MR2095228 (2005f:68050) 68Q19 03B48 03C13 68Q15

Arratia, Argimiro (YV-SBOL); **Ortiz, Carlos E.**

Approximating the expressive power of logics in finite models. (English. English summary)

LATIN 2004: *Theoretical informatics*, 540–556, *Lecture Notes in Comput. Sci.*, 2976, *Springer*, *Berlin*, 2004.

Summary: "We present a probability logic (essentially a first order language extended with quantifiers that count the fraction of elements in a model that satisfy a first order formula) that captures uniform circuit classes such as AC^0 and TC^0 over arithmetic models, namely, finite structures with linear order and arithmetic relations. Furthermore, the semantics of such a logic with respect to an arithmetic model can be closely approximated by giving interpretations of formulas on finite structures where all relations (including the order) are restricted to be 'modular' (i.e. to act subject to an integer modulus). In order to give a precise measure of the proximity between satisfaction of a formula in an arithmetic model and satisfaction of the same formula in the 'approximate' model, we define the approximate formulas and work on a notion of approximate truth. We also indicate how to enhance the expressive power of our probability logic in order to capture polynomial time decidable queries.

"There are various motivations for this work. As of today, there is no known logical description of any computational complexity class below NP which does not requires a built-in linear order. Also, it is widely recognized that many model theoretic techniques for showing definability in logics on finite structures become almost useless when order is present. Hence, if we want to obtain significant lower bound results in computational complexity via logical description we ought to find ways of by-passing the ordering restriction. With this work we take steps towards understanding how well we can approximate, without a true order, the expressive power of logics that capture complexity classes on ordered structures."

{For the entire collection see MR2093690 (2005d:68010)}

MR1937766 (2003i:68034) 68Q15 03B70 03C13 03D15

Arratia, Argimiro A. (YV-SBOL); **Stewart, Iain A.** (4-LSTR-CS)

A note on first-order projections and games. (English. English summary)

Theoret. Comput. Sci. **290** (2003), *no.* 3, 2085–2093.

The paper introduces a general technique for obtaining PSPACE-complete problems via certain games on finite first-order structures.

Suppose we have a first-order relational structure A and a labeling alphabet L. Let $\neg L = \{\neg l \mid l \in L\}$. A labeled structure is a pair (A, λ) where λ assigns to each tuple $\vec{u}$ in a relation R of A at most one label from $L \cup \neg L$.

The game is then played as follows. Player I chooses a label $l \in L$, and erases this label from (A, λ); moreover, he removes every tuple labeled by $\neg l$. A move by Player II is symmetric, with the roles of l and $\neg l$ reversed. If Ω is a class of unlabeled structures, we denote by $LG(\Omega)$ the set of labeled structures such that Player I has a strategy guaranteeing that, after all labels are removed, the resulting structure is in Ω.

The main result of the paper states that under some mild conditions on Ω (there is a first-order projection from transitive closure to Ω), the problem $LG(\Omega)$ is PSPACE-complete under logspace reductions.

Leonid Libkin (3-TRNT-C)

MR1950449 (2003m:68051) 68Q19

Arratia, Argimiro (YV-SBOL)

A uniform presentation of the theory of descriptive computational complexity. (Spanish. English, Spanish summary)

Acta Cient. Venezolana **53** (2002), *no.* 2, 94–118.

Summary: "The theory of descriptive computational complexity deals with computational complexity from the perspective of logic. Among its main goals is the logical characterization of computational complexity classes, traditionally defined in terms of resource bounded Turing machines. The presentation often found of this theory in the current literature follows its historical development; hence, for each particular computational complexity class one finds a particular translation of the associated computational model into the syntactic elements belonging to the logic intended for describing the complexity class. This long and arduous intellectual job can be simplified with a scheme that links a time or space complexity class with a formal language, which requires learning just a single translation of a partic-

ular Turing machine model (namely, the logarithmic space bounded deterministic Turing machine) into formulas of a particular logic (the extension of first order logic with the deterministic transitive closure operator). It is this uniform version of the theory of descriptive computational complexity which we present in this paper."

MR1897179 (2003c:68089) 68Q19 03C80 91A43
Arratia, Argimiro (YV-SBOL)
On the descriptive complexity of a simplified game of Hex. (English. English summary)
Log. J. IGPL **10** (2002), *no.* 2, 105–122.
The goal of descriptive complexity is to provide logical characterizations of complexity classes, as opposed to resource (time/space) based characterizations. The paper under review presents such a characterization for the class PSPACE using first-order logic (FO) extended with a generalized quantifier for a weak version of the game Hex.

The game of Hex is played on a directed graph $G = (A, E)$, where A is the set of nodes, and E is the set of edges. Two nodes, c and d, are fixed. There are two players, I and II, who color nodes in alternating steps. Player I begins by coloring a node red, then player II picks another node and colors it blue, player I picks an uncolored node and colors it red and so on. When the graph is colored, player I wins if there is a path from c to d that consists of red nodes only.

Hex—that is, checking if player I has a winning strategy—has been known to be complete for PSPACE via logspace reductions. In the paper, the author considers a weak version of Hex, called WHEX, and shows that it is complete for PSPACE under a much more restricted class of reductions, namely, quantifier-free projections. WHEX differs from HEX as follows. In the first round, player I starts by picking a node adjacent to c and coloring it red. Suppose in round i, player I picks a node a and colors it red. Then player II must pick an uncolored node b such that $(a, b) \in E$ and color it blue. Then, in round $i+1$, player I picks an uncolored node b' such that $(a, b') \in E$, and colors it red. Intuitively, player I is trying to construct a red path from c to d, and player II is trying to block him.

In addition to proving the completeness of WHEX, the author studies a logic obtained from FO, with a successor relation on the universe, by adding a generalized quantifier for testing whether player I has a winning strategy in WHEX. Such a logic captures the class PSPACE, and enjoys a nice normal form property, in the spirit of normal forms for transitive closure and fixed point logics. Finally, applications to abstract proof systems (modeled as structures with a ternary relation and constants) are considered.

Leonid Libkin (3-TRNT-C)

MR1893554 03C13 03H15

Arratia, Argimiro (YV-SBOL); **Ortiz, Carlos E.**

Nonstandard analysis methods in finite model theory. (Spanish. English, Spanish summary)

Papers from the XIIth Venezuelan Mathematics Conference (Spanish) (Caracas, 1999).

Acta Cient. Venezolana **52** (2001), *suppl.* 2, 2–4.

MR1765053 (2001j:68047) 68Q19 03-02 03C13 03C80 68-02

Arratia, Argimiro A. (YV-SBOL)

★**Introducción a la teoría descriptiva de la computabilidad. (Spanish) [Introduction to descriptive computability theory]**

Universidad de Los Andes, *Facultad de Ciencias*, *Departamento de Matemáticas*, *Mérida*, 2000. *vi*+109 *pp.* *ISBN* 980-261-060-7

This book surveys some of the theory of descriptive complexity, which deals with logical descriptions of complexity classes.

It is divided into six chapters, a bibliography and an index.

Chapter 1 surveys the basics of complexity theory. Chapter 2 gives some results on first-order logic and finite model theory and expressing complexity classes in a first-order language. Chapter 3 treats generalized quantifiers (introduced by P. Lindström [Theoria **32** (1966), 186–195; MR0244012 (39 #5329)]), which can be used to create extensions of first-order logic which capture the classes L, NL and P [see N. Immerman, SIAM J. Comput. **16** (1987), no. 4, 760–778; MR0899700 (88j:68051)]. In Chapter 4 Arratia-Quesada introduces second-order logic and fixed-point logics, which capture NP [see R. Fagin, in *Complexity of computation (Proc. SIAM-AMS Sympos. Appl. Math., New York, 1973)*, 43–73, SIAM-AMS Proc., VII, Amer. Math. Soc., Providence, R.I., 1974; MR0371622 (51 #7840)] and PSPACE [see S. Abiteboul, M. Y. Vardi and V. Vianu, J. ACM **44** (1997), no. 1, 30–56; MR1438464 (98a:68061)]. Chapter 5 discusses methods to determine definability (variations of Ehrenfeucht-Fraïssé games and zero-one laws). Chapter 6 is entitled "P and the dilemma of order", which deals with the (still open) problem of finding a logic for P.

This book is a good introduction for Spanish readers to the study of descriptive complexity.

Ricardo Bianconi (São Paulo)

MR1823264 (2002d:11105) 11M06 11Y60

Arratia, Argimiro (YV-SBOL)

Some youthful ways of evaluating $\zeta(2k)$. (Spanish)

Bol. Asoc. Mat. Venez. **6** (1999), *no.* 2, 167–176.

The present article describes three methods for calculating the values of the Riemann ζ-function at even positive integers.

The well-known first method uses the Fourier series of the even real function $x \to x^{2k}$ and, setting $x=\pi$, delivers a recursion formula, in which $\zeta(2k)$ is expressed by the values of all $\zeta(2k'),\ k'<k$.

The second method, though also elementary, is very nice. From $\sin x \le x \le \tan x\ (0 \le x \le \pi/2)$ one obtains, with $x = x_n = (n/(2m+1))\pi\ (1 \le n \le m)$, the following chain of inequalities, valid for all $k \in \mathbf{N}$:

$$\left(\frac{\pi}{2m+1}\right)^{2k}(\cot^2(x_n))^k \le \frac{1}{n^{2k}} \le \left(\frac{\pi}{2m+1}\right)^{2k}(\cot^2(x_n)+1)^k.$$

Here the values $\cot^2(x_n)$ prove to be the roots a_n of a certain polynomial $p(x)$ of degree m. Summing over $n,\ 1 \le n \le m$, yields a two-sided estimation of the partial sum of $\zeta(2k)$; on both sides (on the right-hand side after a binomial expansion) one may express the sum $\sum_n \alpha_n^k$ of the kth powers of the α_n, using formulae due to Newton, in terms of the coefficients of the polynomial p. Taking $m \to \infty$, lower and upper estimates tend to the same limit and so give the desired value for $\zeta(2k)$.

The third method is of an analytical nature; it starts with the (elementary) transformations

$$\zeta(2) = \frac{4}{3}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2} = \frac{4}{3}\int_0^1\int_0^1 \frac{1}{1-x^2y^2}dxdy = \frac{4}{3}\int_1^{\infty}\frac{\log u}{u^2-1}du$$

and then evaluates the last integral by complex contour integration.

The author remarks that none of the above-mentioned methods applies to the cases of odd k.

{Reviewer's remark: Beukers, in his proof of the irrationality of $\zeta(3)$, succeeds in (successfully) estimating the real triple integral $\int_0^1\int_0^1\int_0^1(1/(1-x^2y^2z^2))dxdydz$.} *Gunter Dufner* (Landau)

MR1737173 (2001h:68050) 68Q19 03C13 03D15

Arratia-Quesada, Argimiro A. (YV-SBOL); **Chauhan, Savita R.** (4-WALS-C); **Stewart, Iain A.** (4-LSTR-CS)

Hierarchies in classes of program schemes. (English. English summary)

J. Logic Comput. **9** (1999), *no.* 6, 915–957.

The authors study program schemes from the point of view of descriptive complexity. Such an approach was undertaken at the early stage of the study of descriptive complexity by several authors [see, e.g., D. Harel and D. Peleg, Inform. and Control **60** (1984), no. 1-3, 86–102; MR0764280 (86h:68110)]. Since then, however, the theory evolved toward "static logic", that is, the extension of first-order logic by various kinds of induction operators or higher-order quantifiers, rather than by program schemes. These authors advocate the use of program schemes as, in their own words, "a model of computation that is amenable to logical analysis yet is closer to the general notion of a program than a logical formula is". The main results concern strictness of the hierarchies within two classes of program schemes that, in descriptive complexity, correspond to the levels NL and P, respectively. As the authors explain, these hierarchies were motivated by the earlier results by Grädel and McColm concerning hierarchies within transitive closure logic (without a built-in ordering relation). The authors emphasize that their proofs are based on combinatorial arguments, and do not use Ehrenfeucht-Fraïssé games, which is the most common technique in the separation results in finite model theory. *Damian Niwiński* (PL-WASW-I)

MR1474140 (98d:68072) 68Q15 03C13

Arratia-Quesada, A. A. (1-WI-CS); **Stewart, I. A.** (4-LSTR-CS)

Generalized hex and logical characterizations of polynomial space. (English. English summary)

Inform. Process. Lett. **63** (1997), *no.* 3, 147–152.

Summary: "We extend a logical characterization of PSPACE due to Makowsky and Pnueli by showing that their logic has a particular normal form which implies that the generalized hex problem is complete for PSPACE via very restricted logical reductions. We also show that this normal form result fails in the absence of a built-in successor relation."