

# On Anaphora and the Binding Principles in Categorical Grammar

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## Abstract

In type logical categorical grammar the analysis of an expression is a resource-conscious proof. Anaphora represents a particular challenge to this approach in that the antecedent resource is multiplied in the semantics. This duplication, which corresponds logically to the structural rule of contraction, may be treated lexically or syntactically. Furthermore, anaphora is subject to constraints, which Chomsky (1981)[1] formulated as Binding Principles A, B, and C. In this paper we consider English anaphora in categorical grammar including reference to the binding principles. We invoke displacement calculus, modal categorical calculus, categorical calculus with limited contraction, and entertain addition of negation as failure.

## 1 Introduction

Principles A, B and C of Chomsky (1981)[1] identify conditions on reflexive and personal pronouns in English. Principle A points to contrasts such as the following:<sup>1</sup>

- (1) a. John<sub>i</sub> likes himself<sub>i</sub>.
- b. \*John<sub>i</sub> thinks Mary likes himself<sub>i</sub>.

According to Principle A a reflexive requires a local c-commanding antecedent. Principle B refers to contrasts such as:

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- (2) a. \*John<sub>i</sub> likes him<sub>i</sub>.  
 b. John<sub>i</sub> thinks Mary likes him<sub>i</sub>.

According to Principle B a personal pronoun must not have a local c-commanding antecedent. Principle C filters examples such as:

- (3) a. \*He<sub>i</sub> likes John<sub>i</sub>.  
 b. \*He<sub>i/j</sub> thinks Bill<sub>i</sub> likes John<sub>j</sub>.

According to Principle C a personal pronoun cannot c-command its antecedent.

In categorial grammar the duplication of the antecedent semantic resource can be performed lexically or syntactically. We consider treating anaphora lexically by assignment of pronouns to higher-order types with lexical semantic contraction in displacement calculus (Morrill and Valentín 2010)[8], and syntactically with the limited syntactic contraction of Jaeger (2005)[5]. In Section 2 we define the displacement calculus **D**, a calculus which deals with discontinuous phenomena. Like the Lambek calculus **L**, which it subsumes, **D** is a sequent logic without structural rules which enjoys Cut-elimination, the subformula property, and decidability. In Section 3 we look at reflexives and Principle A. In Section 4 we consider personal pronouns and Principle B. In Section 5 we look at Principle C.

## 2 Displacement Calculus

The types of the calculus of displacement **D** classify strings over a vocabulary including a distinguished placeholder 1 called the *separator*. The sort  $i \in \mathcal{N}$  of a (discontinuous) string is the number of separators it contains and these punctuate it into  $i + 1$  maximal continuous substrings or *segments*. The types of **D** are sorted into types  $\mathcal{F}_i$  of sort  $i$  by mutual recursion as follows:

$$\begin{array}{llll}
 (4) & \mathcal{F}_j & := & \mathcal{F}_i \backslash \mathcal{F}_{i+j} & \text{under} \\
 & \mathcal{F}_i & := & \mathcal{F}_{i+j} / \mathcal{F}_j & \text{over} \\
 & \mathcal{F}_{i+j} & := & \mathcal{F}_i \cdot \mathcal{F}_j & \text{product} \\
 & \mathcal{F}_0 & := & I & \text{product unit} \\
 & \mathcal{F}_j & := & \mathcal{F}_{i+1} \downarrow_k \mathcal{F}_{i+j}, 1 \leq k \leq i+1 & \text{infix} \\
 & \mathcal{F}_{i+1} & := & \mathcal{F}_{i+j} \uparrow_k \mathcal{F}_j, 1 \leq k \leq i+1 & \text{extract} \\
 & \mathcal{F}_{i+j} & := & \mathcal{F}_{i+1} \odot_k \mathcal{F}_j, 1 \leq k \leq i+1 & \text{discontinuous product} \\
 & \mathcal{F}_1 & := & J & \text{discontinuous product unit}
 \end{array}$$

Where  $A$  is a type we call its sort  $sA$ . The set  $\mathcal{O}$  of *configurations* is defined as follows, where  $\Lambda$  is the empty string and  $[]$  is the metalinguistic separator:

$$(5) \quad \mathcal{O} ::= \Lambda \mid [] \mid \mathcal{F}_0 \mid \mathcal{F}_{i+1} \underbrace{\{\mathcal{O} : \dots : \mathcal{O}\}}_{i+1 \text{ } \mathcal{O}'s} \mid \mathcal{O}, \mathcal{O}$$

Note that the configurations are of a new kind in which some type formulas, namely the type formulas of sort greater than one, label mother nodes rather than leaves, and have a number of immediate subconfigurations equal to their sort. This signifies a discontinuous type intercalated by these subconfigurations. Thus  $A\{\Delta_1 : \dots : \Delta_n\}$  interpreted syntactically is formed by strings  $\alpha_0 + \beta_1 + \dots + \beta_n + \alpha_n$  where  $\alpha_0 + 1 + \dots + 1 + \alpha_n \in A$  and  $\beta_1 \in \Delta_1, \dots, \beta_n \in \Delta_n$ . We call these types *hyperleaves* since in multimodal calculus they would be leaves. We call these new configurations *hyperconfigurations*. The sort of a (hyper)configuration is the number of separators it contains. A *hypersequent*  $\Gamma \Rightarrow A$  comprises an antecedent hyperconfiguration  $\Gamma$  of sort  $i$  and a succedent type  $A$  of sort  $i$ . The *vector*  $\vec{A}$  of a type  $A$  is defined by:

$$(6) \vec{A} = \begin{cases} A & \text{if } sA = 0 \\ A\{\underbrace{[] : \dots : []}_{sA \quad []'s}\} & \text{if } sA > 0 \end{cases}$$

Where  $\Delta$  is a configuration of sort at least  $k$  and  $\Gamma$  is a configuration, the *k-ary wrap*  $\Delta|_k\Gamma$  signifies the configuration which is the result of replacing by  $\Gamma$  the  $k$ th separator in  $\Delta$ . Where  $\Delta$  is a configuration of sort  $i$  and  $\Gamma_1, \dots, \Gamma_i$  are configurations, the *generalized wrap*  $\Delta \otimes \langle \Gamma_1, \dots, \Gamma_i \rangle$  is the result of simultaneously replacing the successive separators in  $\Delta$  by  $\Gamma_1, \dots, \Gamma_i$  respectively. In the hypersequent calculus we use a discontinuous distinguished hyperoccurrence notation  $\Delta\langle \Gamma \rangle$  to refer to a configuration  $\Delta$  and continuous subconfigurations  $\Delta_1, \dots, \Delta_i$  and a discontinuous subconfiguration  $\Gamma$  of sort  $i$  such that  $\Gamma \otimes \langle \Delta_1, \dots, \Delta_i \rangle$  is a continuous subconfiguration of  $\Delta$ . That is, where  $\Gamma$  is of sort  $i$ ,  $\Delta\langle \Gamma \rangle$  abbreviates  $\Delta(\Gamma \otimes \langle \Delta_1, \dots, \Delta_i \rangle)$  where  $\Delta(\dots)$  is the usual distinguished occurrence notation. Technically, whereas the usual distinguished occurrence notation  $\Delta(\Gamma)$  refers to a context containing a *hole* which is a leaf, in hypersequent calculus the distinguished hyperoccurrence notation  $\Delta\langle \Gamma \rangle$  refers to a context containing a hole which may be a hyperleaf, a *hyperhole*.

The hypersequent calculus for the calculus of displacement is given in Figure 1. Observe that the rules for both the concatenating connectives  $\backslash, \cdot, /$  and the wrapping connectives  $\downarrow_k, \odot_k, \uparrow_k$  are just like the rules for Lambek calculus except for the vectorial notation and hyperoccurrence notation; the former are specified in relation to the primitive concatenation represented by the sequent comma and the latter are specified in relation to the defined operations of  $k$ -ary wrap.

Abbreviating here and throughout  $\uparrow_1$  and  $\downarrow_1$  as  $\uparrow$  and  $\downarrow$  respectively, an extensional lexicon for examples of this paper is as follows:

$$(7) \begin{aligned} \mathbf{a} &: ((S\uparrow n_{(110)})\downarrow S)/cn_{(110)} : \lambda A \lambda B \exists C [(A \ C) \wedge (B \ C)] \\ \mathbf{about} &: PP/n_{(98)} : \lambda AA \\ \mathbf{before} &: (S/S)/S : \mathit{before} \\ \mathbf{buys} &: (n_{(101)}\backslash S)/(n_{(106)}\bullet n_{(108)}) : \lambda A ((\mathit{buys} \ \pi_1 A) \ \pi_2 A) \\ \mathbf{coffee} &: n(n) : \mathit{coffee} \end{aligned}$$

$$\begin{array}{c}
\frac{}{\vec{A} \Rightarrow A} id \quad \frac{\Gamma \Rightarrow A \quad \Delta \langle \vec{A} \rangle \Rightarrow B}{\Delta \langle \Gamma \rangle \Rightarrow B} Cut \\
\\
\frac{\Gamma \Rightarrow A \quad \Delta \langle \vec{C} \rangle \Rightarrow D}{\Delta \langle \Gamma, \vec{A} \rangle \langle \vec{C} \rangle \Rightarrow D} \backslash L \quad \frac{\vec{A}, \Gamma \Rightarrow C}{\Gamma \Rightarrow A \setminus C} \backslash R \\
\\
\frac{\Gamma \Rightarrow B \quad \Delta \langle \vec{C} \rangle \Rightarrow D}{\Delta \langle \vec{C} / \vec{B}, \Gamma \rangle \Rightarrow D} /L \quad \frac{\Gamma, \vec{B} \Rightarrow C}{\Gamma \Rightarrow C / B} /R \\
\\
\frac{\Delta \langle \vec{A}, \vec{B} \rangle \Rightarrow D}{\Delta \langle \vec{A} \cdot \vec{B} \rangle \Rightarrow D} \cdot L \quad \frac{\Gamma_1 \Rightarrow A \quad \Gamma_2 \Rightarrow B}{\Gamma_1, \Gamma_2 \Rightarrow A \cdot B} \cdot R \\
\\
\frac{\Delta \langle \Lambda \rangle \Rightarrow A}{\Delta \langle \vec{I} \rangle \Rightarrow A} IL \quad \frac{}{\Lambda \Rightarrow I} IR \\
\\
\frac{\Gamma \Rightarrow A \quad \Delta \langle \vec{C} \rangle \Rightarrow D}{\Delta \langle \Gamma |_k \vec{A} |_k \vec{C} \rangle \Rightarrow D} \downarrow_k L \quad \frac{\vec{A} |_k \Gamma \Rightarrow C}{\Gamma \Rightarrow A \downarrow_k C} \downarrow_k R \\
\\
\frac{\Gamma \Rightarrow B \quad \Delta \langle \vec{C} \rangle \Rightarrow D}{\Delta \langle \vec{C} \uparrow_k \vec{B} |_k \Gamma \rangle \Rightarrow D} \uparrow_k L \quad \frac{\Gamma |_k \vec{B} \Rightarrow C}{\Gamma \Rightarrow C \uparrow_k B} \uparrow_k R \\
\\
\frac{\Delta \langle \vec{A} |_k \vec{B} \rangle \Rightarrow D}{\Delta \langle \vec{A} \odot_k \vec{B} \rangle \Rightarrow D} \odot_k L \quad \frac{\Gamma_1 \Rightarrow A \quad \Gamma_2 \Rightarrow B}{\Gamma_1 |_k \Gamma_2 \Rightarrow A \odot_k B} \odot_k R \\
\\
\frac{\Delta \langle [] \rangle \Rightarrow A}{\Delta \langle \vec{J} \rangle \Rightarrow A} JL \quad \frac{}{[] \Rightarrow J} JR
\end{array}$$

Figure 1: Calculus of displacement **D**





$$\begin{array}{c}
\frac{\frac{\frac{n(m) \Rightarrow n(m)}{\quad} \quad \frac{n(n) \Rightarrow n(n)}{\quad}}{\quad} \bullet R \quad \frac{\frac{n(m) \Rightarrow n(m)}{\quad} \quad \frac{S \Rightarrow S}{\quad}}{\quad} \setminus L}{\frac{n(m), n(n) \Rightarrow n(m) \bullet n(n)}{\quad} \quad \frac{n(m), n(m) \setminus S \Rightarrow S}{\quad}} /L \\
(17) \quad \frac{\frac{\frac{n(m), (n(m) \setminus S) / (n(m) \bullet n(n)), n(m), n(n) \Rightarrow S}{\quad} \setminus R \quad \frac{n(m) \Rightarrow n(m)}{\quad} \quad \frac{S \Rightarrow S}{\quad}}{\frac{(n(m) \setminus S) / (n(m) \bullet n(n)), n(m), n(n) \Rightarrow n(m) \setminus S}{\quad}} \uparrow R \quad \frac{n(m), n(m) \setminus S \Rightarrow S}{\quad}} \setminus L}{\frac{(n(m) \setminus S) / (n(m) \bullet n(n)), [], n(n) \Rightarrow (n(m) \setminus S) \uparrow n(m)}{\quad} \quad \frac{n(m), n(m) \setminus S \Rightarrow S}{\quad}} \downarrow L} \\
\frac{\quad}{n(m), (n(m) \setminus S) / (n(m) \bullet n(n)), ((n(m) \setminus S) \uparrow n(m)) \downarrow (n(m) \setminus S), n(n) \Rightarrow S}
\end{array}$$

This derivation delivers the lexical semantics:

$$(18) \quad (((buys\ j)\ coffee)\ j)$$

Consider further object-oriented reflexivization:

$$(19) \quad \mathbf{mary+talks+to+john+about+himself} : S$$

Lexical lookup for our secondary wrap object-oriented reflexivization type assignment yields the semantically labelled sequent:

$$\begin{array}{l}
(20) \quad n(f) : m, ((n_{256} \setminus S) / PP) / PP : talks, PP / n_{269} : \lambda AA, n(m) : j, \\
\quad \quad PP / n_{298} : \lambda AA, (((n_{337} \setminus S) \uparrow n(m)) \uparrow n(m)) \downarrow_2 ((n_{352} \setminus S) \uparrow n(m)) : \\
\quad \quad \lambda A \lambda B ((A\ B)\ B) \Rightarrow S
\end{array}$$

This has the proof given in Figure 2. This delivers semantics:

$$(21) \quad (((talks\ j)\ j)\ m)$$

However, such extensional types will also allow Principle A violation such as (1b). Modal categorial calculus can be employed to rectify this.

Whatever the details of temporal semantics may turn out to be, it seems clear that the semantics of each lexical item is evaluated at a temporal index bound in the minimal tensed S within which it occurs. Morrill (1990)[7] proposed to characterize such intensionality by adding a modality to Lambek calculus. Let us extend the set of types as follows:

$$(22) \quad \mathcal{F}_i := \Box \mathcal{F}_i$$

The semantic type map  $\tau$  will be such that  $\tau(\Box A) = T \rightarrow \tau(A)$  where  $T$  is the set of time indices, i.e. expressions of type  $\Box A$  are to have as semantics the functional abstraction of the corresponding extensional semantics of the expression of type  $A$ . Thus, we assume the following semantically annotated rules of S4 modality where  $\hat{\ } and  $\check{\ }$  represent temporal intensionalisation and extensionalisation respectively:$

$$(23) \quad \frac{\Box \Gamma \Rightarrow A : \phi}{\Box \Gamma \Rightarrow \Box A : \hat{\phi}} \Box R \quad \frac{\Gamma \langle \vec{A} : x \rangle \Rightarrow B : \phi(x)}{\Gamma \langle \vec{\Box A} : y \rangle \Rightarrow B : \phi(\check{y})}$$

$$\begin{array}{c}
\frac{\frac{n(m)}{PP} \Rightarrow \frac{n(m)}{PP}}{/L} \quad \frac{\frac{n(m)}{PP} \Rightarrow \frac{n(m)}{PP}}{/L} \quad \frac{\frac{n(2576)}{S} \Rightarrow \frac{n(2576)}{S}}{/L} \\
\frac{\frac{n(m)}{PP/n(m), n(m)} \Rightarrow \frac{n(m)}{PP}}{/L} \quad \frac{\frac{n(2576), (n(2576) \setminus S) / PP, PP/n(m), n(m)}{S} \Rightarrow \frac{n(2576) \setminus S}{S}}{/L} \\
\frac{\frac{n(2576), ((n(2576) \setminus S) / PP) / PP, PP/n(m), n(m), PP/n(m), n(m)}{S} \Rightarrow \frac{n(2576) \setminus S}{S}}{/L} \\
\frac{\frac{((n(2576) \setminus S) / PP) / PP, PP/n(m), n(m), PP/n(m), n(m))}{\uparrow R} \Rightarrow \frac{n(2576) \setminus S}{\uparrow R}}{\uparrow R} \\
\frac{\frac{((n(2576) \setminus S) / PP) / PP, PP/n(m), n(m), PP/n(m), n(m), PP/n(m), n(m))}{\uparrow R} \Rightarrow \frac{n(2576) \setminus S}{\uparrow R}}{\uparrow R} \\
\frac{\frac{((n(2576) \setminus S) / PP) / PP, PP/n(m), n(m), PP/n(m), n(m))}{\uparrow R} \Rightarrow \frac{n(2576) \setminus S}{\uparrow R}}{\uparrow R} \\
\frac{\frac{n(f), ((n(2576) \setminus S) / PP) / PP, PP/n(m), n(m), PP/n(m), ((n(2576) \setminus S) \uparrow n(m)) \uparrow_2 (n(f) \setminus S) \uparrow n(m))}{\uparrow_2 L} \Rightarrow \frac{n(f)}{S}}{\uparrow_2 L} \\
\frac{\frac{n(m)}{S} \Rightarrow \frac{n(m)}{S}}{/L} \quad \frac{\frac{n(f) \setminus S}{S} \Rightarrow \frac{n(f) \setminus S}{S}}{/L} \\
\frac{\frac{n(f), n(f) \setminus S}{S} \Rightarrow \frac{n(f), n(f) \setminus S}{S}}{\uparrow L} \\
\frac{\frac{n(f), (n(f) \setminus S) \uparrow n(m) \{n(m)\}}{S} \Rightarrow \frac{n(f), (n(f) \setminus S) \uparrow n(m)}{S}}{\uparrow_2 L}
\end{array}$$

Figure 2: Object-oriented reflexivization



An intensional lexicon for examples of this paper is as follows:

- (28) **after** :  $\Box((S/\Box S)/\Box S) : \hat{\lambda}A\lambda B((\sim \text{after } A) B)$   
**arrives** :  $\Box(n_{(100)}\backslash S) : \text{arrives}$   
**debbie** :  $\Box n(f) : \hat{d}$   
**everyone** :  $\Box((S\uparrow n_{(109)})\downarrow S) : \hat{\lambda}A\forall B[(\sim \text{person } B) \rightarrow (A B)]$   
**herself** :  $\Box(((n(f)\backslash S)\uparrow n(f))\downarrow (n(f)\backslash S)) : \hat{\lambda}A\lambda B((A B) B)$   
**herself** :  $\Box((((n_{(122)}\backslash S)\uparrow n(f))\uparrow_2 n(f))\downarrow_2 ((n_{(137)}\backslash S)\uparrow n(f))) : \hat{\lambda}A\lambda B((A B) B)$   
**likes** :  $\Box((n_{(103)}\backslash S)/n_{(105)}) : \text{likes}$   
**she** :  $\Box n(f)|n(f) : \lambda A \hat{A}$   
**she** :  $\Box(((S\uparrow \Box n(f))\uparrow \Box n(f))\downarrow (S\uparrow \Box n(f))) : \hat{\lambda}A\lambda B((A B) B)$   
**sings** :  $\Box(n_{(100)}\backslash S) : \text{sings}$   
**someone** :  $\Box((S\uparrow \Box n_{(111)})\downarrow S) : \hat{\lambda}A\exists B[(\sim \text{person } B) \wedge (A \hat{B})]$   
**suzy** :  $\Box n(f) : \hat{s}$   
**thinks** :  $\Box((n_{(103)}\backslash S)/\Box S) : \text{thinks}$

## 4 Personal Pronouns and Principle B

Jaeger (2005)[5] presents a syntactic type logical categorial grammar treatment of anaphora inspired by the combinatory categorial grammar treatment of Jacobson (1999)[4]. This uses a type constructor  $B|A$  for an expression of type  $B$  requiring an antecedent of type  $A$ . This was in turn inspired by the syntactic treatment of Hepple (1990)[3] which assigns pronouns the identity function as lexical semantics. Jaeger gives the following left rule for  $|$ :

$$(29) \frac{\Gamma \Rightarrow A : \phi \quad \Delta_1, A : x, \Delta_2, B : y, \Delta_3 \Rightarrow D : \omega(x, y)}{\Delta_1, \Gamma, \Delta_2, B|A : z, \Delta_3 \Rightarrow D : \omega(\phi, (z \phi))} |L$$

Thus in an extensional grammar there is the analysis:

$$(30) \text{john+says+he+walks} : S$$

$$(31) n(m) : j, (n_{(197)}\backslash S)/S : \text{says}, n(m)|n(m) : \lambda AA, n_{(231)}\backslash S : \text{walks} \Rightarrow S$$

$$(32) \frac{\frac{\frac{\overline{n(m) \Rightarrow n(m)} \quad \overline{S \Rightarrow S}}{n(m), n(m)\backslash S \Rightarrow S} \backslash L \quad \frac{\overline{n(m) \Rightarrow n(m)} \quad \overline{S \Rightarrow S}}{n(m), n(m)\backslash S \Rightarrow S} \backslash L}{\overline{n(m) \Rightarrow n(m)} \quad \overline{n(m), (n(m)\backslash S)/S, n(m), n(m)\backslash S \Rightarrow S}}{/L}}{n(m), (n(m)\backslash S)/S, n(m)|n(m), n(m)\backslash S \Rightarrow S} |L$$

$$(33) ((\text{says } (\text{walks } j)) j)$$

Intensionally, and with a quantified antecedent:



$$(38) \mathcal{F}_i := \mathcal{F}_i \& \mathcal{F}_i \mid \mathcal{F}_i + \mathcal{F}_i$$

$$(39) \frac{\Gamma \langle \vec{A} \rangle \Rightarrow C}{\Gamma \langle A \& B \rangle \Rightarrow C} \&L_1 \quad \frac{\Gamma \langle \vec{B} \rangle \Rightarrow C}{\Gamma \langle A \& B \rangle \Rightarrow C} \&L_2$$

$$\frac{\Gamma \Rightarrow A \quad \Gamma \Rightarrow B}{\Gamma \Rightarrow A \& B} \&R$$

$$\frac{\Gamma \langle \vec{A} \rangle \Rightarrow C \quad \Gamma \langle \vec{B} \rangle \Rightarrow C}{\Gamma \langle A + B \rangle \Rightarrow C} +L$$

$$\frac{\Gamma \Rightarrow A}{\Gamma \Rightarrow A + B} +L_1 \quad \frac{\Gamma \Rightarrow B}{\Gamma \Rightarrow A + B} +L_2$$

We propose to introduce into type logical categorial grammar a negation, interpreted in the succedent as non-provability (strong negation, as for example in autoepistemic logic, and Prolog):

$$(40) \mathcal{F}_i := \neg \mathcal{F}_i$$

$$(41) \frac{\not\vdash \Gamma \Rightarrow A}{\Gamma \Rightarrow \neg A} \neg R$$

Thus, to express that *walk* is a non-third person present tense form we might assign it type  $(\exists a N(a) \& \neg N(3(sg))) \setminus S$ .

Treating pronouns by secondary wrap in modal displacement calculus, generating an example like (2b) requires a pronoun type  $\Box(((S \uparrow N) \uparrow_2 \Box N) \downarrow_2 (S \uparrow N))$  in which the pronoun hypothetical subtype is modalized to allow the pronoun in a subordinate clause. But this would also overgenerate (2a). Our proposal is to enforce Principle B by employing the negation:

$$(42) \mathbf{him} : \Box(((S \uparrow N) \uparrow_2 \Box N) \& \neg((J \cdot (N \setminus S)) \uparrow_2 N)) \downarrow_2 (S \uparrow N)) = \Box \alpha$$

Then (2a) is filtered because the negative goal in Figure 3 succeeds; here and henceforth we may abbreviate  $(N \setminus S) / N$  as *TV*. succeeds. Example (2b) is allowed however because the negative goal in Figure 4 fails as required.

## 5 Principle C

We can adopt a similar strategy in order to block Principle C violations in cataphora. The analysis of the following, where the pronoun does not c-command its antecedent, goes through since the negative subgoal fails as required.

$$(43) \mathbf{before+he+walks+every+man+smiles} : S$$

$$\begin{array}{c}
\frac{N \Rightarrow N \quad S \Rightarrow S}{N, VP \Rightarrow S} \backslash L \\
\frac{N \Rightarrow N \quad VP \Rightarrow VP}{VP \Rightarrow VP} \backslash R \\
\frac{N \Rightarrow N \quad VP \Rightarrow VP}{TV, N \Rightarrow VP} /L \\
\frac{[] \Rightarrow J \quad TV, N \Rightarrow VP}{[] \Rightarrow J} JR \quad \frac{TV, N \Rightarrow VP}{\square TV, N \Rightarrow VP} \square L \\
\frac{[] \Rightarrow J \quad \square TV, N \Rightarrow VP}{\square TV, N \Rightarrow J \cdot VP} \cdot R \\
\frac{[] \Rightarrow J \quad \square TV, N \Rightarrow J \cdot VP}{\square TV, [] \Rightarrow (J \cdot VP) \uparrow_2 N} \uparrow_2 R \\
\frac{\vdots \quad \square TV, [] \Rightarrow (S \uparrow N) \uparrow_2 \square N \quad \forall \square TV, [] \Rightarrow \neg((J \cdot VP) \uparrow_2 N)}{\square TV, [] \Rightarrow ((S \uparrow N) \uparrow_2 \square N) \& \neg((J \cdot VP) \uparrow_2 N)} \& R \\
\frac{\square TV, [] \Rightarrow ((S \uparrow N) \uparrow_2 \square N) \& \neg((J \cdot VP) \uparrow_2 N) \quad S \uparrow N \{ \square N \} \Rightarrow S}{\square N, \square TV, \alpha \Rightarrow S} \downarrow_2 L \\
\frac{\square N, \square TV, \alpha \Rightarrow S}{\square N, \square TV, \square \alpha \Rightarrow S} \square L
\end{array}$$

Figure 3: Blocking of  $John_i$  likes  $him_i$  in accordance with Principle B because of the provability of the subgoal which is required to be not provable

Lexical lookup yields the semantically labelled sequent:

$$(44) \ (S/S)/S : \text{before}, (((S \uparrow n(m)) \uparrow n(m)) \& \neg(J \bullet ((n(m) \setminus S) \uparrow n(m)))) \downarrow (S \uparrow n(m))) : \\
\lambda A \lambda B ((\pi_1 A \ B) \ B), n_{(341)} \setminus S : \text{walks}, ((S \uparrow n_{(366)}) \downarrow S) / cn_{(366)} : \\
\lambda A \lambda B \forall C [(A \ C) \rightarrow (B \ C)], cn(m) : \text{man}, n_{(427)} \setminus S : \text{smiles} \Rightarrow S$$

The derivation delivers semantics:

$$(45) \ \forall C [(man \ C) \rightarrow ((before \ (walks \ C)) \ (smiles \ C))]$$

But as required, Principle C violations such as the following will be blocked.

$$(46) \ \text{a. *He}_i \text{ likes John}_i. \\
\text{b. *He}_i \text{ thinks Mary likes John}_i.$$

## 6 Conclusion

This paper offers two innovations in relation to anaphora and the binding principles: we consider the possible application of the generalized discontinuity of the displacement calculus to various forms of anaphora, and we make the technical innovation of introducing negation as failure into categorial logic, and apply this to the capture of binding principles.

As regards the Cut rule and negation as failure, note that by using them both together we would get undesirable derivations such as the following:



$$(47) \frac{\frac{N \Rightarrow S/(N \setminus S) \quad \frac{\not\vdash S/(N \setminus S) \Rightarrow N}{S/(N \setminus S) \Rightarrow \neg N} \neg R}{N \Rightarrow \neg N} \text{Cut}}{N \Rightarrow \neg N}$$

Adding the negation as failure (right) rule brings our categorial logic to the realms of non-monotonic reasoning where the transitivity of the consequence relation must be dropped. The other connectives used in this paper, the displacement connectives, S4 modality, and additives, enjoy Cut-elimination. But in the presence of negation as failure, the Cut rule must be considered not just eliminable, but inadmissible. However, the subformula property holds of all the connectives used here: the sequent presentation is such that for every rule, the formula occurrences in the premises are always subformulas of those in the conclusion. Given this state of affairs, the Cut-free backward chaining sequent search space turns out to be finite and hence the categorial logic used in this paper is decidable. Thus the system considered here is *implementable*; indeed some derivation examples used in this document have been generated automatically from a Prolog implementation.

Concerning the negation connective, let us remark the following aspects. On the one hand, as far as we are aware, no left sequent rule for negation as failure is known. This seems to be an open problem. On the other hand, as the reader may have noticed, the polarity of the negated subtypes in our applications is always positive, consistent with the absence of a left rule.

Much remains to be said and done on anaphora and the binding principles in English and other languages and we have only been able to touch on a few points here. The account of Principle A in terms of modal categorial logic was introduced twenty years ago but it appears that no other categorial account of locality has been developed in detail. Here we have suggested that parts of Principles B (antilocality) and C may be treated in the grammar by means of negation, in particular negation as failure. We hope this may be a first indication of how categorial approaches may be sensitized to these negative conditions directly in the grammar while preserving as much as possible the good theoretical properties of the logic.

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