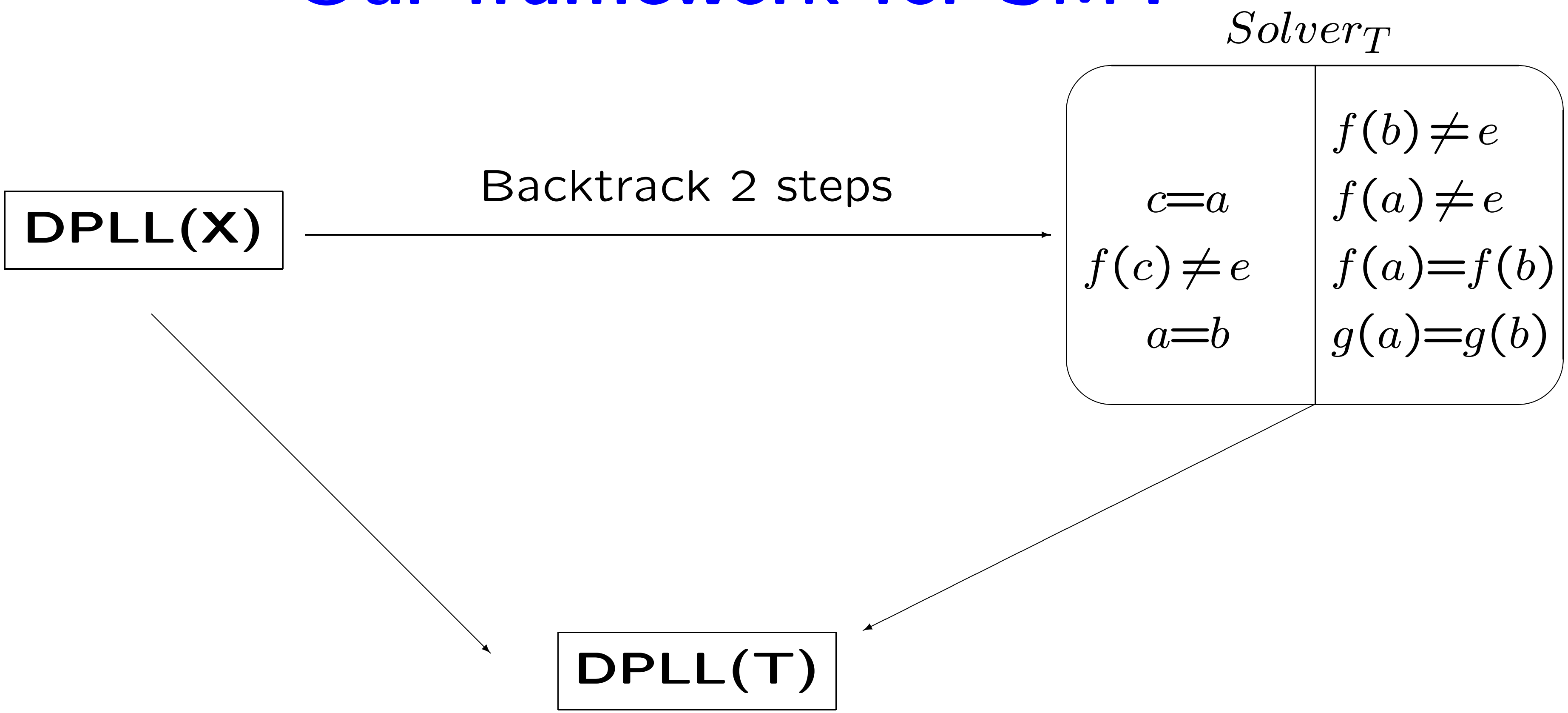

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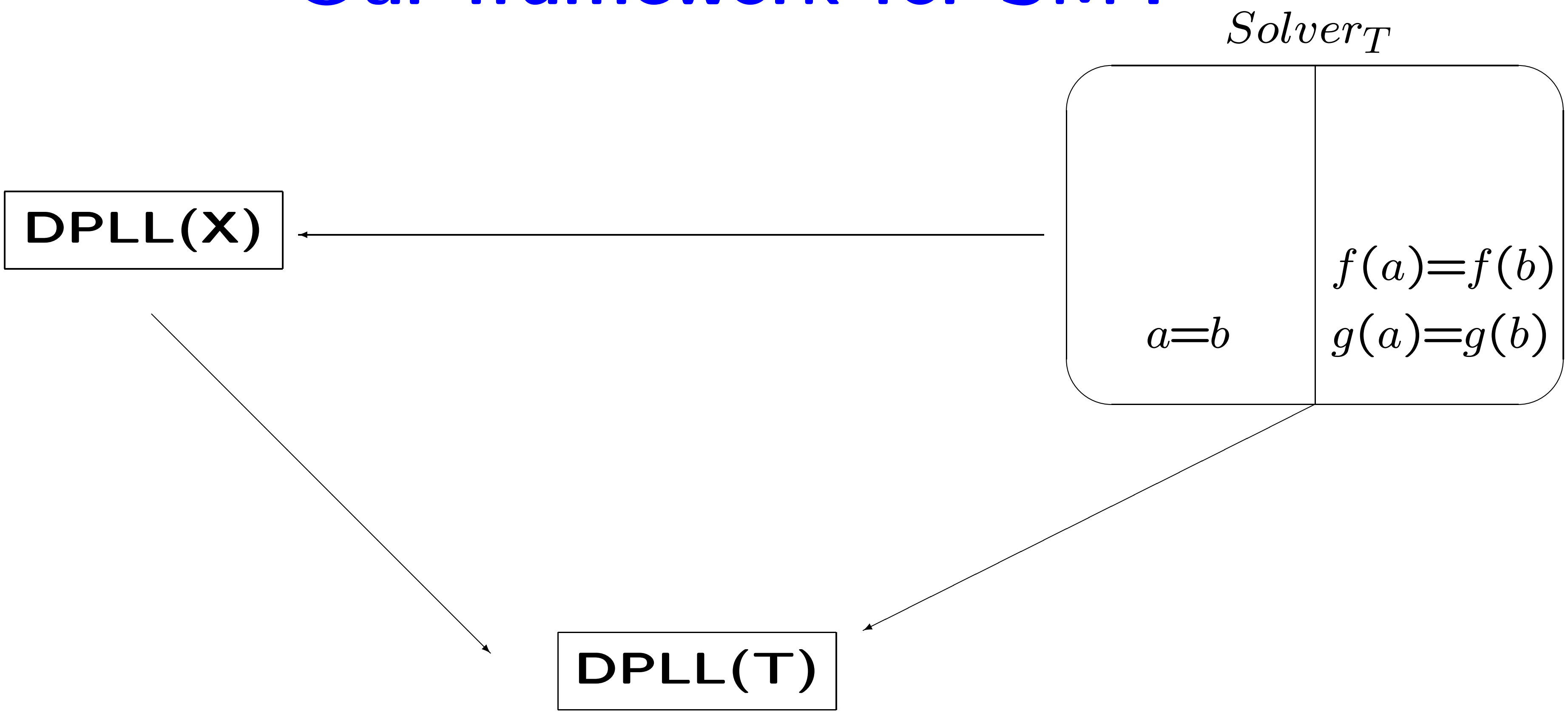

The Davis-Putnam algorithm (DPLL)

- Depth-first search algorithm with backtracking
- At each point, the algorithm keeps a partial interpretation and tries to extend it
- Three successful mechanisms to speed up the search
  - Branching heuristic: determines the literal to extend the interpretation
  - Unit propagation: prunes the search space
  - Conflict Analysis: indicates where to backtrack to and adds lemmas

Branching heuristics

- Unassigned literal with the highest score is selected
- New literals introduced in CNF translation can be selected
- VSIDS heuristic [Moskewicz et al '01]

T-based Branching heuristics

- Unassigned literal with the highest score is selected
- New literals introduced in CNF translation can be selected
- VSIDS heuristic [Moskewicz et al '01]
- Theory-dependent heuristics

Unit Propagation

- A literal appearing in a unit clause has to be true
- EXAMPLE:
  - Consider the binary clause

$$a \neq d \quad \vee \quad g(c)=h(a)$$

  - Now add $a=d$ to the interpretation.
  - The binary clause becomes unit and $g(c)=h(a)$ is added to the interpretation
- State-of-the-art mechanism to detect unit clauses: two watched literal scheme [Moskewicz'01]

T-based Unit Propagation

- A literal appearing in a unit clause has to be true
- EXAMPLE:
  - Consider the binary clause

$$c \neq d \ \vee \ g(c) = h(a)$$

  - Now add $a = d$ to the current interpretation $I = \{a = c\}$.
  - The binary clause becomes unit due to the theory and $g(c) = h(a)$ is added to the interpretation
- Literals returned by SetTrue allow $DPLL$(X) to detect these unit clauses

Conflict analysis

- Analysis performed on the implication graph
- Literals true due to
  - decision (no antecedent in the graph)
  - Unit propagation
- Learning schemes: decision scheme, 1UIP, 2UIP, AllUIP

T-based Conflict analysis

- Analysis performed on the implication graph
- Literals true due to
  - decision (no antecedent in the graph)
  - T-based unit propagation
- Learning schemes similar to decision scheme, 1UIP, 2UIP, AllUIP
- UIP-based learning schemes do not lift with non-exhaustive solvers

Comparison with existing approaches

- Neither loss of structure nor blowup in size
- Theory information used to drive the search
- General framework
- Benefits from improvements in SAT technology

A concrete case: EUF with offsets

- Extension of EUF, but not full CLU
- The sintax is:

$$\begin{aligned} formula :== \;& \mathbf{true} \mid \mathbf{false} \mid predicateSymbol(int\_term, \ldots, int\_term) \\ & \mid \neg formula \mid (formula \vee formula) \\ & \mid (formula \wedge formula) \mid (int\_term = int\_term) \end{aligned}$$

$$\begin{aligned} int\_term :== \;& functionSymbol(int\_term, \ldots, int\_term) \\ & \mid ite(formula, int\_term, int\_term) \\ & \mid succ(int\_term) \mid pred(int\_term) \end{aligned}$$

A solver for EUF with offsets

- New DST-like algorithm for CC with offsets [Nieuwenhuis and Oliveras '03] is the key ingredient
- Two initial transformations at the formula level done once and for all
- After that, only (dis)equalities between constants

A solver for EUF with offsets

The full solver is an extension of the CC algorithm:

- Deals with disequalities
- Incremental and backtrackable
- Explanations based on CC with proof extraction [Nieuwenhuis and Oliveras '04]

Experimental results

Comparison with lazy approaches:

Family	SVC	ICS	DPLL(T)
Buggy Cache	(1 T) 6000	179	7
Code Validation	57	55	4
DLX processor	17	4	1
Elf processor	(1 T) 6078	(4 T) 24001	575
OOO-rf	(2 T) 12666	(2 M) 12458	6385
OOO-tag	(4 T) 28768	(2 M, 2 T) 24050	1979
Load-Store	(3 T) 18475	(1 M, 1 T) 12167	30
Cache Protocol	(4 T) 26112	(5 T) 32022	3601
Two queues	1872	(2 M) 12175	74

T: timeout (more than 6000s.)

M: out of memory, counted as timeout

Experimental results

Comparison with eager approaches (using BerkMin):

Family	SD	Hybrid	DPLL(T)
Buggy Cache	2	3	7
Code Validation	45	28	4
DLX processor	10	13	1
Elf processor	5882	3182	575
OOO-rf	(2 T) 18211	(1 T) 10126	6385
OOO-tag	247	6918	1979
Load-Store	51	45	30
Cache Protocol	4151	209	3601
Two queues	407	793	74

Experimental results

Comparison with eager approaches (using Siege):

Family	SD	Hybrid	DPLL(T)
Buggy Cache	2	4	7
Code Validation	34	28	4
DLX processor	12	13	1
Elf processor	3585	1653	575
OOO-rf	(3 T) 18689	(2 T) 13180	6385
OOO-tag	211	(1 T) 7600	1979
Load-Store	54	45	30
Cache Protocol	4594	228	3601
Two queues	858	(1 T) 6809	74

Conclusions and future work

Conclusions:

- New approach for SMT
- Combines advantages of lazy and eager approaches
- Experimental tests are highly positive

Future work:

- Experiment with more theories
- Define isolated core functionalities of the $DPLL$(X) engine
- Extend to non-quantifier-free formulas