Islands, coordination and parasitic gaps*

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Mind the gap ... Mind the gap
London Transport

Abstract

We present type logical grammar of left extraction with attention to islands, coordination and parasitic gaps. We consider implementation in proof nets. The account suggests a measure of the semigrammaticality of island violations.

0. Data

Coordination might be thought a test for constituency, but it is possible to coordinate elements which are non-constituents on most any theory. Consider for example Right Node Raising (Postal 1974; Bresnan 1974) and Left Node Raising (Schachter and Mordechay 1983):

(1) a. [John liked and Mary disliked] London.
 b. Bill met [John on Monday and Sue on Tuesday].

The Right Node Raising in (1a) suggests that a subject can form a constituent with a transitive verb. A certain prosodic markedness accompanies the construction. Generally it might be assumed that the conjunts are sentences with gaps in object position, and that this situation is signalled prosodically. However the Left Node Raising (1b) suggests, even more radically, that an object can form a constituent with an adverbial, and this time there is no prosodic markedness.

Coordination might also be thought a test for category identity, but coordination of apparently unlike categories is possible (Sag et al. 1985):

(2) 007 is Bond and teetotal.

In (2) the conjunts are respectively a proper name and an adjective.

In these respects coordination conflicts with common basic assumptions and motivates type logical grammar (Morrill 1994, Moortgat 1997, Carpenter 1997) which characterizes all of the above as indeed coordination of like constituents. Our concern in this paper is with left extraction including relations to coordination, and including parasitic gaps.

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Although left extraction such as interrogativization, topicalization and relativization is unbounded in distance, it is not unconstrained. Coordinate structures are *islands* to extraction (Coordinate Structure Constraint, Ross 1967):

(3) *man that$_i$ [Suzy met Bill and Mary married e_i]

Extraction becomes grammatical if it is Across-the-Board (ATB):

(4) man that$_i$ [Suzy met e_i and Mary married e_i]

However, not even ATB extraction is possible of entire conjunts:

(5) *man that$_i$ Suzy met [the friend of e_i and e_i]

Adverbials and nominal subjects are weak islands (Adverbial Island Constraint; Subject Condition, Chomsky 1973):

(6) a. ?man that$_i$ [the friends of e_i] smiled
    b. ?paper that$_i$ John slept [without reading e_i]

Extraction from weak islands becomes fine if accompanied by a cobound non-island extraction:

(7) a. man that$_i$ [the friends of e_i] admire e_i
    b. paper that$_i$ John filed e_i [without reading e_i]

This is referred to as *parasitic extraction* (Ross 1967; Taraldsen 1979; Engdahl 1981, 1983; Sag 1983). The idea is to say that parasitic gaps in islands are dependent on or licensed by non-island host gaps. Note that in judging (6) we even experience pressure to force a transitive reading on the intransitive verbs.

We assume here that, as the term suggests, a parasitic gap *must* fall within an island:

(8) *slave that$_i$ John sold e_i to e_i

Thus we must consider the co-bound extraction in (9) (cf. Postal 1993 (8a)), where neither gap falls within an island, some other kind of "symbiotic" extraction.

(9) man who$_i$ Mary convinced e_i that John wanted to visit e_i

A host gap can licence more than one parasitic gap, but only in different islands:

(10) a. paper that$_i$ [the editor of e_i] filed e_i [without reading e_i]
     b. *slave who$_i$ [the fact that John sold e_i to e_i] surprised e_i

A host gap cannot directly licence a parasitic gap in an island within an island (Postal's 1994 Island Condition). However, it seems that one parasitic gap can in turn be host to another:

(11) a. man who$_i$ [the fact that [the friends of e_i] admire e_i] surprises e_i
     b. ?paper that$_i$ John published e_i [without [the editor of e_i] rereading e_i]
     c. ?man who$_i$ [the fact that [the friends of e_i] admire e_i [without praising e_i]] surprises e_i

Sentential subjects are strong islands and do not seem to allow parasitic gaps:

(12) a. ??man who$_i$ [that Mary likes e_i] surprises John
     b. ?man who$_i$ [that Mary likes e_i] surprises e_i

Fixed Subject Constraint (Bresnan 1972; Chomsky and Lasnik 1977) violations are out:

(13)*man who$_i$ John said that e_i left

The subjects of uncomplementized embedded sentences, however, can be extracted, and can licence parasitic gaps:

(14) a. man that$_i$ John thinks e_i left
b. man that$_i$ [the friends of e_i] think e_i left

Main subject relativization seems not to license parasitic gaps:

(15)?man that$_i$ left [without John meeting e_i]

As well as involving the puzzling distribution of parasitic gaps, these data are challenging because some judgements are not categorical: a full story will need to say something about degrees of acceptability despite ungrammaticality, or degrees of unacceptability despite grammaticality, or both. Our labels ?-, ??-, ..., * leave open the question as to whether the ambivalence arises from the former or the latter. In our account it will sometimes be due to the one, sometimes to the other, potentially sometimes to both.

Section 1 presents the basic framework. Section 2 considers islands, section 3 coordination, section 4 left extraction, and section 5 parasitic gaps. In section 6 we consider proof nets for the account developed and in section 7 we consider semigrammaticality and acceptability.

1. Type Logical Grammar

A *($|\Sigma|$-)sorted algebra* $\langle\{D_\sigma\}_{\sigma\in\Sigma}, \{\circ_{o_\Phi}\}_{o_\Phi\in\Omega,\Phi\in\Sigma*\to\Sigma}\rangle$ comprises a Σ-indexed family $\{D_\sigma\}_{\sigma\in\Sigma}$ of domains, a set Ω of operators, and an Ω-indexed set $\{\circ_{o_\Phi}\}_{o_\Phi\in\Omega,\Phi\in\Sigma*\to\Sigma}\rangle$ of operations where $\circ_{o_{\sigma_1,\ldots,\sigma_n\to\sigma}}$ is an n-place operation mapping from $D_{\sigma_1},\ldots,D_{\sigma_n}$ into D_σ. An *algebra* $\langle D, \{\circ_{o_i}\}_{o_i\in\Omega,i\in\mathcal{N}}\rangle$ is a 1-sorted algebra where $\circ_{o_i}$ is an i-place operation on the domain D; we call a list of the arities of the operations the *arity* of the algebra.

Given sets X and Y, the *functional exponentiation* X^Y is the set of all functions from Y to X; the *Cartesian product* $X\times Y$ is the set of all ordered pairs of elements from X (first) and Y (second); the *disjoint sum* $X\uplus Y$ is $(\{0\}\times X)\cup(\{1\}\times Y)$. A *frame* is a family of domains which is closed under disjoint sum, Cartesian product and functional exponentiation, i.e an algebra $\langle\{D\},\uplus,\times,(\_)^-\rangle$.[1] Given some set d of *atomic semantic type indices* (e.g. $\{e,o\}$ for entities and propositions) we define *semantic type indices* $\mathcal{T}$ by:

(16) $\mathcal{T} ::= d \mid \mathcal{T}{+}\mathcal{T} \mid \mathcal{T}\&\mathcal{T} \mid \mathcal{T}{\to}\mathcal{T}$

A *semantic algebra* is a sorted algebra:

(17) $\langle\{D_\tau\}_{\tau\in\mathcal{T}}, \{\iota_{1\tau\to\tau+\tau'}, \iota_{2\tau'\to\tau+\tau'}, (\_,\_)_{\tau,\tau'\to\tau\&\tau'}, (\_\ \_)_{\tau'\to\tau,\tau'\to\tau}\}_{\tau,\tau'\in\mathcal{T}}\rangle$

where

$$
(18)\quad \begin{array}{rcl}
D_{\tau+\tau'} & = & D_\tau \uplus D_{\tau'}\\
D_{\tau\&\tau'} & = & D_\tau \times D_{\tau'}\\
D_{\tau'\to\tau} & = & D_\tau^{D_{\tau'}}
\end{array}
$$

That is, $\{D_\tau\}_{\tau\in\mathcal{T}}$ is a $\mathcal{T}$-indexed frame; and $\iota_1 m = \langle 0, m\rangle$ and $\iota_2 m = \langle 1, m\rangle$ (first and second injection), $(m, m') = \langle m, m'\rangle$ (ordered pairing), and $(m\ m') = m(m')$ (functional application).

A *semigroup* is an algebra $\langle L, +\rangle$ of arity (2) the operation of which is *associative*:

(19) $s_1{+}(s_2{+}s_3) = (s_1{+}s_2){+}s_3$

[1] I cannot resist observing the cardinal homomorphism from the algebra of frames to the algebra of arithmetic: $|X\uplus Y| = |X| + |Y|, |X\times Y| = |X|\times|Y|, |X^Y| = |X|^{|Y|}$.

Because + is associative, we can omit its parentheses.

We call a semigroup $\langle L, +\rangle$ the *underlying prosodic algebra* of the *prosodic algebra*:

(20) $\langle \mathcal{P}(L), \backslash, \bullet, /, \wedge, \vee\rangle$

where

$$\begin{array}{rcl} A \bullet B & = & \{s+s' |\ s \in A \text{ and } s' \in B\} \\ A\backslash B & = & \{s|\ \text{for all } s' \in A, s'+s \in B\} \\ B/A & = & \{s|\ \text{for all } s' \in A, s+s' \in B\} \\ A \wedge B & = & \{s|\ s \in A \text{ and } s \in B\} \\ A \vee B & = & \{s|\ s \in A \text{ or } s \in B\} \end{array}$$

We call (20) the *powerset residuated lattice algebra* of the underlying prosodic algebra.[2]

Suppose partial knowledge of a prosodic algebra, for example:[3]

(21)

dreaming	:	N\S
John	:	N
knows	:	(N\S)/S
letter	:	CN
likes	:	(N\S)/N
man	:	CN
Mary	:	N
reading	:	(N\S)/N
ran	:	N\S
slept	:	N\S
that	:	(CN\CN)/(S/N)
the	:	N/CN
without	:	((N\S)\(N\S))/((N\S)∨S))

Then further facts regarding the prosodic algebra are entailed, for example:

(22) a. **John+knows+Mary+knows+John+ran**: S

b. **man+that+John+knows+Mary+likes**: CN

c. **John+slept+without+dreaming**: S

[2] A *residuated semigroup* is a structure $\langle D, \to, \cdot, \leftarrow; \leq\rangle$ of arity (2, 2, 2; 2) such that $B \leq A \to C$ iff $A \cdot B \leq C$ iff $A \leq C \leftarrow B$ (equivalently, $A \cdot (A \to B) \leq B \leq A \to (A \cdot B)$ and $(B \leftarrow A) \cdot A \leq B \leq (B \cdot A) \leftarrow A$), and $\leq$ is a partial order; we say that $(\to, \cdot, \leftarrow)$ is a *residuated triple* with respect to $\leq$. We see that $\langle \mathcal{P}(L), \backslash, \bullet, /; \subseteq\rangle$ is a residuated semigroup and that $(\backslash, \bullet, /)$ is a residuated triple.

[3] We ignore here features; see e.g. Morrill (1994, ch. 6).

There is the following natural deduction calculus for the prosodic algebra:

(23)

$$\dfrac{\begin{array}{cc}\vdots & \vdots\\ \alpha\colon A & \gamma\colon A\backslash B\end{array}}{\alpha{+}\gamma\colon B}\,\backslash\mathrm{E} \qquad \dfrac{\begin{array}{c}\overline{a\colon A}^{\,n}\\ \vdots\\ a{+}\gamma\colon B\end{array}}{\gamma\colon A\backslash B}\,\backslash\mathrm{I}^{n}$$

$$\dfrac{\begin{array}{cc}\vdots & \vdots\\ \gamma\colon B/A & \alpha\colon A\end{array}}{\gamma{+}\alpha\colon B}\,/\mathrm{E} \qquad \dfrac{\begin{array}{c}\overline{a\colon A}^{\,n}\\ \vdots\\ \gamma{+}a\colon B\end{array}}{\gamma\colon B/A}\,/\mathrm{I}^{n}$$

$$\dfrac{\begin{array}{ccc} & \overline{a\colon A}^{\,n} & \overline{b\colon B}^{\,n}\\ \vdots & \vdots & \vdots\\ \delta\colon A{\bullet}B & & \gamma(a{+}b)\colon C\end{array}}{\gamma(\delta)\colon C}\,{\bullet}\mathrm{E}^{n} \qquad \dfrac{\begin{array}{cc}\vdots & \vdots\\ \alpha\colon A & \beta\colon B\end{array}}{\alpha{+}\beta\colon A{\bullet}B}\,{\bullet}\mathrm{I}$$

$$\dfrac{\begin{array}{c}\vdots\\ \gamma\colon A\wedge B\end{array}}{\gamma\colon A}\,\wedge\mathrm{E} \qquad \dfrac{\begin{array}{c}\vdots\\ \gamma\colon A\wedge B\end{array}}{\gamma\colon B}\,\wedge\mathrm{E} \qquad \dfrac{\begin{array}{cc}\vdots & \vdots\\ \gamma\colon A & \gamma\colon B\end{array}}{\gamma\colon A\wedge B}\,\wedge\mathrm{I}$$

$$\dfrac{\begin{array}{ccc} & \overline{a\colon A}^{\,n} & \overline{b\colon B}^{\,n}\\ \vdots & \vdots & \vdots\\ \delta\colon A\vee B & \gamma(a)\colon C & \gamma(b)\colon C\end{array}}{\gamma(\delta)\colon C}\,\vee\mathrm{E}^{n} \qquad \dfrac{\begin{array}{c}\vdots\\ \alpha\colon A\end{array}}{\alpha\colon A\vee B}\,\vee\mathrm{I} \qquad \dfrac{\begin{array}{c}\vdots\\ \beta\colon B\end{array}}{\beta\colon A\vee B}\,\vee\mathrm{I}$$

Observe that there are rules of elimination (E) and introduction (I), in which operators are eliminated and introduced respectively reading from premises to conclusion. These reflect the necessary and sufficient conditions for a prosodic object to belong to the type in question. The calculus is sound and complete for the $\backslash$, $\bullet$, $/$ fragment **L** (for completness see Buszkowski 1986).[4]

There is, for example, the following derivation:

(24)

$$\dfrac{\mathbf{man}\colon \mathrm{CN} \qquad \dfrac{\mathbf{that}\colon (\mathrm{CN}\backslash\mathrm{CN})/(\mathrm{S}/\mathrm{N}) \qquad \dfrac{\dfrac{\mathbf{John}\colon \mathrm{N} \qquad \dfrac{\mathbf{knows}\colon (\mathrm{N}\backslash\mathrm{S})/\mathrm{S} \qquad \dfrac{\mathbf{Mary}\colon \mathrm{N} \qquad \dfrac{\mathbf{likes}\colon (\mathrm{N}\backslash\mathrm{S})/\mathrm{N} \qquad \overline{a\colon \mathrm{N}}^{\,i}}{\mathbf{likes}{+}a\colon \mathrm{N}\backslash\mathrm{S}}\,/\mathrm{E}}{\mathbf{Mary}{+}\mathbf{likes}{+}a\colon \mathrm{S}}\,\backslash\mathrm{E}}{\mathbf{knows}{+}\mathbf{Mary}{+}\mathbf{likes}{+}a\colon \mathrm{N}\backslash\mathrm{S}}\,/\mathrm{E}}{\mathbf{John}{+}\mathbf{knows}{+}\mathbf{Mary}{+}\mathbf{likes}{+}a\colon \mathrm{S}}\,\backslash\mathrm{E}}{\mathbf{John}{+}\mathbf{knows}{+}\mathbf{Mary}{+}\mathbf{likes}\colon \mathrm{S}/\mathrm{N}}\,/\mathrm{I}^{i}}{\mathbf{that}{+}\mathbf{John}{+}\mathbf{knows}{+}\mathbf{Mary}{+}\mathbf{likes}\colon \mathrm{CN}\backslash\mathrm{CN}}\,/\mathrm{E}}{\mathbf{man}{+}\mathbf{that}{+}\mathbf{John}{+}\mathbf{knows}{+}\mathbf{Mary}{+}\mathbf{likes}\colon \mathrm{CN}}\,\backslash\mathrm{E}$$

[4] This fragment is the calculus of Lambek (1958). That Lambek calculus has the weak generative capacity of context-free grammar, *Chomsky's conjecture*, was eventually proved by Pentus (1993). Whether the computational complexity of deciding **L**-validity is polynomial remains an open question.

A *syntactic algebra* is the product of a prosodic algebra and a semantic algebra under the *type map* T defined by:

$$(25)\quad \begin{array}{rcl} T(A\bullet B) & = & T(A)\&T(B)\\ T(A\backslash B) & = & T(A)\to T(B)\\ T(B/A) & = & T(A)\to T(B)\\ T(A\wedge B) & = & T(A)\&T(B)\\ T(A\vee B) & = & T(A)+T(B) \end{array}$$

Thus, a syntactic algebra is:

$$(26)\quad \langle\{L\times D_\tau\}_{\tau\in\mathcal{T}}, \{\backslash_{\tau,\tau'\to\tau\to\tau'}, \bullet_{\tau,\tau'\to\tau\&\tau'}, /_{\tau,\tau'\to\tau'\to\tau}, \wedge_{\tau,\tau'\to\tau\&\tau'}, \vee_{\tau,\tau'\to\tau+\tau'}\}_{\tau,\tau'\in\mathcal{T}}\rangle$$

where

$$\begin{array}{rcl} A\bullet B & = & \{\langle s+s', \langle m, m'\rangle\rangle|\ \langle s, m\rangle\in A \text{ and } \langle s', m'\rangle \in B\}\\ A\backslash B & = & \{\langle s, m\rangle|\text{ for all } \langle s', m'\rangle\in A, \langle s'+s, m(m')\rangle\in B\}\\ B/A & = & \{\langle s, m\rangle|\text{ for all } \langle s', m'\rangle\in A, \langle s+s', m(m')\rangle\in B\}\\ A\wedge B & = & \{\langle s, \langle m, m'\rangle\rangle|\ \langle s, m\rangle\in A \text{ and } \langle s, m'\rangle\in B\}\\ A\vee B & = & \{\langle s, \langle 0, m\rangle\rangle|\ \langle s, m\rangle\in A\}\cup\{\langle s, \langle 1, m'\rangle\rangle|\ \langle s, m'\rangle\in B\} \end{array}$$

We call this an **Ll**-syntactic algebra: L for Lambek (1958, 1988), l for lattice; for lattice operations semantically interpreted by pairing and injection see Morrill (1990).

We extend the semantic terms with functional abstraction such that $(\lambda x\phi\ \psi) = \phi\{\psi/x\}$ if $\phi\{\psi/x\}$ is free,[5] first and second projection such that $\pi_1(\phi,\psi) = \phi$ and $\pi_2(\phi,\psi) = \psi$, and case branching such that $(\iota_1\chi \to x.\phi; y.\psi) = \phi\{\chi/x\}$ and $(\iota_2\chi \to x.\phi; y.\psi) = \psi\{\chi/y\}$ if $\phi\{\chi/x\}$ and $\psi\{\chi/y\}$ are free.

The *syntactic type indices* will include some set of *atomic syntactic type indices*, for example $\{\mathrm{N}, \mathrm{S}, \mathrm{CN}\}$ for referring nominals, declarative sentences and count nouns:

$$(27)\quad \begin{array}{rcl} T(\mathrm{N}) & = & e\\ T(\mathrm{S}) & = & o\\ T(\mathrm{CN}) & = & e\to o \end{array}$$

Suppose partial knowledge of a syntactic algebra in the form of a *lexicon* of *lexical entries* such as the following:

$$(28)\quad \begin{array}{rcl} \textbf{dreaming}\text{-}\mathit{dream} & : & \mathrm{N\backslash S}\\ \textbf{John}\text{-}j & : & \mathrm{N}\\ \textbf{knows}\text{-}\mathit{know} & : & \mathrm{(N\backslash S)/S}\\ \textbf{letter}\text{-}\mathit{letter} & : & \mathrm{CN}\\ \textbf{likes}\text{-}\mathit{like} & : & \mathrm{(N\backslash S)/N}\\ \textbf{man}\text{-}\mathit{man} & : & \mathrm{CN}\\ \textbf{Mary}\text{-}m & : & \mathrm{N}\\ \textbf{reading}\text{-}\mathit{read} & : & \mathrm{(N\backslash S)/N}\\ \textbf{runs}\text{-}\mathit{run} & : & \mathrm{N\backslash S}\\ \textbf{slept}\text{-}\mathit{sleep} & : & \mathrm{N\backslash S}\\ \textbf{that}\text{-}\lambda x\lambda y\lambda z[(y\ z)\wedge(x\ z)] & : & \mathrm{(CN\backslash CN)/(S/N)}\\ \textbf{the}\text{-}\mathit{THE} & : & \mathrm{N/CN}\\ \textbf{without}\text{-}\lambda x\lambda y\lambda z[(y\ z)\wedge\neg(x\to u.(u\ z); v.v)] & : & \mathrm{((N\backslash S)\backslash(N\backslash S))/((N\backslash S)\vee S)} \end{array}$$

($[\phi\wedge\psi]$ and $\neg\phi$ abbreviate $((AND\ \phi)\ \psi)$ and $(NOT\ \phi)$.) Then further facts regarding the syntactic algebra are entailed, for example (where $(\phi\ \psi_1\ \ldots\ \psi_n)$ abbreviates $((\ldots(\phi\ \psi_1)\ \ldots)\ \psi_n)$):

(29) a. **John+knows+Mary+knows+John+runs**, $(\mathit{know}\ (\mathit{know}\ (\mathit{run}\ j)\ m)\ j)$: S

b. **man+that+John+knows+Mary+likes**, $\lambda z[(\mathit{man}\ z)\wedge(\mathit{know}\ (\mathit{like}\ z\ m)\ j)]$: CN

c. **John+slept+without+dreaming**, $[(\mathit{sleep}\ j)\wedge\neg(\mathit{dream}\ j)]$: S

[5] $\phi\{\psi/x\}$ is the result of substituting the free occurrences of x in ϕ by ψ; it is free iff no variable becomes bound in the process.

There is the following natural deduction calculus for reasoning about syntactic algebra:

(30)

$$\frac{\begin{array}{c}\vdots\\ \alpha\text{-}\phi\colon A\end{array}\quad\begin{array}{c}\vdots\\ \gamma\text{-}\chi\colon A\backslash B\end{array}}{\alpha{+}\gamma\text{-}(\chi\ \phi)\colon B}\backslash\mathrm{E}\qquad\qquad\frac{\begin{array}{c}\overline{a\text{-}x\colon A}^{\,n}\\ \vdots\\ a{+}\gamma\text{-}\psi\colon B\end{array}}{\gamma\text{-}\lambda x\psi\colon A\backslash B}\backslash\mathrm{I}^n$$

$$\frac{\begin{array}{c}\vdots\\ \gamma\text{-}\chi\colon B/A\end{array}\quad\begin{array}{c}\vdots\\ \alpha\text{-}\phi\colon A\end{array}}{\gamma{+}\alpha\text{-}(\chi\ \phi)\colon B}/\mathrm{E}\qquad\qquad\frac{\begin{array}{c}\overline{a\text{-}x\colon A}^{\,n}\\ \vdots\\ \gamma{+}a\text{-}\psi\colon B\end{array}}{\gamma\text{-}\lambda x\psi\colon B/A}/\mathrm{I}^n$$

$$\frac{\begin{array}{c}\vdots\\ \delta\text{-}\omega\colon A\bullet B\end{array}\quad\begin{array}{c}\overline{a\text{-}x\colon A}^{\,n}\quad\overline{b\text{-}y\colon B}^{\,n}\\ \vdots\qquad\qquad\vdots\\ \gamma(a{+}b)\text{-}\chi(x,y)\colon C\end{array}}{\gamma(\delta)\text{-}\chi(\pi_1\omega,\pi_2\omega)\colon C}\bullet\mathrm{E}^n\qquad\qquad\frac{\begin{array}{c}\vdots\\ \alpha\text{-}\phi\colon A\end{array}\quad\begin{array}{c}\vdots\\ \beta\text{-}\psi\colon B\end{array}}{\alpha{+}\beta\text{-}(\phi,\psi)\colon A\bullet B}\bullet\mathrm{I}$$

$$\frac{\begin{array}{c}\vdots\\ \gamma\text{-}\chi\colon A\wedge B\end{array}}{\gamma\text{-}\pi_1\chi\colon A}\wedge\mathrm{E}\qquad\frac{\begin{array}{c}\vdots\\ \gamma\text{-}\chi\colon A\wedge B\end{array}}{\gamma\text{-}\pi_2\chi\colon B}\wedge\mathrm{E}\qquad\frac{\begin{array}{c}\vdots\\ \gamma\text{-}\phi\colon A\end{array}\quad\begin{array}{c}\vdots\\ \gamma\text{-}\psi\colon B\end{array}}{\gamma\text{-}(\phi,\psi)\colon A\wedge B}\wedge\mathrm{I}$$

$$\frac{\begin{array}{c}\vdots\\ \delta\text{-}\omega\colon A\vee B\end{array}\quad\begin{array}{c}\overline{a\text{-}x\colon A}^{\,n}\\ \vdots\\ \gamma(a)\text{-}\chi_1\colon C\end{array}\quad\begin{array}{c}\overline{b\text{-}y\colon B}^{\,n}\\ \vdots\\ \gamma(b)\text{-}\chi_2\colon C\end{array}}{\gamma(\delta)\text{-}(\omega\to x.\chi_1;y.\chi_2)\colon C}\vee\mathrm{E}^n\qquad\frac{\begin{array}{c}\vdots\\ \alpha\text{-}\phi\colon A\end{array}}{\alpha\text{-}\iota_1\phi\colon A\vee B}\vee\mathrm{I}\qquad\frac{\begin{array}{c}\vdots\\ \beta\text{-}\psi\colon B\end{array}}{\beta\text{-}\iota_2\psi\colon A\vee B}\vee\mathrm{I}$$

For example, there are the following derivations:

(31) man that John knows Mary likes: CN

$$\frac{\mathbf{m}\text{-}m\colon \mathrm{CN}\quad\frac{\mathbf{t}\text{-}\lambda x\lambda y\lambda z[(y\ z)\wedge(x\ z)]\colon (\mathrm{CN}\backslash\mathrm{CN})/(\mathrm{S}/\mathrm{N})\quad\frac{\frac{\mathbf{J}\text{-}j\colon \mathrm{N}\quad\frac{\mathbf{k}\text{-}k\colon (\mathrm{N}\backslash\mathrm{S})/\mathrm{S}\quad\frac{\mathbf{M}\text{-}m\colon \mathrm{N}\quad\frac{\mathbf{l}\text{-}l\colon (\mathrm{N}\backslash\mathrm{S})/\mathrm{N}\quad\overline{a\text{-}x\colon \mathrm{N}}^{\,i}}{\mathbf{likes}{+}a\text{-}(l\ x)\colon \mathrm{N}\backslash\mathrm{S}}/\mathrm{E}}{\mathbf{Mary}{+}\mathbf{likes}{+}a\text{-}(l\ x\ m)\colon \mathrm{S}}\backslash\mathrm{E}}{\mathbf{knows}{+}\mathbf{Mary}{+}\mathbf{likes}{+}a\text{-}(k\ (l\ x\ m))\colon \mathrm{N}\backslash\mathrm{S}}/\mathrm{E}}{\mathbf{John}{+}\mathbf{knows}{+}\mathbf{Mary}{+}\mathbf{likes}{+}a\text{-}(k\ (l\ x\ m)\ j)\colon \mathrm{S}}\backslash\mathrm{E}}{\mathbf{John}{+}\mathbf{knows}{+}\mathbf{Mary}{+}\mathbf{likes}\text{-}\lambda x(k\ (l\ x\ m)\ j)\colon \mathrm{S}/\mathrm{N}}/\mathrm{I}^i}{\mathbf{that}{+}\mathbf{John}{+}\mathbf{knows}{+}\mathbf{Mary}{+}\mathbf{likes}\text{-}\lambda y\lambda z[(y\ z)\wedge(k\ (l\ z\ m)\ j)]\colon \mathrm{CN}\backslash\mathrm{CN}}/\mathrm{E}}{\mathbf{man}{+}\mathbf{that}{+}\mathbf{John}{+}\mathbf{knows}{+}\mathbf{Mary}{+}\mathbf{likes}\text{-}\lambda z[(m\ z)\wedge(k\ (l\ z\ m)\ j)]\colon \mathrm{CN}}\backslash\mathrm{E}$$

(32) John slept without dreaming: S

$$\dfrac{\mathbf{J}\text{-}j\text{: N} \qquad \dfrac{\mathbf{s}\text{-}s\text{: N}\backslash\text{S} \qquad \dfrac{\mathbf{w}\text{-}\lambda x\lambda y\lambda z[(y\ z)\wedge\neg(x\to u.(u\ z);v.v)]\text{: }((\text{N}\backslash\text{S})\backslash(\text{N}\backslash\text{S}))/((\text{N}\backslash\text{S})\vee\text{S}) \qquad \dfrac{\mathbf{d}\text{-}d\text{: N}\backslash\text{S}}{\mathbf{d}\text{-}\iota d\text{: }(\text{N}\backslash\text{S})\vee\text{S}}\,\vee\text{I}}{\mathbf{w}\text{+}\mathbf{d}\text{-}\lambda y\lambda z[(y\ z)\wedge\neg(d\ z)]\text{: }(\text{N}\backslash\text{S})\backslash(\text{N}\backslash\text{S})}\,/\text{E}}{\mathbf{s}\text{+}\mathbf{w}\text{+}\mathbf{d}\text{-}\lambda z[(s\ z)\wedge\neg(d\ z)]\text{: N}\backslash\text{S}}\,\backslash\text{E}}{\mathbf{J}\text{+}\mathbf{s}\text{+}\mathbf{w}\text{+}\mathbf{d}\text{-}[(s\ j)\wedge\neg(d\ j)]\text{: S}}\,\backslash\text{E}$$

We refer to the theory of **Ll**-syntactic algebras as *type logic* **Ll**. An **Ll**-*type logical grammar* is an **Ll**-lexicon. We see that **Ll** already affords a very rudimentary account of the unboundedness of left extraction.

2. Brackets for islands

With a view to islands, let us generalize the underlying prosodic algebra to a "bracket semigroup" $\langle L, +, b\rangle$ of arity (2, 1) (Morrill 1992). Then the prosodic algebra determined becomes:

(33) $\langle \mathcal{P}(L), \backslash, \bullet, /, [\,], [\,]^{-1}, \wedge, \vee\rangle$
where we add

$$\begin{array}{rcl} [\,]A & = & \{b(s) \mid s \in A\} \\ [\,]^{-1}A & = & \{s \mid b(s) \in A\} \end{array}$$

We again call this prosodic algebra the powerset residuated lattice algebra of the underlying algebra.[6]

There are the following rules of natural deduction:

(34)

$$\dfrac{\begin{array}{c}\vdots\\ \beta\text{: }[\,]A\end{array} \quad \begin{array}{c}\overline{a\text{: }A}^{\,n}\\ \vdots\\ \gamma(b(a))\text{: }C\end{array}}{\gamma(\beta)\text{: }C}\,[\,]\text{E}^n \qquad\qquad \dfrac{\begin{array}{c}\vdots\\ \alpha\text{: }A\end{array}}{b(\alpha)\text{: }[\,]A}\,[\,]\text{I}$$

$$\dfrac{\begin{array}{c}\vdots\\ \beta\text{: }[\,]^{-1}A\end{array}}{b(\beta)\text{: }A}\,[\,]^{-1}\text{E} \qquad\qquad \dfrac{\begin{array}{c}\vdots\\ b(\beta)\text{: }A\end{array}}{\beta\text{: }[\,]^{-1}A}\,[\,]^{-1}\text{I}$$

We consider syntactic algebra under the type map as before, together with:

(35) $T([\,]A) = T([\,]^{-1}A) = T(A)$

The syntactic algebra becomes (Morrill 1992) the **Llb**-syntactic algebra:

(36) $\langle\{\mathcal{P}(L)\times D_\tau\}_{\tau\in\mathcal{T}}, \{\backslash, \bullet, /, [\,]_{\tau\to\tau}, [\,]^{-1}{}_{\tau\to\tau}, \wedge, \vee\}_{\tau\in\mathcal{T}}\rangle$
where we add

$$\begin{array}{rcl} [\,]A & = & \{\langle b(s), m\rangle \mid \langle s, m\rangle \in A\} \\ [\,]^{-1}A & = & \{\langle s, m\rangle \mid \langle b(s), m\rangle \in A\} \end{array}$$

We add the following natural deduction rules for **Llb**-syntactic algebra:

[6] A pair of operations $(\mathbf{B}, \mathbf{B}^{-1})$ is a *residuated pair* with respect to a partial order $\leq$ iff $\mathbf{B}A \leq C$ iff $A \leq \mathbf{B}^{-1}C$ (equivalently, $\mathbf{B}\mathbf{B}^{-1}A \leq A \leq \mathbf{B}^{-1}\mathbf{B}A$). We see that $([\,], [\,]^{-1})$ is a residuated pair with respect to the partial order $\subseteq$.

(37)

$$\dfrac{\begin{array}{c}\vdots\\ \beta\text{-}\psi\colon [\,]A\end{array}\quad \begin{array}{c}\overline{a\text{-}x\colon A}^{\,n}\\ \vdots\\ \gamma(b(a))\text{-}\chi(x)\colon C\end{array}}{\gamma(\beta)\text{-}\chi(\psi)\colon C}\,[\,]\mathrm{E}^n \qquad \dfrac{\begin{array}{c}\vdots\\ \alpha\text{-}\phi\colon A\end{array}}{b(\alpha)\text{-}\phi\colon [\,]A}\,[\,]\mathrm{I}$$

$$\dfrac{\begin{array}{c}\vdots\\ \beta\text{-}\psi\colon [\,]^{-1}A\end{array}}{b(\beta)\text{-}\psi\colon A} \qquad \dfrac{\begin{array}{c}\vdots\ [\,]^{-1}\mathrm{E}\\ b(\beta)\text{-}\phi\colon A\end{array}}{\beta\text{-}\phi\colon [\,]^{-1}A}\,[\,]^{-1}\mathrm{I}$$

We call the theory of **Llb**-syntactic algebras type logic **Llb**.

Now, nominal subjects and adverbials will be defined as bracketed domains by assignments such as the following:

(38)

$$\begin{array}{rcl}\mathbf{likes}\text{-}\mathit{like} & : & ([\,]\mathrm{N}\backslash\mathrm{S})/\mathrm{N}\\ \mathbf{runs}\text{-}\mathit{run} & : & [\,]\mathrm{N}\backslash\mathrm{S}\\ \mathbf{without}\text{-}\lambda x\lambda y\lambda z[(y\ z)\wedge\neg(x\to u.(u\ z);v.v)] & : & [\,]^{-1}(([\,]\mathrm{N}\backslash\mathrm{S})\backslash([\,]\mathrm{N}\backslash\mathrm{S})/(([\,]\mathrm{N}\backslash\mathrm{S})\vee\mathrm{S})\end{array}$$

For example, there is the following derivation:

(39) John slept without dreaming: S

$$\dfrac{\dfrac{\mathbf{J}\text{-}j\colon \mathrm{N}}{b(\mathbf{J})\text{-}j\colon [\,]\mathrm{N}}\,[\,]\mathrm{I} \quad \dfrac{\mathbf{s}\text{-}s\colon [\,]\mathrm{N}\backslash\mathrm{S} \quad \dfrac{\dfrac{\mathbf{w}\text{-}\lambda x\lambda y\lambda z[(y\ z)\wedge\neg(x\to u.(u\ z);v.v)]\colon [\,]^{-1}(([\,]\mathrm{N}\backslash\mathrm{S})\backslash([\,]\mathrm{N}\backslash\mathrm{S}))/(([\,]\mathrm{N}\backslash\mathrm{S})\vee\mathrm{S}) \quad \dfrac{\mathbf{d}\text{-}d\colon [\,]\mathrm{N}\backslash\mathrm{S}}{\mathbf{d}\text{-}\iota d\colon ([\,]\mathrm{N}\backslash\mathrm{S})\vee\mathrm{S}}\,\vee\mathrm{I}}{\mathbf{w}+\mathbf{d}\text{-}\lambda y\lambda z[(y\ z)\wedge\neg(d\ z)]\colon [\,]^{-1}(([\,]\mathrm{N}\backslash\mathrm{S})\backslash([\,]\mathrm{N}\backslash\mathrm{S}))}\,/\mathrm{E}}{b(\mathbf{w}+\mathbf{d})\text{-}\lambda y\lambda z[(y\ z)\wedge\neg(d\ z)]\colon ([\,]\mathrm{N}\backslash\mathrm{S})\backslash([\,]\mathrm{N}\backslash\mathrm{S})}\,[\,]^{-1}\mathrm{E}}{\mathbf{s}+b(\mathbf{w}+\mathbf{d})\text{-}\lambda z[(s\ z)\wedge\neg(d\ z)]\colon [\,]\mathrm{N}\backslash\mathrm{S}}\,\backslash\mathrm{E}}{b(\mathbf{J})+\mathbf{s}+b(\mathbf{w}+\mathbf{d})\text{-}[(s\ j)\wedge\neg(d\ j)]\colon \mathrm{S}}\,\backslash\mathrm{E}$$

The bracketing prevents extraction from adverbial islands as in (6b):

(40)?paper that$_i$ John slept [without reading e_i]

The reason why is that we cannot equate:

(41) b(**John**)+**slept**+b(**without**+**reading**+a)
and
b(**John**)+**slept**+b(**without**+**reading**)+a

3. Coordination

For coordination we may assume assignment schemata like the following for Boolean conjunction:

(42) $\mathbf{and}\text{-}\lambda x\lambda y\lambda z_{\tau_1}\ldots\lambda z_{\tau_n}[(y\ z_{\tau_1}\ \ldots\ z_{\tau_n})\wedge(x\ z_{\tau_1}\ \ldots\ z_{\tau_n})]\colon ([\,]X\backslash X)/[\,]X$
where $T(X)=\tau_1\to(\cdots\to(\tau_n\to o)\cdots),\ n\geq 0$

This semantics is that given in Keenan and Faltz (1985), Gazdar (1980) and Partee and Rooth (1983). This gives rise to coordination of constituents such as the following (but it does not address the number of subject coordination as in (43d)).

(43) a. [John arrived and Mary left].
b. John left [quickly and without complaining].
c. John [picked up his bag and left].
d. [John and Mary] left.

$$
\dfrac{\dfrac{\mathbf{J}\text{-}j\text{: N}}{b(\mathbf{J})\text{-}j\text{: []N}}\,[\,]\mathrm{I}
\quad
\dfrac{\mathbf{l}\text{-}l\text{: []N}\backslash\mathrm{S}
\quad
\dfrac{\dfrac{\mathbf{q}\text{-}q\text{: }([]N\backslash S)\backslash([]N\backslash S)}{b(\mathbf{q})\text{-}q\text{: }[](([]N\backslash S)\backslash([]N\backslash S))}\,[\,]\mathrm{I}
\quad
\dfrac{\mathbf{a}
\quad
\dfrac{\dfrac{\dfrac{\mathbf{w}\text{-}\lambda x\lambda y\lambda z[(y\ z)\wedge\neg(x\to u.(u\ z);v.v)]\text{: }[]^{-1}((([]\mathrm{N}\backslash\mathrm{S})\backslash([]\mathrm{N}\backslash\mathrm{S}))/(([]\mathrm{N}\backslash\mathrm{S})\vee\mathrm{S})
\quad
\dfrac{\mathbf{c}\text{-}c\text{: }[]\mathrm{N}\backslash\mathrm{S}}{\mathbf{c}\text{-}\iota c\text{: }([]\mathrm{N}\backslash\mathrm{S})\vee\mathrm{S}}\,\vee\mathrm{I}}{\mathbf{w}+\mathbf{c}\text{-}\lambda y\lambda z[(y\ z)\wedge\neg(c\ z)]\text{: }[]^{-1}(([]\mathrm{N}\backslash\mathrm{S})\backslash([]\mathrm{N}\backslash\mathrm{S}))}\,/\mathrm{E}}{b(\mathbf{w}+\mathbf{c})\text{-}\lambda y\lambda z[(y\ z)\wedge\neg(c\ z)]\text{: }([]\mathrm{N}\backslash\mathrm{S})\backslash([]\mathrm{N}\backslash\mathrm{S})}\,[\,]^{-1}\mathrm{E}}{b(b(\mathbf{w}+\mathbf{c}))\text{-}\lambda y\lambda z[(y\ z)\wedge\neg(c\ z)]\text{: }[](([]\mathrm{N}\backslash\mathrm{S})\backslash([]\mathrm{N}\backslash\mathrm{S}))}\,[\,]\mathrm{I}}{\mathbf{a}+b(b(\mathbf{w}+\mathbf{c}))\text{-}\lambda y\lambda z\lambda w[(y\ z\ w)\wedge(z\ w)\wedge\neg(c\ w)]\text{: }[]((([]\mathrm{N}\backslash\mathrm{S})\backslash([]\mathrm{N}\backslash\mathrm{S}))\backslash(([]\mathrm{N}\backslash\mathrm{S})\backslash([]\mathrm{N}\backslash\mathrm{S})))}\,/\mathrm{E}}{b(\mathbf{q})+\mathbf{a}+b(b(\mathbf{w}+\mathbf{c}))\text{-}\lambda z\lambda w[(q\ z\ w)\wedge[(z\ w)\wedge\neg(c\ w)]]\text{: }([]\mathrm{N}\backslash\mathrm{S})\backslash([]\mathrm{N}\backslash\mathrm{S})}\,\backslash\mathrm{E}}{\mathbf{l}+b(\mathbf{q})+\mathbf{a}+b(b(\mathbf{w}+\mathbf{c}))\text{-}\lambda w[(q\ l\ w)\wedge[(l\ w)\wedge\neg(c\ w)]]\text{: }[]\mathrm{N}\backslash\mathrm{S}}\,\backslash\mathrm{E}}{b(\mathbf{J})+\mathbf{l}+b(\mathbf{q})+\mathbf{a}+b(b(\mathbf{w}+\mathbf{c}))\text{-}[(q\ l\ j)\wedge[(l\ j)\wedge\neg(c\ j)]]\text{: S}}\,\backslash\mathrm{E}
$$

Figure 1: Natural deduction derivation of 'John left quickly and without complaining': S

See for example figure 1 and (44).

(44) John and Mary left: S

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\mathbf{J}\text{-}j\text{: N}}{b(\mathbf{J})\text{-}j\text{: }[\,]\mathrm{N}} \quad \dfrac{}{a\text{-}x\text{: }[\,]\mathrm{N}\backslash\mathrm{S}}\,i}{\mathbf{J}+a\text{-}(x\ j)\text{: S}}\,\backslash\mathrm{E}}{\mathbf{J}\text{-}\lambda x(x\ j)\text{: }\mathrm{S}/([\,]\mathrm{N}\backslash\mathrm{S})}\,/\mathrm{I}^{i}}{b(\mathbf{J})\text{-}\lambda x(x\ j)\text{: }[\,](\mathrm{S}/([\,]\mathrm{N}\backslash\mathrm{S}))}\,[\,]\mathrm{I} \quad \dfrac{\mathbf{a}\text{-}\lambda x\lambda y\lambda z[(y\ z)\wedge(x\ z)] \quad \dfrac{\dfrac{\dfrac{\dfrac{\mathbf{M}\text{-}m\text{: N}}{b(\mathbf{M})\text{-}m\text{: }[\,]\mathrm{N}} \quad \dfrac{}{b\text{-}y\text{: }[\,]\mathrm{N}\backslash\mathrm{S}}\,j}{\mathbf{M}+b\text{-}(y\ m)\text{: S}}\,\backslash\mathrm{E}}{\mathbf{M}\text{-}\lambda y(y\ m)\text{: }\mathrm{S}/([\,]\mathrm{N}\backslash\mathrm{S})}\,/\mathrm{I}^{j}}{b(\mathbf{M})\text{-}\lambda y(y\ m)\text{: }[\,](\mathrm{S}/([\,]\mathrm{N}\backslash\mathrm{S}))}\,[\,]\mathrm{I}}{\mathbf{a}+b(\mathbf{M})\text{-}\lambda y\lambda z[(y\ z)\wedge(z\ m)]\text{: }[\,](\mathrm{S}/([\,]\mathrm{N}\backslash\mathrm{S}))\backslash(\mathrm{S}/([\,]\mathrm{N}\backslash\mathrm{S}))}\,/\mathrm{E}}{b(\mathbf{J})+\mathbf{a}+b(\mathbf{M})\text{-}\lambda z[(z\ j)\wedge(z\ m)]\text{: }\mathrm{S}/([\,]\mathrm{N}\backslash\mathrm{S})}\,\backslash\mathrm{E} \quad \mathbf{l}\text{-}l\text{: }[\,]\mathrm{N}\backslash\mathrm{S}}{b(\mathbf{J})+\mathbf{a}+b(\mathbf{M})+\mathbf{l}\text{-}[(l\ j)\wedge(l\ m)]\text{: S}}\,/\mathrm{E}$$

However, it also gives rise to coordination of non-constituents such as Right Node Raising (RNR), see Steedman (1985):

(45) a. [John liked and Mary disliked] London.
b. [the belief and the hope] that they would come back
c. [John was and Mary is] extremely angry.
d. [John arrived and Mary left] without making salutations.
e. [a man and a woman] outside
f. I [have been wondering whether, but wouldn't positively want to state that], your theory is correct. (Bresnan 1974)
g. [John tried and Mary managed] to finish writing within the six weeks.
h. ?He thinks [that John or that Mary] tried to deceive him.
i. ?I [liked this but preferred that] sofa.
j. [a red or a green] tee-shirt.

For example:

(46) John liked and Mary disliked London: S

$$\dfrac{\dfrac{\dfrac{\mathbf{John}\text{-}j\text{: N}}{b(\mathbf{John})\text{-}j\text{: }[\,]\mathrm{N}}\,[\,]\mathrm{I} \quad \dfrac{\mathbf{liked}\text{-}like\text{: }([\,]\mathrm{N}\backslash\mathrm{S})/\mathrm{N} \quad \dfrac{}{a\text{-}x\text{: N}}\,i}{\mathbf{liked}+a\text{-}(like\ x)\text{: }[\,]\mathrm{N}\backslash\mathrm{S}}\,/\mathrm{E}}{b(\mathbf{John})+\mathbf{liked}+a\text{-}(like\ x\ j)\text{: S}}\,\backslash\mathrm{E}}{b(\mathbf{John})+\mathbf{liked}\text{-}\lambda x(like\ x\ j)\text{: S/N}}\,/\mathrm{I}^{i}$$

Similarly, the coordination assignment schema yields Left Node Raising (LNR), see Dowty (1988):

(47) a. Bill met [John on Monday and Sue on Tuesday]
b. John is [good natured on Fridays but moody on Mondays]
c. He wanted [to stay on Monday and to go on Tuesday]

For example:

(48) Bill met John on Monday and Sue on Tuesday: S

$$\dfrac{\dfrac{\dfrac{\dfrac{}{a\text{-}x\text{: }([\,]\mathrm{N}\backslash\mathrm{S})/\mathrm{N}}\,i \quad \mathbf{John}\text{-}j\text{: N}}{a+\mathbf{John}\text{-}(x\ j)\text{: }[\,]\mathrm{N}\backslash\mathrm{S}}\,/\mathrm{E} \quad \dfrac{\dfrac{\mathbf{on}\text{-}on\text{: }[\,]^{-1}(([\,]\mathrm{N}\backslash\mathrm{S})\backslash([\,]\mathrm{N}\backslash\mathrm{S}))/\mathrm{N} \quad \mathbf{Monday}\text{-}monday\text{: N}}{\mathbf{on}+\mathbf{Monday}\text{-}(on\ monday)\text{: }[\,]^{-1}(([\,]\mathrm{N}\backslash\mathrm{S})\backslash([\,]\mathrm{N}\backslash\mathrm{S}))}\,/\mathrm{E}}{b(\mathbf{on}+\mathbf{Monday})\text{-}(on\ monday)\text{: }([\,]\mathrm{N}\backslash\mathrm{S})\backslash([\,]\mathrm{N}\backslash\mathrm{S})}\,[\,]^{-1}\mathrm{E}}{a+\mathbf{John}+b(\mathbf{on}+\mathbf{Monday})\text{-}(on\ monday\ (x\ j))\text{: }[\,]\mathrm{N}\backslash\mathrm{S}}\,\backslash\mathrm{E}}{\mathbf{John}+b(\mathbf{on}+\mathbf{Monday})\text{-}\lambda x(on\ monday\ (x\ j))\text{: }(([\,]\mathrm{N}\backslash\mathrm{S})/\mathrm{N})\backslash([\,]\mathrm{N}\backslash\mathrm{S})}\,\backslash\mathrm{I}^{i}$$

This account of coordination generates ATB extraction from coordinate structures:

(49) man who$_i$ [Suzy met e_i and Mary married e_i]

However, extraction of an entire conjunct is correctly blocked:

(50)*man that Suzy met [the friend of e_i and e_i]

This is blocked because the coordinator expects brackets around its conjunts and here there is no right conjunt around which to put any brackets. The example would be derivable if the underlying prosodic algebra was assumed to contain an identity element ϵ such that $\epsilon+s = s = s+\epsilon$; then the empty right conjunt could be analyzed as $b(\epsilon)$: [](((([]N\S)/N)\(([]N\S)/N)). Note that (50) would not be blocked by an unbracketed coordinator type $(X\backslash X)/X$. CSC violations like (3) are also blocked (but would not be by an unbracketed coordinator type $(X\backslash X)/X$):

(51)*man that$_i$ [Suzy met Bill and Mary married e_i]

Mysteriously, the CSC violation in (52) is quite acceptable:

(52)?food that$_i$ Mary [went shopping and bought e_i]

Consider now the apparent coordination of unlike types (2):

(53) 007 is Bond and teetotal.

Assume the following "polymorphic" type assignment to the copula (Morrill 1990, 1996):

(54) **is**-$\lambda x\lambda y(x \to z.[z = y]; w.(w\ \lambda u[u = y]\ y))$: (N\S)/(N∨(CN/CN))

($[\phi = \psi]$ abbreviates $((EQ\ \phi)\ \psi)$). Morrill (1990) then observes that the coordination of "unlike types" is generated by like-type coordinator assignment schemata, see figure 2.

Finally, by way of further motivation of bracket operators, consider the following example, due to Paul Dekker.

(55)*[Bill thinks and the brother of] John walks.

Without bracket operators, the two conjunts share the type (S/(N\S))/N so that the assignment schema we have given would predict that the unacceptable coordination is grammatical. But with subjects bracketed, the second conjunt has no such type, consistent with the non-coordination (56), and (55) is not generated.

(56)*The brother of [John walks and Mary talks].

4. Left extraction

Because S/N means an S lacking an N *at its right periphery*, the relative pronoun type (CN\CN)/(S/N) will not generate medial extraction:

(57) man that$_i$ Mary showed e_i Exmoor

The reason why is that we cannot equate:

(58) b(**Mary**)+**showed**+a+**Exmoor**
and
b(**Mary**)+**showed**+**Exmoor**+a

$$\dfrac{\mathbf{007} \qquad \dfrac{\mathbf{is} \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{a\text{-}s\text{: } (\mathrm{N}\backslash\mathrm{S})/(\mathrm{N}\vee(\mathrm{CN}/\mathrm{CN}))}\,i \qquad \mathbf{B}\text{-}\iota B\text{: } \mathrm{N}\vee(\mathrm{CN}/\mathrm{CN})}{a+\mathbf{B}\text{-}(s\ \iota B)\text{: } \mathrm{N}\backslash\mathrm{S}}\,/\mathrm{E}}{\mathbf{B}\text{-}\lambda s(s\ \iota B)\text{: } ((\mathrm{N}\backslash\mathrm{S})/(\mathrm{N}\vee(\mathrm{CN}/\mathrm{CN})))\backslash(\mathrm{N}\backslash\mathrm{S})}\,\backslash\mathrm{I}^{i} \qquad \mathbf{a} \qquad \dfrac{\dfrac{\dfrac{}{b\text{-}t\text{: } (\mathrm{N}\backslash\mathrm{S})/(\mathrm{N}\vee(\mathrm{CN}/\mathrm{CN}))}\,j \qquad \mathbf{t}\text{-}\iota'\lambda u\lambda v[(t\ v)\wedge(u\ v)]\text{: } \mathrm{N}\vee(\mathrm{CN}/\mathrm{CN})}{b+\mathbf{t}\text{-}(t\ \iota'\lambda u\lambda v[(t\ v)\wedge(u\ v)])\text{: } \mathrm{N}\backslash\mathrm{S}}\,/\mathrm{E}}{\mathbf{t}\text{-}\lambda t(t\ \iota'\lambda u\lambda v[(t\ v)\wedge(u\ v)])\text{: } ((\mathrm{N}\backslash\mathrm{S})/(\mathrm{N}\vee(\mathrm{CN}/\mathrm{CN})))\backslash(\mathrm{N}\backslash\mathrm{S})}\,\backslash\mathrm{I}^{j}}{b(\mathbf{B})+\mathbf{a}+b(\mathbf{t})\text{-}\lambda z\lambda w[(z\ \iota B\ w)\wedge(z\ \iota'\lambda u\lambda v[(t\ v)\wedge(u\ v)]\ w)]\text{: } ((\mathrm{N}\backslash\mathrm{S})/(\mathrm{N}\vee(\mathrm{CN}/\mathrm{CN})))\backslash(\mathrm{N}\backslash\mathrm{S})}}{\mathbf{is}+b(\mathbf{B})+\mathbf{a}+b(\mathbf{t})\text{-}\lambda y([B = y]\wedge[(t\ y)\wedge[y = y]])\text{: } \mathrm{N}\backslash\mathrm{S}}\,\backslash\mathrm{E}}{\mathbf{007}+\mathbf{is}+b(\mathbf{B})+\mathbf{a}+b(\mathbf{t})\text{-}[B = \mathit{007}]\wedge[(t\ \mathit{007})\wedge[\mathit{007} = \mathit{007}]]\text{: } \mathrm{S}}\,\backslash\mathrm{E}$$

Figure 2: Natural deduction derivation of '007 is Bond and teetotal': S

That is, we want some kind of *commutativity* ($s{+}s' = s'{+}s$). With a view to this, we generalize the underlying prosodic algebra from a bracket semigroup to a "bi-bracket semigroup" $\langle L, +, b, i\rangle$ of arity (2, 1, 1) where:

(59) $i(s){+}s' = s'{+}i(s)$ $\qquad$ i-commutativity

Then the prosodic algebra, i.e. the powerset residuated lattice algebra, becomes:

(60) $\langle \mathcal{P}(L), \backslash, \bullet, /, [\,], [\,]^{-1}, [i], [i]^{-1}, \wedge, \vee\rangle$
where

$$\begin{array}{rcl}[i]A & = & \{i(s)|\ s \in A\}\\ [i]^{-1}A & = & \{s|\ i(s) \in A\}\end{array}$$

There are the following natural deduction rules, just as before for the plain bracket operators:

(61)

$$\dfrac{\begin{array}{c}\vdots\\ \beta\colon [\mathrm{i}]A\end{array} \qquad \begin{array}{c}\overline{a\colon A}^{\,n}\\ \vdots\\ \gamma(i(a))\colon C\end{array}}{\gamma(\beta)\colon C}\,[\mathrm{i}]\mathrm{E}^n \qquad\qquad \dfrac{\begin{array}{c}\vdots\\ \alpha\colon A\end{array}}{i(\alpha)\colon [\mathrm{i}]A}\,[\mathrm{i}]\mathrm{I}$$

$$\dfrac{\begin{array}{c}\vdots\\ \beta\colon [\mathrm{i}]^{-1}A\end{array}}{i(\beta)\colon A}\,[\mathrm{i}]^{-1}\mathrm{E} \qquad\qquad \dfrac{\begin{array}{c}\vdots\\ i(\beta)\colon A\end{array}}{\beta\colon [\mathrm{i}]^{-1}A}\,[\mathrm{i}]^{-1}\mathrm{I}$$

In addition we allow the equation (59) of i-commutativity to apply to labels.
The type map is extended by:

(62) $T([i]A) = T([i]^{-1}A) = T(A)$

The syntactic algebra becomes:

(63) $\langle\{\mathcal{P}(L) \times D_\tau\}_{\tau\in\mathcal{T}}, \{\backslash, \bullet, /, [\,], [\,]^{-1}, [i]_{\tau\to\tau}, [i]^{-1}_{\tau\to\tau}, \wedge, \vee\}_{\tau\in\mathcal{T}}\rangle$
where

$$\begin{array}{rcl}[i]A & = & \{\langle i(s), m\rangle|\ \langle s, m\rangle \in A\}\\ [i]^{-1}A & = & \{\langle s, m\rangle|\ \langle i(s), m\rangle \in A\}\end{array}$$

I.e. everything is generalized exactly as before for the bracket operation and bracket operators. Just to be completely explicit, here are the natural deduction rules:

(64)

$$\dfrac{\begin{array}{c}\vdots\\ \beta\text{-}\psi\colon [\mathrm{i}]A\end{array} \qquad \begin{array}{c}\overline{a\text{-}x\colon A}^{\,n}\\ \vdots\\ \gamma(i(a))\text{-}\chi(x)\colon C\end{array}}{\gamma(\beta)\text{-}\chi(\psi)\colon C}\,[\mathrm{i}]\mathrm{E}^n \qquad\qquad \dfrac{\begin{array}{c}\vdots\\ \alpha\text{-}\phi\colon A\end{array}}{i(\alpha)\text{-}\phi\colon [\mathrm{i}]A}\,[\mathrm{i}]\mathrm{I}$$

$$\dfrac{\begin{array}{c}\vdots\\ \beta\text{-}\psi\colon [\mathrm{i}]^{-1}A\end{array}}{i(\beta)\text{-}\psi\colon A}\,[\mathrm{i}]^{-1}\mathrm{E} \qquad\qquad \dfrac{\begin{array}{c}\vdots\\ i(\beta)\text{-}\phi\colon A\end{array}}{\beta\text{-}\phi\colon [\mathrm{i}]^{-1}A}\,[\mathrm{i}]^{-1}\mathrm{I}$$

The only novelty relative to plain bracketing is that i-commutativity (59) on labels is added.

We assign the relative pronoun type $(\mathrm{CN}\backslash\mathrm{CN})/(\mathrm{S}/[i][i]^{-1}\mathrm{N})$, and the medial extraction (57) can now be derived:

(65) Mary showed Exmoor: $\mathrm{S}/[i][i]^{-1}\mathrm{N}$

$$\dfrac{\dfrac{}{c\text{-}z\text{: }[\mathrm{i}][\mathrm{i}]^{-1}\mathrm{N}}\,j \qquad \dfrac{\dfrac{\dfrac{\dfrac{\mathbf{Mary}\text{-}m\text{: N}}{b(\mathbf{Mary})\text{-}m\text{: }[\,]\mathrm{N}}\,[\,]\mathrm{I} \qquad \dfrac{\mathbf{showed}\text{-}s\text{: }([\,]\mathrm{N}\backslash\mathrm{S})/(\mathrm{N}\bullet\mathrm{N}) \qquad \dfrac{\dfrac{\dfrac{}{a\text{-}x\text{: }[\mathrm{i}]^{-1}\mathrm{N}}\,i}{i(a)\text{-}x\text{: N}}\,[\mathrm{i}]^{-1}\mathrm{E} \qquad \mathbf{Exmoor}\text{-}e\text{: N}}{i(a)+\mathbf{Exmoor}\text{-}(x,e)\text{: }\mathrm{N}\bullet\mathrm{N}}}{\mathbf{showed}+i(a)+\mathbf{Exmoor}\text{-}(s\ (x,e))\text{: }[\,]\mathrm{N}\backslash\mathrm{S}}\,/\mathrm{E}}{\mathbf{Mary}+\mathbf{showed}+i(a)+\mathbf{Exmoor}\text{-}(s\ (x,e)\ m)\text{: S}}\,\backslash\mathrm{E}}{\mathbf{Mary}+\mathbf{showed}+\mathbf{Exmoor}+i(a)\text{-}(s\ (x,e)\ m)\text{: S}}\,i\text{–comm}}{\dfrac{\mathbf{Mary}+\mathbf{showed}+\mathbf{Exmoor}+c\text{-}(s\ (z,e)\ m)\text{: S}}{\mathbf{Mary}+\mathbf{showed}+\mathbf{Exmoor}\text{-}\lambda z(s\ (z,e)\ m)\text{: }\mathrm{S}/[\mathrm{i}][\mathrm{i}]^{-1}\mathrm{N}}\,[\mathrm{i}]^{-1}E^i}\,[\mathrm{i}]^{-1}E^i$$

Note that we could just as well have assigned the relative pronoun to $(\mathrm{CN}\backslash\mathrm{CN})/([i][i]^{-1}\mathrm{N}\backslash\mathrm{S})$ (and/or 'showed' type $((([\,]\mathrm{N}\backslash\mathrm{S})/\mathrm{N})/\mathrm{N})$). Peripheral extraction is of course still obtained; the islandhood of adverbials and nominal subjects as in (6) is ensured because although $i(s)$ commutes under $+$, it does not penetrate $b(\_)$, i.e. we dot not (yet) have anything like $b(s') + i(s) = b(s' + i(s))$.

The idea, inspired by linear logic, of using unary operators to licence commutation-like structural operations on the gap subtypes of fillers goes back to Morrill et al. (1990). The idea of using a residuated pair (such that $[\,][\,]^{-1}A \Rightarrow A$) appears in Moortgat (1999), but in the context of an underlying prosodic structure which employs a binary relation instead of our unary operation (and a ternary relation instead of our binary semigroup operation). That kind of *relational* model can clearly simulate any *algebraic* model of the kind considered here because an n-ary operation is an n+1-ary relation. We stick to the algebraic methodology in an endeaver to shed light on what form of the more general relational models might be necessary.

5. Parasitic gaps (type logic Lbi)

The bi-bracket type logic given so far does not support parasitic extraction, such as (7b):

(66)?paper that$_i$ John filed e_i [without reading e_i]

The reason why is that we cannot equate, say:

(67) $b(\mathbf{John})+\mathbf{filed}+i(a)+b(\mathbf{without}+\mathbf{reading}+i(a))$
and
$b(\mathbf{John})+\mathbf{filed}+i(a)+b(\mathbf{without}+\mathbf{reading})$

Earlier categorial formulations involved substitution combinators (Szabolcsi 1983, Steedman 1987). From the perspective under development, we see that for parasitic extraction we want some kind of *idempotency* (s+s = s) (cf. also Morrill et al. 1990). However, parasitic gaps *must* occur within bracketed island domains, and with only one per bracketed domain. In view of these considerations we specialize the underlying bi-bracket semigroup algebra thus:

(68)
$$\begin{array}{rcll} b(s' + i(s)) & = & b(b(s')) + i(i(s)) & bi\text{-distributivity} \\ i(i(s)) + i(s) & = & i(s) & i\text{-idempotency} \end{array}$$

We speak accordingly of an **Lbi**-algebra and of type logic **Lbi**. Parasitic extraction as in

(7b) is now derived thus:

(69) John filed without reading: S/[i][i]$^{-1}$N

$$
\dfrac{
\dfrac{}{c\text{-}z: [\mathrm{i}][\mathrm{i}]^{-1}\mathrm{N}}\,j
\quad
\dfrac{
\dfrac{
\mathbf{J}\text{-}j: \mathrm{N}
\quad
\dfrac{
\dfrac{
\mathbf{f}\text{-}f: ([\,]\mathrm{N}\backslash\mathrm{S})/\mathrm{N}
\quad
\dfrac{\dfrac{}{a\text{-}x: [\mathrm{i}]^{-1}\mathrm{N}}\,i}{i(a)\text{-}x: \mathrm{N}}\,[\mathrm{i}]^{-1}\mathrm{E}
}{\mathbf{f}+i(a)\text{-}(f\ x): [\,]\mathrm{N}\backslash\mathrm{S}}\,/\mathrm{E}
\quad
\dfrac{
\dfrac{
\mathbf{without}
\quad
\dfrac{
\dfrac{
\mathbf{r}\text{-}r: (\mathrm{N}\backslash\mathrm{S})/\mathrm{N}
\quad
\dfrac{\dfrac{}{a: [\mathrm{i}]^{-1}\mathrm{N}}\,i}{i(a)\text{-}x: \mathrm{N}}\,[\mathrm{i}]^{-1}\mathrm{E}
}{\mathbf{r}+i(a)\text{-}(r\ x): \mathrm{N}\backslash\mathrm{S}}\,/\mathrm{E}
}{\mathbf{r}+i(a)\text{-}\iota(r\ x): (\mathrm{N}\backslash\mathrm{S})\vee\mathrm{S}}\,\vee\mathrm{I}
}{\mathbf{w}+\mathbf{r}+i(a)\text{-}\lambda y\lambda z[(y\ z)\wedge\neg(r\ x\ z)]: [\,]^{-1}(([\,]\mathrm{N}\backslash\mathrm{S})\backslash([\,]\mathrm{N}\backslash\mathrm{S}))}\,/\mathrm{E}
}{b(\mathbf{w}+\mathbf{r}+i(a))\text{-}\lambda y\lambda z[(y\ z)\wedge\neg(r\ x\ z)]: ([\,]\mathrm{N}\backslash\mathrm{S})\backslash([\,]\mathrm{N}\backslash\mathrm{S})}\,[\,]^{-1}\mathrm{E}
}{\mathbf{f}+i(a)+b(\mathbf{w}+\mathbf{r}+i(a))\text{-}\lambda z[(f\ x\ z)\wedge\neg(r\ x\ z)]: [\,]\mathrm{N}\backslash\mathrm{S}}\,/\mathrm{E}
}{b(\mathbf{J})+\mathbf{f}+i(a)+b(\mathbf{w}+\mathbf{r}+i(a))\text{-}[(f\ x\ j)\wedge\neg(r\ x\ j)]: \mathrm{S}}\,\backslash\mathrm{E}
}{b(\mathbf{J})+\mathbf{f}+b(b(\mathbf{w}+\mathbf{r}))+i(a)\text{-}[(f\ x\ j)\wedge\neg(r\ x\ j)]: \mathrm{S}}
}{
\dfrac{b(\mathbf{J})+\mathbf{f}+b(b(\mathbf{w}+\mathbf{r}))+c\text{-}[(f\ z\ j)\wedge\neg(r\ z\ j)]: \mathrm{S}}{b(\mathbf{J})+\mathbf{f}+b(b(\mathbf{w}+\mathbf{r}))\text{-}\lambda z[(f\ z\ j)\wedge\neg(r\ z\ j)]: \mathrm{S}/[\mathrm{i}][\mathrm{i}]^{-1}\mathrm{N}}\,/\mathrm{I}^j
}\,[\mathrm{i}]\mathrm{E}^i
$$

Example (7a) is derived similarly and the "double" parasitic extraction (10a) is derived:

(70) the editor of filed without reading: S/[i][i]$^{-1}$N

$$
\dfrac{
\dfrac{
\dfrac{}{\mathbf{c}\text{-}z: [\mathrm{i}][\mathrm{i}]^{-1}\mathrm{N}}\,j
\quad
\dfrac{
\dfrac{
\dfrac{\mathbf{the\ editor\ of}\quad \dfrac{}{a\text{-}x: [\mathrm{i}]^{-1}\mathrm{N}}\,i}{b(\mathbf{t}+\mathbf{e}+\mathbf{o}+i(a))\text{-}(T\ (o\ x\ e)): [\,]\mathrm{N}}
\quad
\dfrac{\mathbf{filed}\quad \dfrac{}{a\text{-}x: [\mathrm{i}]^{-1}\mathrm{N}}\,i \quad \mathbf{without}\quad \mathbf{reading}\quad \dfrac{}{a\text{-}x: [\mathrm{i}]^{-1}\mathrm{N}}\,i}{\mathbf{f}+i(a)+b(\mathbf{w}+\mathbf{r}+i(a))\text{-}\lambda z[(f\ x\ z)\wedge\neg(r\ x\ z)]: [\,]\mathrm{N}\backslash\mathrm{S}}
}{b(\mathbf{t}+\mathbf{e}+\mathbf{o}+i(a))+\mathbf{f}+i(a)+b(\mathbf{w}+\mathbf{r}+i(a))\text{-}[(f\ x\ (T\ (o\ x\ e)))\wedge\neg(r\ x\ (T\ (o\ x\ e)))]: \mathrm{S}}
}{b(b(\mathbf{t}+\mathbf{e}+\mathbf{o}))+\mathbf{f}+b(b(\mathbf{w}+\mathbf{r}))+i(a)\text{-}[(f\ x\ (T\ (o\ x\ e)))\wedge\neg(r\ x\ (T\ (o\ x\ e)))]: \mathrm{S}}
}{b(b(\mathbf{t}+\mathbf{e}+\mathbf{o}))+\mathbf{f}+b(b(\mathbf{w}+\mathbf{r}))+c\text{-}[(f\ z\ (T\ (o\ z\ e)))\wedge\neg(r\ z\ (T\ (o\ z\ e)))]: \mathrm{S}}
}{b(b(\mathbf{t}+\mathbf{e}+\mathbf{o}))+\mathbf{f}+b(b(\mathbf{w}+\mathbf{r}))\text{-}\lambda z[(f\ z\ (T\ (o\ z\ e)))\wedge\neg(r\ z\ (T\ (o\ z\ e)))]: \mathrm{S}/[\mathrm{i}][\mathrm{i}]^{-1}\mathrm{N}}
$$

However, as required parasitic gaps must occur within islands. Consider example (8):

(71)*slave that$_i$ John sold e_i to e_i

This is blocked because the nature of *i*-idempotency and *bi*-distribution cannot equate, say:

(72) b(**John**)+**sold**+$i(a)$+**to**+$i(a)$
and
b(**John**)+**sold**+$i(a)$+**to**

Nor is it possible to obtain two parasitic gaps in a single island: extraction of the first transforms a single bracketing to a double bracketing, which will then block any extraction of a second.

The reader may check that multiple parasitic extractions like (11) are derived. What has been presented so far may already suggest possible variations according to interpretations of judgements leading to different analytical generalizations for English or other languages. The reader may also have some thoughts regarding semigrammaticality and islandhood. The formal account of parasitic extraction that we have given resides in a few algebraic definitions. Our fundamental analytical generalizations are embodied in the principles of *i*-idempotency, and *bi*-distribution governing the underlying prosodic algebra.

It is often repeated that in Montague grammar semantic calculation is metagrammatical, having no role in defining the object language model specified by a grammar. Let us note that in type logical grammar the same is true of prosodic and syntactic calculation (Morrill 1994). We have presented natural deduction calculus purely for the convenience of displaying formal derivations of the predictions of grammar.

If we interpret grammar computationally or psychologically, however, we are interested in such calculus attributed with significance in relation to processing. Insofar as representational and derivational economics is concerned, natural deduction is perhaps not entirely without merit, but a reflection indicates some redundancy. Thus, we have basically trees or graphs with entire syntactic types at each node when, typically, immediately connected nodes stand in an immediate subtype relation. It is natural to ask as to the deep and essential structure of these derivations, and a beautiful answer comes from the direction of the proof nets of linear logic (Girard 1987).

In addition to their compelling candidature as the deep structures of type logical competence, simple considerations of proof nets yields a theory of acceptability in performance (Johnson 1998, Morrill 2000) including centre embedding, garden pathing, heavy noun phrase shift, left-to-right quantifier scoping, passivization and right association. Psychological interpretation of grammar requires great caution, but the results so far motivate the formulation in terms of proof nets of the analysis we have given, and invite investigation of whether something can be said in relation to the ambivalence of certain judgements. We turn to this in the next sections.

6. Proof nets

We consider first proof nets for type logic **L**. We define the input ($^\bullet$) and output ($^\circ$) *polar type trees* of a type A as the result of unfolding it into *links* as follows:

(73)

$$\frac{A^\bullet \quad B^\bullet}{A{\bullet}B^\bullet}\,i \qquad \frac{A^\circ \quad B^\bullet}{A\backslash B^\bullet}\,ii \qquad \frac{B^\bullet \quad A^\circ}{B/A^\bullet}\,ii$$

$$\frac{B^\circ \quad A^\circ}{A{\bullet}B^\circ}\,ii \qquad \frac{B^\circ \quad A^\bullet}{A\backslash B^\circ}\,ii \qquad \frac{A^\bullet \quad B^\circ}{B/A^\circ}\,ii$$

The unfoldings can be compared to the rules of natural deduction: input ($^\bullet$) for elimination (E) and output ($^\circ$) for introduction (I). $A{\bullet}B^\bullet$ states that if you have $A{\bullet}B$, then you have A and B. $A\backslash B^\bullet$ states that if you have $A\backslash B$, then if you can show A, then you have B. $B/A^\bullet$ is similar. $A{\bullet}B^\circ$ states that to show $A{\bullet}B$, it is sufficient to show A and B. $A\backslash B^\circ$ states that to show $A\backslash B$, it is sufficient to show B on the assumption A. B/A° is similar.

Note that in the output-unfoldings, the left-to-right order of the subtypes is reversed. This is because each polarity is implicitly the negation of the other and we choose to treat input as positive and output as negative. Consider[7] the negation of going first from i to j and then from j to k; it is to go first from k to j and then from j to i, i.e. de Morgan's law has the form $\neg(A \wedge B) = \neg B \vee \neg A$. When conjunction/disjunction is commutative it can be written $\neg(A \wedge B) = \neg A \vee \neg B$, but word order is not commutative, and we respect the interaction of negation and order in this context.[8]

The classification into i-links and ii-links reflects whether the upper "premises" belong to the same (i-) or different (ii-) subproofs in the rules of natural deduction. The links of polar type trees correspond to steps of derivation. A (correct) derivation as a whole will be the result of connecting these together (in a valid way).

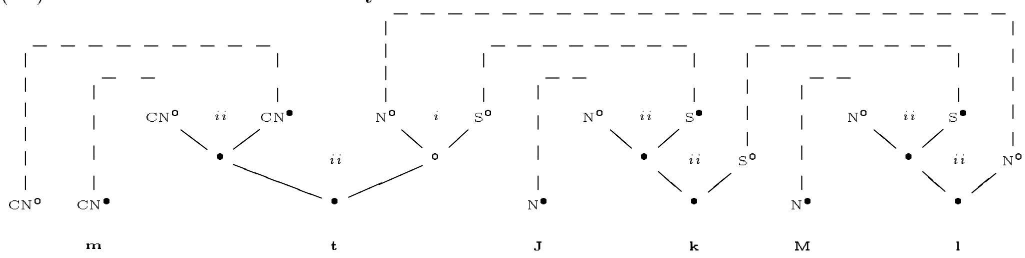
Suppose we are interested in knowing whether words of types $A_1, \ldots, A_n$ constitute (in that order) an expression of type A. Then we form a *proof frame* which is the sequence of polar type trees $A^\circ, A^\bullet, \ldots, A_n{}^\bullet$. A *proof structure* is a proof frame in which every leaf is connected (by an *axiom link*) to exactly one other with the same atom but of opposite polarity. A *proof net* is a proof structure which satisfies certain condicions.

[7] Philippe de Groote (p.c.)

[8] We could have chosen instead to treat input as negative and output as positive. But then our word order would be represented right-to-left rather than left-to-right on the page.

By way of example, there is the following proof net corresponding to (24).

(74) man that John knows Mary likes: CN



Note that the proof net in (74) is *planar*, i.e. its links can be drawn (in the half-plane) without any crossing. In fact every **L** proof net is planar, but planarity, though a necessary condition, is not a sufficient condition for a proof structure to be a proof net.[9]

A necessary and sufficient criterion for a proof structure to be a proof net is provided by planarity plus *acyclicity*. A *switching* of a proof structure is the result of removing edges from i-links such that only one edge remains in each i-link. Acyclicity is that every switching of a proof structure is an acyclic graph. Call a *i-free* path in a proof structure a path which does not traverse two edges of any i-link; call a *vicious cycle* a cycle which is a i-free path. Then acyclicity can be expressed as requiring that there be no vicious cycle.

The reasoning behind this acyclicity is complex (Girard 1987; Danos and Regnier 1989; Bellin and van de Wiele 1995; Lecomte and Retoré 1995). However, it suggests an attractive computational procedure: attempt to build a proof net incrementally left-to-right by placing (planar) axiom links on the proof frame growing in time with words coming in on the right. Planarity is can be implemented by a simple stack automaton. To ensure global acyclicity, it is enough just to check for each successive axiom link placement that it does not introduce a vicious cycle, i.e. that no axiom link is placed between leaves connected by a i-free path. That there is no i-free path between two leaves can be checked in linear time.

Assuming such a procedure, a natural complexity profile is induced, that giving the number of open leaves between successive words. For example, the complexity profile of (74) is:

(75)

3			a		a	
2	a	a		a		
1						
0						a
	man	that	John	knows	Mary	likes

In fact, a whole range of performance phenomena appear to be explained by such a metric (Morrill 2000). And if an absolute upper bound on stack depth is assumed (e.g. the capacity of short term memory), the processing becomes linear time, i.e. potentially real time. Indeed, there does appear to be a complete breakdown beyond 7–9 unresolved dependencies.

Compare the effect of applying this processing model to phrase structure grammar. English is primarily right-branching, so it would be predicted that processing crashes shortly after 7–9 words! This model of language processing, incremental, bottom-up, with a bounded stack, is the first one anyone would think of, but assuming phrase structure, it makes entirely the wrong predictions. Many complex alternatives have been entertained sustaining phrase structure, typically divorcing competence from performance. One wonders whether, if the technology of proof nets had been available in the past, the processing model would have stayed, and the phrase structure would have gone.

[9] If it were, we would have a polynomial decision procedure for **L**: planar linking is a Dyck language, i.e. a language of well-bracketing, and context-free; so just construct the proof frame (linear time) and memoise context-free recognition on the leaves by a Dyck grammar (less than cubic time).

Furthermore, the proof nets make very good sense semantically. Remarkably, the different proof nets that may be built on a proof frame correspond one-to-one with its different semantic readings so that, in this context, the problem of "spurious ambiguity" is dissolved. Deterministic travel instructions deliver the corresponding lambda terms in beta-eta-long normal form by successive left-to-right generation of the symbols of the terms' textual notation (de Groote and Retoré 1996; Morrill 1999). The travel starts upwards at the output root, visits each node exactly twice, once moving upwards and once moving downwards, and terminates downwards back at the origin in time $2n$, n the number of nodes in the proof net (cf. the preorder traversal of a tree).

With such computational properties, logical rationalle and linguistic interpretations, the theoretical subtilty of acyclicity of proof nets seems a small price to pay. As Einstein put it, we must make things as simple as possible, but no simpler. It seems important to investigate the generalization of proof nets to type logical grammar more widely. We sketch considerations for some of our categorial operations, though not for the lattice operacions, in the following subsections.

6.1 Bracket proof nets (Lb)

Consider type logic **Lb**. We add unfoldings:[10]

(76)
$$\frac{[^\bullet \quad A^\bullet \quad]^\bullet}{[\,]A^\bullet}\,i \qquad \frac{[^\circ \quad A^\bullet \quad]^\circ}{[\,]^{-1}A^\bullet}\,ii$$

$$\frac{]^\circ \quad A^\circ \quad [^\circ}{[\,]A^\circ}\,ii \qquad \frac{]^\bullet \quad A^\circ \quad [^\bullet}{[\,]^{-1}A^\circ}\,i$$

An example like (32) (but without the join operator) has the proof net analysis in figure 3.

Extraction from adverbial islands such as (6) is blocked; the extraction and bracketing dependencies cannot be kept planar:

(77)?paper that$_i$ John slept [without reading e_i]

6.2. Bi-bracket proof nets (Lbi)

For the bi-bracket type logic **Lbi** we repeat the same pattern of bracket unfolding:

(78)
$$\frac{[i^\bullet \quad A^\bullet \quad i]^\bullet}{[\mathrm{i}]A^\bullet}\,i \qquad \frac{[i^\circ \quad A^\bullet \quad i]^\circ}{[\mathrm{i}]^{-1}A^\bullet}\,ii$$

$$\frac{i]^\circ \quad A^\circ \quad [i^\circ}{[\mathrm{i}]A^\circ}\,ii \qquad \frac{i]^\bullet \quad A^\circ \quad [i^\bullet}{[\mathrm{i}]^{-1}A^\circ}\,i$$

Thus medial extraction like (57) has the proof net analysis in figure 4; of course i-commutativity means that proof nets might not be planar, as in this case; the precise geometry is a question for further investigation.

However, for parasitic extraction there is a further issue still. Proof nets as we have seen them so far are *linear*, as is reflected in the one-to-one matching of leaves by axiom

[10] This formulation of proof nets for bracketing, which remains tentative, is inspired by the translation $|[\,]A| = [\bullet|A|\bullet], |[\,]^{-1}A| = [\backslash|A|/]$ from **Lb** to **L** which I think appears in the 1996 PhD. thesis of Kuhn Versmissen, and the proposal of Fadda (2002) to treat brackets by means of "channels" in pregroup grammars.

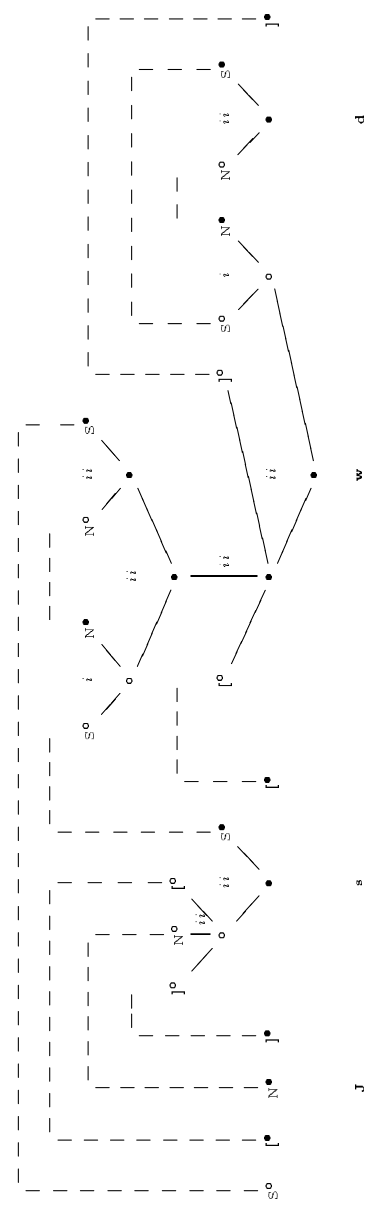


Figure 3: Proof net analysis of ‘John slept without dreaming’: S

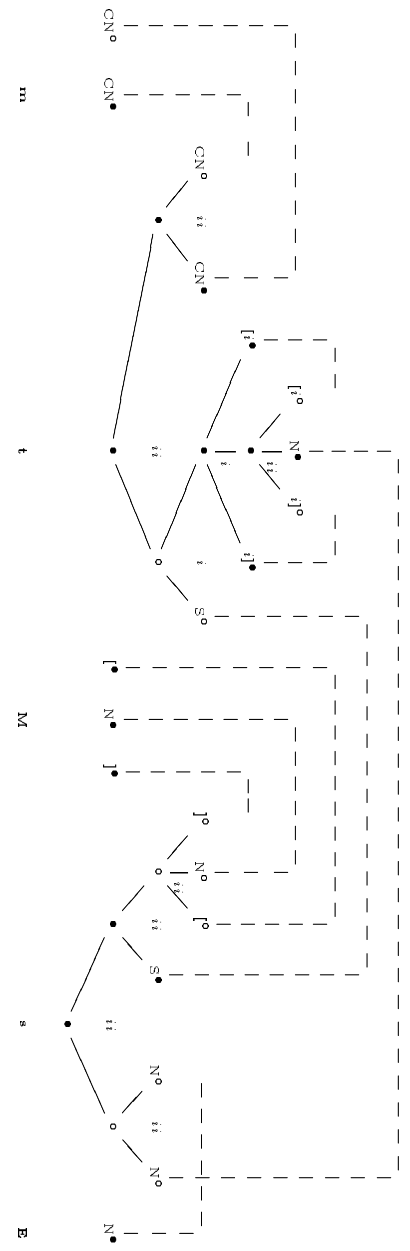


Figure 4: Proof net analysis of ‘man that$_i$ Mary showed e_i Exmoor’: CN

links, and associativity and i-commutativity are linear equations, with one occurrence of each variable on each side of the equation. But i-idempotency is a *non-linear* equation, with its variable occuring twice on one side:

(79) $i(i(s)) + i(s) = i(s)$ $\qquad$ i-commutativity

This means that something more is required for proof nets.

The general form of a treatment already exists within linear logic. Linear logic contains a unary operator ! for which there is an idempotent law $!A \otimes !A \equiv !A$. Danos (1990) accomodates this in proof nets by making available the unfolding:

(80)
$$\frac{!A^{\bullet} \qquad !A^{\bullet}}{!A^{\bullet}}$$

Note that this unfolding is recursive and potentially infinite, so that in processing it will have to be performed on-line on a call-by-need basis. The full theory involves devices called *boxes* which are viewed from outside as single links which must satisfy usual criteria outwardly, and from inside as complete subproofs which must satisfy usual criteria inwardly, in a recursive fashion; see for example de Groote and Retoré (1996).

We assume, then, the i-idempotent unfolding:

(81)
$$\frac{[\mathrm{i}][\mathrm{i}]A^{\bullet} \qquad [\mathrm{i}]A^{\bullet}}{[\mathrm{i}]A^{\bullet}}$$

Note that semantic trips will now visit certain nodes an *even* number of times, an equal number travelling upwards and downwards, and that the corresponding lambda terms will be *multiple*-binding rather than just *linear*-binding as before. This will constitute the multiple abstraction in the semantics of parasitic extraction. For instance, (7b) has the proof net analysis in figure 5, where we do not attempt to cope with the brackets.

Finally, the reader may like to construct proof net analyses of multiple parasitic extractions like (10a) and (11):

(82) paper that$_i$ [the editor of e_i] filed e_i [without reading e_i]

(83) a. man who$_i$ [the fact that [the friends of e_i] admire e_i] surprises e_i
b. ?paper that$_i$ John published e_i [without [the editor of e_i] rereading e_i]
c. ?man who$_i$ [the fact that [the friends of e_i] admire e_i [without praising e_i]] surprises e_i

The complexity profiles may be compared along the lines of Morrill (2000) and it will be seen that a degradation of acceptability is predicted from (83a) to (83b), where parasitic extraction further to the right requires unresolved dependencies to be held open longer in time. Degradation is also predicted from (83a) to (83c) where higher multiplicity of parasitic gaps requires more temporarily unresolved idempotent filler unfoldings. Our account treats parasitic multiplicity of any degree as grammatical, though deteriorating in acceptability with degree and rightwardness.

7. Semigrammaticality

On the account given, weak island violations like (6) are blocked:

(84) a. ?man that$_i$ [the friends of e_i] smiled
b. ?paper that$_i$ John slept [without reading e_i]

This is because we cannot equate, say:

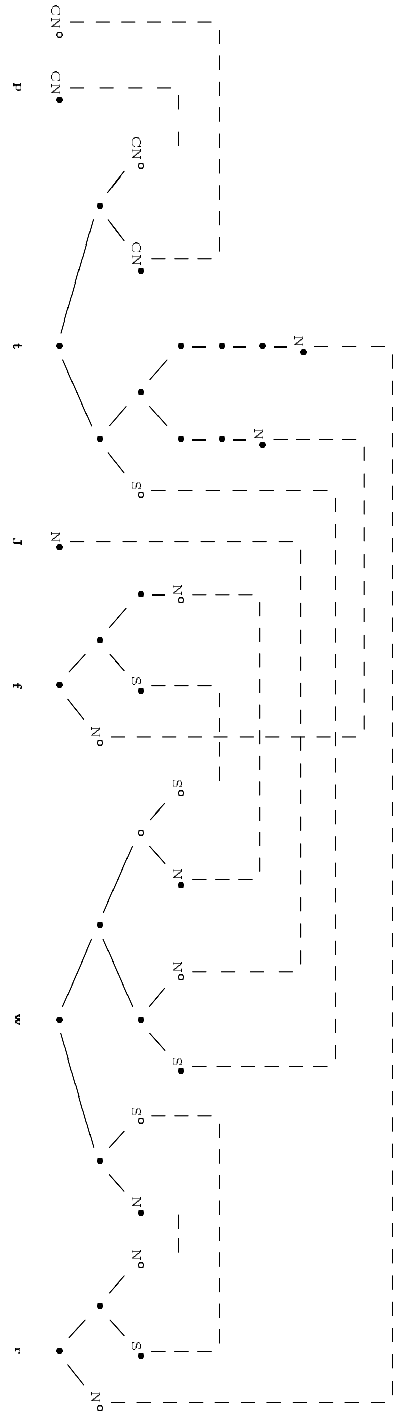


Figure 5: Proof net analysis of ‘paper that$_i$ John filed e_i [without reading e_i]’

(85) b(**John**)+**slept**+b(**without**+**reading**+$i(a)$)
and
b(**John**)+**slept**+b(**without**+**reading**)+$i(a)$

However, such examples are not *that* bad, and they are *almost* yielded by *bi*-distributivity; assume the following "semiequation" obtained by collapsing $i(i(s))$ in *bi*-distributivity to $i(s)$:

(86) $?b(s'+i(s)) = b(b(s'))+i(s)$ $\qquad$?*bi*-distributivity

Then the weak island violations of (6) are each processable under a single application of the semiequation of ?*bi*-distribution, which we may regard as characterising them as ?-.

Consider now strong islands like sentential subjects as in (12a):

(87)??man who$_i$ [that Mary likes e_i] surprises John

We assume assignments such as **annoys**: ([][]CP\S)/N with double brackets for strong islands. Then (87) also becomes processable under ?*bi*-distribution, but only under *two* applications of the semiequation, characterising it as ??-.

Extraction from within two weak islands is similarly characterised as ??-:

(88) a. ??man who$_i$ [the fact that [the friends of e_i] slept] annoys John
b. ??man who$_i$ [the fact that Mary left [without meeting e_i]] annoys John

Correspondingly, (89) are characterised as ???-:

(89) a. ???man who$_i$ [that Mary slept [without meeting e_i]] annoys John
b. ???man who$_i$ [that [the friends of e_i] slept] annoys John

And so forth.

However, even with ?*bi*-distribution, fixed subject constraint violations like (13) are blocked, at least if we assume no identity element in the underlying prosodic algebra.

(90)*man who$_i$ John said that e_i left

The example is underivable because there is simply nowhere to place the brackets that 'left' expects on its subject. The only way this could be derived by ?*bi*-distribution would be to assume an identity element in the underlying semigroup. The unacceptability of (90) argues against the existence of such an element. Thus alongside the scale of ω degrees of semigrammaticality: ?-, ??-, ???-, ..., there is the absolute ungrammaticality of, say, fixed subject constraint violations.

By the same token, however, the account as developed so far does not yield extraction of the subjects of uncomplementized embedded sentences, which can furthermore host parasitic gaps ((14)):

(91) a. man that$_i$ John thinks e_i left
b. man that$_i$ [the friends of e_i] think e_i left

Such extractions are generated if we generalize a sentence-embedding assignment such as **thinks**: (N\S)/S to **thinks**: (N\S)/(([]N∨[i][i]$^{-1}$N)•([]N\S)).

The relative pronoun type (CN\CN)/(S/[i][i]$^{-1}$N) does not yield main subject relativisation. This is appropriate for 'whom'. For 'who' and 'that' we could assume in addition (CN\CN)/([]N\S), which appropriately does not allow parasitics ((15)):

(92)?man that$_i$ left [without John meeting e_i]

The two assignments to the latter relative pronouns can be collapsed to the single polymorphic type (CN\CN)/(([]N∧[i][i]$^{-1}$N)\S).

Finally, a brief consideration reveals that parasitic extraction from a strong island is derivable by one application of ?*bi*-distribution:

(93)?man who$_i$ [that Mary likes e_i] surprises e_i

Parasitic gaps outside islands as in (8) are not possible even with ?*bi*-distribution:

(94)*slave that$_i$ John sold e_i to e_i

However, this could be derived independently of *bi*-distribution by a semiequation of the form $?i(s)+i(s) = i(s)$ obtained by collapsing $i(i(s))$ in *i*-idempotency to $i(s)$. Parasitic extraction such as (94) is possible in some Scandanavian languages, so we might posit such a (semi)equation in those cases.

8. Conclusion

We have given a precise grammar of left extraction including weak and strong islands, inviolable constraints on extraction and coordination, and parasitic gaps. A close relation between grammar and language processing is implemented in the notion of proof net, of computational and logical interest and importance in its own right. This discrete algebraic, logical and computational theory appears to deliver formal grammar together with a prospect for explanations of ambivalent judgements in terms of both acceptability and semigrammaticality, in an intimate and integrated theory of competence and performance.

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