Modality-based Argumentation Using Possibilistic Stable Models

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Abstract
In many fields of automated information processing it becomes crucial to consider together imprecise, uncertain or inconsistent information. Modalities are terms which indicate the level of certainty with which a claim can be made. Argumentation theory is a suitable framework for practical and uncertain reasoning, where arguments could support conclusions. We present a modality-based argumentation approach, where the modalities categorize the encoded knowledge and allow building arguments which express levels of certainty. This approach is based on the concept of possibilistic stable models.

1 Introduction
Argumentation has proved to be a useful tool for representing and dealing with domains in which rational agents are not able to decide by themselves about something, and may encounter other agents with different preference values. The ability to reason about what is the best or most appropriate course of action to take in a given situation is an essential activity for a rational agent. A rational agent may also use argumentation techniques to perform its individual reasoning as it needs to make rational decisions under complex preferences policies, or to reason about its commitments, its goals, etc.

Since Aristotle’s Metaphysics, modalities have been an object of study for logicians especially in relation with the construction of arguments. Modalities are terms which indicate the level of certainty with which a claim can be made. According to Merriam-Webster Dictionary a modality is:

“The classification of logical propositions according to their asserting or denying the possibility, impossibility, contingency, or necessity of their content”.

Research on rational agents has raised further questions about modalities in the context of argumentation, and the roles that arguments play in the pursuit of an agent’s goals and plans.

In the medical domain, there are different sources of examples of argumentation where the evidence/possibility plays a central role in order to make decisions [Fox and Modgil, Currently in press]. The main objective is to discover the acceptable set of arguments that support a given claim in a given context. This is a purposeful process where the validity of arguments and the evidence of premises are both approached. For instance, in the process of organ transplanting, there is small amount of information available w.r.t. the viability criteria which are applied whether a particular organ is viable to be transplanted. However, there is a high-level of detail and quality information w.r.t. each medical case. Since medical decision-making is susceptible to the evidence of the information, it is not always natural to quantify the medical knowledge in a numerical way. For instance in [Szolovits, 1982], it is pointed out that the chief disadvantages of the decision theory approach are the difficulties of obtaining reasonable estimates of probabilities and utilities for a particular analysis. Although techniques such as sensitivity analysis help greatly to indicate which potential inaccuracies are unimportant, the lack of adequate data often forces artificial simplifications of the problem and lowers confidence in the outcome of the analysis.

To build a unifying framework, argumentation and evidence have been explored by different points of view [Bonet and Geffner, 1996; Krause et al., 1995; Amgoud and Prade, 2004]. However, most of the proposals suggest lack of a versatile specification language for encoding the available knowledge and the evidence involved.

The use of logic specification languages is a successful approach for encoding knowledge. In the last two decades, one of the most successful logic programming approach has been Answer Set Programming (ASP). ASP is the realization of much theoretical work on Non-monotonic Reasoning and Artificial Intelligence applications. It represents a new paradigm for logic programming that allows, using the concept of negation as failure, to handle problems with default knowledge and produce non-monotonic reasoning. The efficiency of the answer set solvers have allowed to increase the list of ASP’s practical applications e.g., planning, logical agents and Artificial Intelligence [DLV, 1996; SMODELS, 1995].

In [Nicolas et al., 2005], an extension of ASP was proposed which permits to take into account a certainty level, expressed in terms of necessity measure, on each rule of a possibilistic normal logic program. The semantics of a possibilistic nor-
nal logic program based on possibilistic stable models.

By considering the ASP’s language and a variation of the possibilistic stable models\(^1\), we present a modality based argumentation approach where the knowledge is quantified by modalities. We understand a modality as a category of linguistic meaning having to do with the expression of possibility and necessity like: possible, probable, plausible, supported and open\(^2\). Thus, the concept of modality argument is proposed where each modality argument has a quantifier that represents confidence in its conclusion. Since managing inconsistent information is a natural feature of our approach, the argumentation-based inference consists of two steps: constructing modality arguments and managing conflict between modality arguments.

Our novel modality-based argumentation approach represents one of the most expressive argumentation approach defined until now which permits to express levels of uncertainty based on modalities. Since, humans currently use arguments for explaining choices which are already made, or for evaluating potential choices, this approach contributes to the study of defining fundamental mechanisms for modeling decision making process based on arguments.

The main contributions of this paper are: 1.- The generalization of the possibilistic stable models in order to manage non-numerical uncertain degrees about the real world and to use strong negation in the specification language. 2.- The definition of a novel modality-based argumentation approach for building arguments by considering possibilistic stable models. 3.- The definition of a suitable approach for managing conflicts between modality arguments which offers suitable features for handling inconsistency information. To the best of our knowledge, our approach is the first work which considers two kinds of negations in the specification language in order to build arguments.

The rest of the paper is structured as follows: In §2, we put forward the syntax to be used. In §3, we introduce our specification language. In §4, we define a variation of the possibilistic stable models. In §5, we introduce the concept of a modality argument and define how to manage conflicts between modality arguments. In §7, we present a short overview of the related works to our approach, our future work and we outline our conclusions.

## 2 Background

A signature $\mathcal{L}$ is a finite set of elements that we call atoms. A literal is an atom, $a$, or the negation of an atom $\neg a$. The complement of a literal is defined as $\bar{a} = \neg a$ and $\neg \bar{a} = a$. Given a set of literals $\{l_1, \ldots, l_n\}$, we write $\text{not} \{l_1, \ldots, l_n\}$ in order to denote $\{\text{not}~l_1, \ldots, \text{not}~l_n\}$. An extended normal clause, $C$, is denoted by

$$l \leftarrow l_1, \ldots, l_j, \text{not}~l_{j+1}, \ldots, \text{not}~l_n$$

where $l$ is a literal and $n \geq 0$, each $l_i$ is a literal. When $n = 0$ the extended normal clause is an abbreviation of $l \leftarrow \top$, where $\top$ and $\bot$ are the ever true and ever false propositions respectively. An extended normal program is a finite set of extended normal clauses.

Sometimes, we denote a clause $C$ by $l \leftarrow B^+, \text{not}~B^-$, where $B^+$ contains all the positive body literals and $B^-$ contains all the negative body literals. $C^+$ and $C^-$ denote $l \leftarrow B^+$ and $l \leftarrow \text{not}~B^-$ respectively. We also use $\text{body}(C)$ to denote $B^+ \cup \text{not}~B^-$. When $B^+ = \emptyset$, the clause $C$ is called extended definite clause. An extended definite program is a finite set of extended definite clauses.

We denote by $\mathcal{L}_P$ the extended signature of $P$, i.e. the set of literals that occurs in $P$. We point out that we understand the negation $\neg$ as the so called classical negation (or strong negation) by the ASP’s community and the negation not as the negation as failure [Baral, 2003].

In the following sections, we assume familiarity with basic concepts in lattice theory. A good introductory treatment of the relevant concepts can be found in the text [Davey and Priestly, 2002]. Given a complete lattice $(Q, \leq)~$ and $S \subseteq Q$, $\text{LUB}(S)$ denotes the least upper bound of $S$, $\text{GLB}(S)$ denotes the greatest lower bound of $S$, $\text{TOP}_Q$ denotes the top of $Q$, and $\text{BOT}_Q$ denotes the bottom of $Q$.

## 3 Modality specifications

First of all, we present the syntax of our specification language. The basic concept of our language is a modality clause.

### Definition 1 (Modality clause)

Let $(Q, \leq)$ be a complete lattice. A modality clause is denoted by: $\text{Modality} \colon C$, where $\text{Modality} \in Q$ and $C$ is an extended normal clause. When $C$ is an extended definite clause, the modality clause is called definite modality clause.

Notice that by using a complete lattice $Q$, a modality clause categorizes the sentence which is expressed in the extended normal clause $C$. This means that a modality clause locates a sentence in the domain of $Q$.

We understand a modality as a category of certain meaning having to do with the expression of possibility. Therefore, we are categorizing a set of possibilities by a complete lattice. For instance, if $S$ is the set of labels $\{\text{Certain}, \text{Confirmed}, \text{Probable}, \text{Plausible}, \text{Supported}, \text{Open}\}$ such that the labels hold the relations expressed in Figure 1, then $S$ could be regarded as a set of modalities where each label is a possible category of beliefs.

Formally, we understand a modality logic program $P$ as a tuple of the form $\langle (Q, \leq), \text{Modality}_C \rangle$, where $(Q, \leq)$ is a complete lattice and $\text{Modality}_C$ is a set of modality clauses such that: $\forall (q : C) \in \text{Modality}_C, q \in Q$.

For instance, one possible modality logic program (in the context of medical domain) with its intuitive meaning could be described as follows (in this program, we assume the complete lattice of Figure 1):

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\(^1\)Namely, we consider a complete lattice in order to define possibilistic stable models and we extend the language by using strong negation. It is worth mentioning that according to Section 4.3 of [Dubois et al., 1994], basically for the Possibilistic Logic’s inference what is needed is a complete lattice.

\(^2\)In [Fintel, 2006] a study of the kinds of modal meaning can be found.
Figure 1: A lattice where the following relations hold:
Open ⊆ Supported, Supported ⊆ Plausible,
Supported ⊆ Probable, Probable ⊆ Confirmed,
Plausible ⊆ Confirmed, and Confirmed ⊆ Certain.

It is confirmed that the donor has been infected by streptococcus viridans.
Confirmed: dsve.

It is plausible that if the donor has been infected by streptococcus viridans, then the recipient could be infected too.
Plausible: risv ← dsve.

To define the semantics of the modality programs, we shall require to project part of the modality clauses as follows: If $r := (\text{Modality} : C)$ is a modality clause, then $r^* = C$ and $r^+ = \text{Modality}$. It is easy to see that for any modality program $P$, there is a normal/definite logic program $\Delta(P) := \{r^+ | r \in P \}$. In the case that $\forall \alpha \in \Delta(P)$, $c$ is an extended definite clause, then $P$ is called definite modality program.

4 Modality program’s semantics

In this section, the semantics of the modality programs is presented. This semantics is based on a variation of the possibilistic stable semantics presented in [Nicolas et al., 2005]. The main differences of our approach w.r.t. Nicolas’ et al. approach [Nicolas et al., 2005] are: 1.- We consider a complete lattice instead of only the interval $[0, 1]$ for representing the degree of uncertainty about the real world. Notice that, since $[0, 1]$ is a complete lattice, our approach is a generalization of Nicolas’ approach. 2.- Also we extend the language of the possibilistic stable semantics presented in [Nicolas et al., 2005] by introducing strong negation in the framework of possibilistic answer set programming.

First of all, we start defining some relevant concepts. A possibilistic literal is a pair $l = (a, q) \in L \times Q$, where $L$ is a finite set of literals and $(Q, \leq)$ is a complete lattice. We apply the projections $*$ and $\gamma$ to the possibilistic literals as follows: $l^* = a$ and $l^\gamma = q$. Given a set of possibilistic literals $S$, we define the generalization of $*$ and $\gamma$ over $S$ as follows: $S^* = \{l^* | l \in S \}$ and $S^\gamma = \{l^\gamma | l \in S \}$. Three basic operations between sets of possibilistic literals are formalized as follows:

Definition 2 Let $L$ be a finite set of literals and $(Q, \leq)$ be a finite complete lattice. Consider $A = 2^{L \times Q}$ the finite set of all the possibilistic literal sets induced by $L$ and $Q$. $\forall A, B \in A$, we define:

\[
A \cap B = \{(x, GLB\{q_1, q_2\}) | (x, q_1) \in A \land (x, q_2) \in B\}
\]

\[
A \cup B = \{(x, q) | (x, q) \in A \land x \notin B^\circ \} \cup \{(x, q) | x \notin A^\circ \land (x, q) \in B\} 
\]

\[
A \subseteq B \iff A^* \subseteq B^*, \text{ and } \forall (x, q) \in A \land (x, q_1) \in B \text{ then } q_1 \leq q_2 
\]

The following proposition is straightforward.

Proposition 1 $(A, \subseteq)$ is a complete lattice.

The semantics of the modality programs is based on its possibilistic stable models. In the case of definite modality programs, the semantics is given by the fix-point of the operator $\Pi_T$. The operator $\Pi_T$ is based on the evaluation $\text{App}(P, L, x)$ which is defined as follows:

Definition 3 Let $\tau = (\alpha : l \leftarrow l_1, \ldots, l_j)$ be an extended definite modality clause. $(Q, \leq)$ be a complete lattice such that $\alpha \in Q$ and $L$ be a set of possibilistic literals.

- $r$ is $\alpha$-applicable in $L$ if $\text{body}(r^*) = \emptyset$
- $r$ is $\beta$-applicable in $L$ if $\beta = \text{GLB}\{\alpha, \alpha_1, \ldots, \alpha_j\}$ and \((\{l_1, \alpha_1\}, \ldots, (l_j, \alpha_j)\) $\subseteq L$, $r$ is $\text{BOTQ}$-applicable otherwise.

Given an extended definite modality program $P$ and a literal $x, \text{App}(P, L, x) = \{ r \in P | \text{head}(r^*) = x, r \text{ is } v\text{-applicable in } L \}$

Basically, $\text{App}(P, L, x)$ finds the modality clauses which define the modality of the literal $x$ w.r.t. $L$. Now, the semantics of any definite modality program is defined as follows:

Definition 4 Let $P = ((Q, \leq), \text{Modality}_\text{Clauses})$ be an extended definite modality program and $L$ be a set of possibilistic literals. The operator $\Pi_T(P, L) = \{ (x, q) | x \in \text{head}(\Delta(P)), \text{App}(P, L, x) \neq \emptyset, q = \text{LUB}_{r \in \text{App}(P, L, x)} \{v | r \text{ is } v\text{-applicable in } L \} \}$

then the iterated operator $\Pi_T^n(P, L)$ is defined by $\Pi_T^0(P, L) = 0$ and $\Pi_T^{n+1}(P, L) = \Pi_T(P, \Pi_T^n(P, L))$, $\forall n \geq 0$.

The operator $\Pi_T$ behaves exactly as in [Nicolas et al., 2005]. If one conclusion is obtained by different rules, its modality is equal to the greatest certainty value which is obtained by LUB. The following proposition guarantees that the operator $\Pi_T$ always has a fix-point.

Proposition 2 Let $P$ be an extended definite modality program. Then $\Pi_T$ has at least fix-point $\Pi_T^n(P, L)$ that we called the set of possibilistic consequences of $P$ and we denote by $\Pi C n(P, L)$.

We will define the reduction of a modality program w.r.t. a set of atoms in order to define the possibilistic stable model semantics.

Definition 5 Let $P$ be a modality program and $A$ be a set of literals. The possibilistic reduction of $P$ w.r.t. $A$ is the definite modality program $P^A = \{ (r^*)^+, r^\gamma) | r \in P$ and $\text{body}^-(r^*) \cap A = \emptyset \}$
Intuitively, $P^\Delta$ is obtained from $P$ by removing all the modality clauses whose bodies have negated literals that also are in $A$ and considering only the positive part of the clauses’ bodies of the rest of the program $P$. Remember, that if $C$ is the clause $l \leftarrow B^+, \text{not } B^-$, then $C^\Delta$ denotes $l \leftarrow B^-$. By considering the reduction $P^\Delta$, the semantics of any modality program is defined as follows:

**Definition 6** Let $P$ be a modality program, and $S$ be a set of possibilistic literals. $S$ is a possibilistic stable model of $P$ if and only if $S = \PiCN(P^{(S^+)})$.

In order to illustrate the definitions, let us consider the following example.

**Example 1** Let us consider again the lattice presented in Figure 1, and the following proposition atoms: $a = \text{‘donor is HIV+’}$; $b = \text{‘the organ is viable for transplanting’}$; and $c = \text{‘the organ has correct functions and correct structure’}$. Let $P_1$ be the following single modality logic program:

\[
\text{It is probable that donor is HIV+} \\
\text{Probable: } a.
\]

It is supported that if donor is HIV+ and there is no evidence that the organ has correct functions and correct structure, then the organ is not viable for transplanting.

\[
\text{Supported: } \neg a, \text{not } c.
\]

and $S = \{(a, \text{Probable}), (\neg b, \text{Supported})\}$. It is easy to see that $P_1^{(S^+)}$ is:

\[
\text{Probable: } a. \quad \text{Supported: } \neg b \leftarrow a
\]

Then, $\PiCN(P_1^{(S^+)}) = \{(a, \text{Probable}), (\neg b, \text{Supported})\}$. Therefore, $S = \PiCN(P_1^{(S^+)})$. This means that $S$ is a possibilistic stable model of $P_1$.

5 Argumentation based inference

The argumentation-based inference procedure consists of two steps: constructing modality arguments and managing conflict between modality arguments. Thus, we shall start by defining how to build arguments from a modality program.

5.1 Building arguments

A modality argument is based on possibilistic stable modes and is defined as follows:

**Definition 7 (Modality argument)** Let $P = \langle(Q, \leq), \text{Modality Clauses} \rangle$ be a modality logic program. An argument $A$ w.r.t. $P$ is a tuple of the form $A = \langle \text{Claim}, \text{Support}, q \rangle$, such that there is a possibilistic stable model $M$ of $P$, $(\text{Claim}, q) \in M$ and the following conditions hold:

1. Support $\subseteq P$;
2. $(\text{Claim}, q) \in \PiCN(\text{Support}(M^+))$; and
3. Support is minimal w.r.t. set inclusion.

$A$ gathers all the modality arguments which can be constructed from $P$.

Notice that by definition of possibilistic stable model, $\text{Claim}$ is a literal and $q \in Q$, $q$ is consider a modality qualifier which has the objective of quantifying the level of certainty of the argument. $\text{Support}$ is the minimal subset of $P$ such that applying the possibilistic reduction of $P$ w.r.t. $M$, one infers $(\text{Claim}, q)$.

In order to illustrate the definition, let us consider the following example.

**Example 2** Let $P$ be the modality program $P_1$ from Example 1 plus the following modality clause:

\[
\text{It is confirmed that if an organ has explicitly bad functions and bad structure then the organ is not viable for transplanting} \\
\text{Confirmed: } \neg b \leftarrow \neg c.
\]

Then $P$ is:

\[
\text{Probable: } a. \\
\text{Supported: } \neg b \leftarrow a, \text{not } c. \\
\text{Confirmed: } \neg b \leftarrow \neg c.
\]

We can see that $M = \{(a, \text{Probable}), (\neg b, \text{Supported})\}$ is a possibilistic stable model of $P$. Let us build a modality argument in order to support the conclusion that the organ is not viable for transplanting $(\neg b, \text{Supported})$. Two possible sets of modality clauses are: $S_1 = \{(\text{Probable} : a), (\text{Supported} : \neg b \leftarrow \neg c)\}$ and $S_2 = \{(\text{Probable} : a), (\text{Supported} : \neg b \leftarrow a, \text{not } c)\}$. By applying the possibilistic reduction $S_1^{(M^+)}$ and the fix point $\PiCN(S_1^{(M^+)})$, it is not possible to infer $(\neg b, \text{Supported})$. Now, by applying the possibilistic reduction $S_2^{(M^+)}$ and the fix point $\PiCN(S_2^{(M^+)})$, one can see that $(\neg b, \text{Supported})$ is inferred from $S_2$. Then, a modality argument which supports $\neg b$ is: $(\neg b, \{(\text{Probable} : a), (\text{Supported} : \neg b \leftarrow a, \text{not } c)\}, \text{Supported})$.

5.2 Managing conflict between modality arguments

In the case that a rational agent’s knowledge base is inconsistent, there is a possibilistic stable model $M$ such that $\{(a, q_1), (\neg a, q_2)\} \subseteq M$, then one can construct two modality arguments of the form: $A_{q_1} = \{(\text{Support}, q_1)\}$ and $A_{q_2} = \{(\text{Support}, q_2)\}$. This means that these arguments attack each other, then there is a conflict between them. The conflicts between modality arguments are formalized by the following definitions.

**Definition 8** Let $A_{q_1}, A_{q_2} \in \mathcal{A}$ such that $A_{q_1} = \langle \text{Claim}, \text{Support}, q_1 \rangle$ and $A_{q_2} = \langle \text{Claim}, \text{Support}, q_2 \rangle$. $A_{q_1}$ attacks $A_{q_2}$ if $\text{Claim} = l$ and $\text{Claim} = \tilde{l}$.

**Definition 9** Let $A_{q_1}, A_{q_2} \in \mathcal{A}$. $A_{q_1} = \langle \text{Claim}, \text{Support}, q_1 \rangle$ undercuts $A_{q_2} = \langle \text{Claim}, \text{Support}, q_2 \rangle$ if and only if $\exists q : l \leftarrow B^+, \text{not } B^-$ is in $\text{Support}$ such that $\text{Claim} \in B^-$. Notice that, the concept of undercut is just over literals negated by negation as failure, this means that if $A_{q_2}$ undercuts $A_{q_1}$, then $A_{q_1}$ is attacking $A_{q_2}$’s assumptions. Two arguments are compared by considering their certainty levels as follows:
Definition 10 Let \( \text{Arg}_1, \text{Arg}_2 \in \mathcal{ARG} \) such that \( \text{Arg}_1 = \langle \text{Claim}_1, \text{Support}_1, q_1 \rangle \) and \( \text{Arg}_2 = \langle \text{Claim}_2, \text{Support}_2, q_2 \rangle \). \( \text{Arg}_1 \) is preferred to \( \text{Arg}_2 \) if and only if \( q_1 \geq q_2 \).

Once is identified a conflict between arguments, it is important to identify which argument wins. Then, the concept of defeat is defined as follows:

Definition 11 Let \( \text{Arg}_1, \text{Arg}_2 \in \mathcal{ARG} \) such that \( \text{Arg}_1 = \langle \text{Claim}_1, \text{Support}_1, q_1 \rangle \) and \( \text{Arg}_2 = \langle \text{Claim}_2, \text{Support}_2, q_2 \rangle \). \( \text{Arg}_2 \) defeats \( \text{Arg}_1 \) if \( \text{Arg}_1 \) attacks/undercuts \( \text{Arg}_2 \) and it is not the case that \( \text{Arg}_2 \) is preferred to \( \text{Arg}_1 \).

Notice that, if \( \text{Arg}_1 \) defeats \( \text{Arg}_2 \), then \( \text{Arg}_1 \)'s claim has a support with more evidence/certainty that \( \text{Arg}_2 \). In order to illustrate those definitions, let us consider the following example.

Example 3 Let \( S \) be an ordered set such that \( S = \{\text{certain}, \text{likely}, \text{maybe}, \text{unlikely}, \text{false}\} \) and the following relations hold: \( \text{false} \preceq \text{unlikely}, \text{unlikely} \preceq \text{maybe}, \text{maybe} \preceq \text{likely}, \text{likely} \preceq \text{certain} \). Also, let us consider the following predicates with their respective intended meanings:

\[
ele(X,O) : \text{The recipient } X \text{ is eligible for transplanting organ } O; 
\text{compatible}(X,O,L) : \text{The recipient } X \text{ is histocompatible with organ } O \text{ in a level } L; 
\text{urgency}(X,E) : \text{The recipient } X \text{ has an urgency } E \text{ in order to be transplanted}; 
\text{temperature}(X,T) : \text{The recipient } X \text{ has a temperature } T.
\]

Now, let us suppose that there are two possible recipients \( r_1 \) and \( r_2 \) and we want to assign a heart. Then, let us consider the following grounded modality program \( P \):

\[
\text{It is true that if the heart is assigned to } r_2, \text{ then } r_1 \text{ will not be eligible for transplanting and vice versa}. \\
\text{Certain: } \text{elec}(r_1, \text{heart}) \leftarrow \neg \text{elec}(r_2, \text{heart}). \\
\text{Certain: } \text{elec}(r_2, \text{heart}) \leftarrow \neg \text{elec}(r_1, \text{heart}).
\]

\[
\text{It is possible that if the receptor } r_1 \text{ has a histocompatibility with the heart, then } r_1 \text{ will be eligible for transplanting}. \\
\text{Maybe: } \text{elec}(r_1, \text{heart}) \leftarrow \text{compatible}(r_1, \text{heart, high}).
\]

\[
\text{It is true that } r_1 \text{ has a high histocompatibility with the heart}. \\
\text{Certain: } \text{compatible}(r_1, \text{heart, high}).
\]

\[
\text{It is very likely that if the receptor } r_1 \text{ is in 0-urgency, then } r_1 \text{ will be eligible for transplanting}. \\
\text{Likely: } \text{elec}(r_1, \text{heart}) \leftarrow \text{urgency}(r_1, 0-urgency)
\]

\[
\text{It is very unlikely that } r_1 \text{ will be in 0-urgency}. \\
\text{Unlikely: } \text{urgency}(r_1, 0-urgency).
\]

\[
\text{It is likely that if } r_1 \text{ has high temperature, then } r_1 \text{ will not be eligible for transplanting}. \\
\text{Likely: } \neg \text{elec}(r_1, \text{heart}) \leftarrow \text{temperature}(r_1, \text{high}).
\]

\[
\text{(It is true that the recipient } r_1 \text{ has high temperature)} \\
\text{Certain: } \text{temperature}(r_1, \text{high}).
\]

By considering the program \( P \), we want to know who is eligible for transplanting (recipient \( r_1 \) or recipient \( r_2 \)). First, we can see that the only \( P \)'s possibilistic stable model is:

\[
M = \{(\text{compatible}(r_1, \text{heart, high}), \text{Certain}), \\
(\text{urgency}(r_1, 0-urgency), \text{Unlikely}), \\
(\text{temperature}(r_1, \text{high}), \text{Certain}), \\
(\text{elec}(r_2, \text{heart}), \text{likely}),\quad (\text{elec}(r_1, \text{heart}), \text{maybe}), \\
(\neg \text{elec}(r_1, \text{heart}), \text{likely})\}.
\]

Notice that from \( M \), we can build a modality argument \( \text{Arg}_1 \) which suggests that it is expected that \( r_2 \) will be eligible for transplanting.

\[
\text{Arg}_1 = \{\text{elec}(r_2, \text{heart}), \{\text{Certain : temperature}(r_1, \text{high}), \text{likely}\}, \\
(\text{Likely : } \neg \text{elec}(r_1, \text{heart}) \leftarrow \text{temperature}(r_1, \text{high}), \text{likely}\}, \\
(\text{Certain : elec}(r_2, \text{heart}) \leftarrow \neg \text{elec}(r_1, \text{heart}), \text{likely})\}.
\]

However, it is possible to build a modality argument \( \text{Arg}_2 \) which suggests that the recipient \( r_1 \) can be eligible for transplanting because he has a high histocompatibility with the organ.

\[
\text{Arg}_2 = \{\text{elec}(r_1, \text{heart}), \{\text{Certain : compatible}(r_1, \text{heart, high}), \text{likely}\}, \\
(\text{Likely : } \neg \text{elec}(r_1, \text{heart}) \leftarrow \text{temperature}(r_1, \text{high}), \text{likely}\})\}.
\]

But, there is another argument \( \text{Arg}_3 \) which is stronger than \( \text{Arg}_2 \) and it suggests that \( r_1 \) must not be selected for transplanting because he has high temperature(fever).

\[
\text{Arg}_3 = \{\neg \text{elec}(r_1, \text{heart}), \{\text{Certain : temperature}(r_1, \text{high}), \text{likely}\}. \\
(\text{Likely : } \neg \text{elec}(r_1, \text{heart}) \leftarrow \text{temperature}(r_1, \text{high}), \text{likely})\}.
\]

This means, that \( \text{Arg}_3 \) defeats \( \text{Arg}_2 \). Therefore, the best recipient for transplanting is the recipient \( r_2 \).

Formally, we extend Dung’s approach [Dung, 1995] in order to solve the conflicts of a set of modality arguments. We will define the concept of modality argumentation framework as follows:

Definition 12 (Modality Argumentation framework)
A modality argumentation framework \( AF \) is the tuple \( AF = \langle \mathcal{ARG}, \text{Attacks}, \text{Undercuts, preferred_to} \rangle \) where \( \text{Attacks} \) contains the relations of attack between arguments, \( \text{Undercuts} \) contains the relations of undercut between arguments, and \( \text{preferred_to} \) contains the preferred relations between arguments.

In order to illustrate the definition, let us consider only the arguments of Example 3. Then \( AF_{\text{Example3}} \) is the tuple \( \{\text{Arg}_1, \text{Arg}_2, \text{Arg}_3\}, \{(\text{Arg}_2, \text{Arg}_3), (\text{Arg}_3, \text{Arg}_2)\}\) . Now, we define the notion of acceptable argument w.r.t. a set of modality arguments.

Definition 13 Let \( AF = \langle \mathcal{ARG}, \text{Attacks, Undercuts, preferred_to} \rangle \) to be a modality argumentation framework and \( S \subseteq \mathcal{ARG} \). A modality argument \( A \in \mathcal{ARG} \) is acceptable w.r.t. \( S \) (acceptable\( (A,S) \)), if \( (\forall X)((X \in \mathcal{ARG} \land (\text{defeat}(X,A))) \rightarrow (\exists Y) (Y \in S \land \text{defeat}(Y,X))) \).

For instance, let us consider \( AF_{\text{Example3}} \). If \( S = \{\text{Arg}_3\} \), then \( \text{Arg}_3 \) is acceptable w.r.t. \( S \) because the only argument which attacks \( \text{Arg}_3 \) is \( \text{Arg}_2 \), but \( \text{Arg}_3 \) defeats \( \text{Arg}_2 \). In order to define the semantics of a modality argumentation framework we also generalize the Dung’s definitions of conflict free set and admissible set.
Definition 14 Let $AF = (\mathcal{ARG}, \text{Attacks}, \text{Undercut}, \text{preferred_to})$ be a modality argumentation framework and $S \subseteq \mathcal{ARG}$. $S$ is a conflict-free if $((\forall X)(\forall Y)((X \in S \land Y \in S) \rightarrow ((X,Y) \notin \text{Attacks} \land (X,Y) \notin \text{Undercuts})).$

For example, if $S = \{Arg_2, Arg_3\}$, then $S$ is not a conflict-free set because $Arg_2$ and $Arg_3$ attack each other.

Definition 15 A conflict-free set of arguments $S$ is admissible if $((\forall X)(X \in S \rightarrow \text{acceptable}(X,S))).$

If we consider $S = \{Arg_1, Arg_2\}$, then $S$ is an admissible set. Finally, we shall present how to get the acceptable arguments from a modality argumentation framework.

Definition 16 Let $AF = (\mathcal{ARG}, \text{Attacks}, \text{Undercut}, \text{preferred_to})$ be a modality argumentation framework and $S \subseteq \mathcal{ARG}$. $S$ is a modality preferred extension if and only if it is a maximal (w.r.t. set inclusion) admissible set of $AF$.

For instance, let us consider again $AF_{Example3}$, we can see that it has four admissible set: $S_1 = \{\}$, $S_2 = \{Arg_1\}$, $S_3 = \{Arg_2\}$, and $S_4 = \{Arg_1, Arg_2\}$. The maximal admissible set w.r.t. set inclusion is $S_4$, therefore $S_4$ is a modality preferred extension of $AF_{Example3}$.

In domains of high-risk, as medical domain, it is important to infer sound information. The modality preferred semantics implies consistent information. This property is formalized with the following theorem:

Theorem 1 (Consistency Information) Let $AF = (\mathcal{ARG}, \text{Attacks}, \text{Undercut}, \text{preferred_to})$ be a modality argumentation framework and $S \subseteq \mathcal{ARG}$. If $S$ is a modality preferred extension, then the following condition holds:

- If $Cs = \{\text{Claim}|\text{Claim, Support}, q \in S\}$, then $Cs$ is a consistent set of literals.

6 Discussion

Even thought humans currently use arguments for explaining choices which are already made, or for evaluating potential choices, there are few proposals based on arguments for handling decision making where evidence/uncertainty plays a central role. In fact, we can point out three main approaches on this topic: Bonet and Geffner [Bonet and Geffner, 1996], works based on Logic of Argumentation (LA) [Krause et al., 1995] and more recently works based on the Possibilistic Logic (PL) [Amgoud and Prade, 2004]. From our point of view, all these approaches have relevant properties. However, their expressive power is quite limited.

To find a representation of the information under evidence/uncertainty has been subject of much debate. For those steeped in probability, there is only one appropriate model for numeric uncertainty, and that is probability. But probability has its problems. For one thing, the numbers are not always available. For another, the commitment to numbers means that any two events must be comparable in terms of probability: either one event is more probable than the other, or they have equal probability [Halpern, 2005]. In fact, in [McCarthy and Hayes, 1969], McCarthy and Hayes pointed out that attaching probabilities to a statement has the following objections:

1. It is not clear how to attach probabilities to statements containing quantifiers in a way that corresponds to the amount of conviction people have.
2. The information necessary to assign numerical probabilities is not ordinary available. Therefore, a formalism that required numerical probabilities would be epistemologically inadequate.

In [Carofiglio, 2004], it was proposed an interesting argumentation approach where the degree of belief in the argument’s conclusion depends on the degree of belief in the argument’s premises. This approach is so useful when the application domain permits to define probability links between premises and conclusions of an arguments.

In our modality argumentation definition, we also make a direct relation between the degree of belief in the argument’s conclusion and the degree of belief in the argument’s premises like [Carofiglio, 2004]’s approach. However, our approach does not depend of probability relations. Mainly, it takes relevance when in an application domain it is difficult to define probability relations as it is the case in the medical domain. It is important to point out that sometimes when we are using a probability approach, one of the hardest parts for solving a problem is to identify the probability relations. However, sometimes it is enough to have just relative likelihoods (modalities) for modeling different levels of evidence/uncertainty e.g. possible, probable, plausible, supported and open, where each relative likelihood is a possible world/class of believes. Also by considering a partial order ≤ for ordering the relative likelihoods, we can provide a likelihood ordering for the worlds/classes of believes (see Figure 1).

By the lack of space, it is difficult to present in this paper a long example where we could show all the features of our possibilistic approach. We are expecting to have a long version of this paper in a short term.

7 Conclusions and future work

In this paper we present an argumentation approach which has a rich specification language for encoding knowledge under imprecise or uncertain information. For instance, our approach permits to use two kinds of negation: strong negation and negation as failure, instead of only one, strong negation, as it is the case of all the known approaches. In fact, our approach is the result of the combination of a successful non-monotonic approach (Answer Set Programming [Baral, 2003]) and some standard ideas of the most representative argumentation approach e.g. Dung’s approach [Dung, 1995], LA’s approach [Krause et al., 1995], and PL’s approach [Amgoud and Prade, 2004].

Strictly speaking, our definition of modality argument is an argument in favor of a belief. Thus, if we want to give arguments in favor of a goal that a rational agent has to complete, the reader could see [Halpern, 2005], where it is presented a discussion of some of the problems to find a numerical representation for uncertainty.
it is necessary to distinguish between arguments in favor of beliefs and arguments in favor of goals. Therefore, our future work is addressed to extend our approach in order to achieve goals.

We propose a modality based argumentation approach by understanding a modality as a category of certain meaning having to do with the expression of possibility. This approach has a rich specification language based on the ASP’s language. The specification language permits to encode knowledge which expresses levels of certainty. By using a modality specification language and the concept of possibilistic stable model, we define a novel approach of modality arguments. A modality argument emphasizes in the evidence/uncertainty knowledge that support its conclusion. By considering the evidence of each argument, it is presented a conflict managing approach between modality arguments. This approach also manages the inconsistency of a knowledge base.

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References


