Exercises on logic synthesis*

Deadline: January 10th, 2007

1. Calculate the quotient and remainder obtained by the algebraic division of F by D, where

$$\begin{aligned} F &= abrs + abrt + abd + abe + abu + ghrs + ghrw + ghd + ghe + ghu + dp + eq + rstuw \\ D &= ab + gh \end{aligned}$$

2. Consider a library including the following cells: {AND2 with cost 4; OR2 with cost 5; INV with cost 1}. Draw the pattern trees for these cells using NAND2 and INV as base functions. Then consider the function $f = a\overline{b} + b\overline{c}\overline{d}$. determine the subject graph for f using the same base functions. Find a minimum cost cover of the subject graph using dynamic programming and the inverter-pair heuristic. *Hint*: use the following decomposition:

$$f = \text{NAND2}(p, q);\ \ p = \text{NAND2}(a, \overline{b});\ \ q = \text{NAND2}(\overline{c}, \overline{r});\ \ r = \text{NAND2}(\overline{d}, b).$$

3. We can define a lexicographical order of the vertices in $\mathbb{B}^n$ by using the order of the binary value they represent. For instance, for $\mathbb{B}^2$ we have the order

$$\overline{x_1}\,\overline{x_2} <_L \overline{x_1}x_2 <_L x_1\overline{x_2} <_L x_1x_2.$$

Given two functions, f and g, we say that $f <_L g$ if $f(X) < g(X)$ in the first lexicographical vertex X in which they differ. For instance, let f, g, and h be the following functions:

x_1x_2	f	g	h
00	1	0	0
01	0	1	1
10	0	1	0
11	1	0	1

We have that $h <_L g <_L f$, since $h(1,0) < g(1,0)$ and $g(0,0) < f(0,0)$.

Design a BDD algorithm for detecting that $f <_L g$. The algorithm should receive two BDDs as inputs and produce a Boolean as output. *Hint:* Use the recursive paradigm; take the skeleton of the AND function of two BDDs and modify it to design this one.

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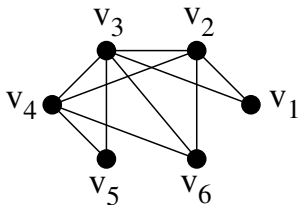
4. Let $G(V, E)$ be a graph with $V = \{v_1, \ldots, v_n\}$.

   - Derive a CNF formula that characterizes all cliques of the graph [Hint: use one variable for each vertex, and consider all edges that are not in the graph.]
   - Assume that the previous formula is represented by a BDD. How would you find the largest clique in the graph?
   - Finding the largest clique of a graph is an NP-complete problem? Where is the complexity "hidden" when solving the problem with BDDs?
   - Draw the BDD (without complement edges) that characterizes all cliques of the graph in the figure and find the largest clique.



5. Synthesize the asynchronous specification depicted in Fig. 1. Assume that a and d are input signals, whereas b, c and x are output signals.

   - Derive the graph of reachable states with their corresponding encoding (clue: the graph has 18 states).
   - Calculate the *don't care* set of the reachability space and represent it in a Karnaugh map.
   - Derive the Karnaugh maps for the next-state functions of b and c, identifying the excitation and quiescent regions. Use 5-variable Karnaugh maps.
   - Derive a logic implementation for b and c.

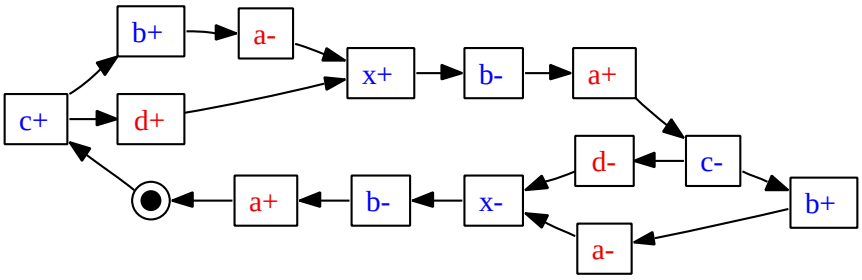


Figure 1: STG specification of an asynchronous circuit.