Tower of Hanoi

Mathematical game invented by the French mathematician Edouard Lucas on 1883.

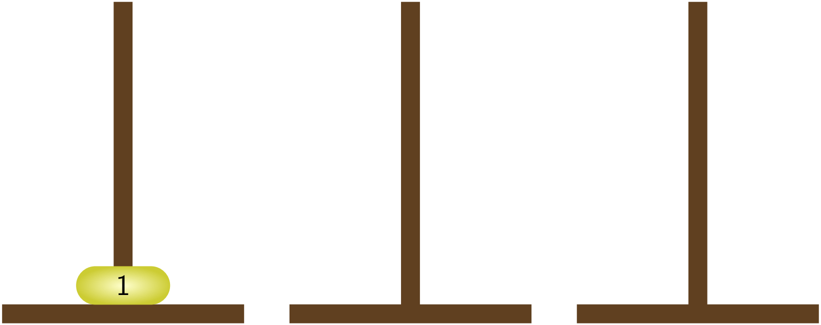
It consists of three rods and a number of disks of various diameters, which can slide onto any rod. The puzzle begins with the disks stacked on one rod in order of decreasing size, the smallest at the top.

The objective of the puzzle is to move the entire stack to another rod, obeying the following rules:

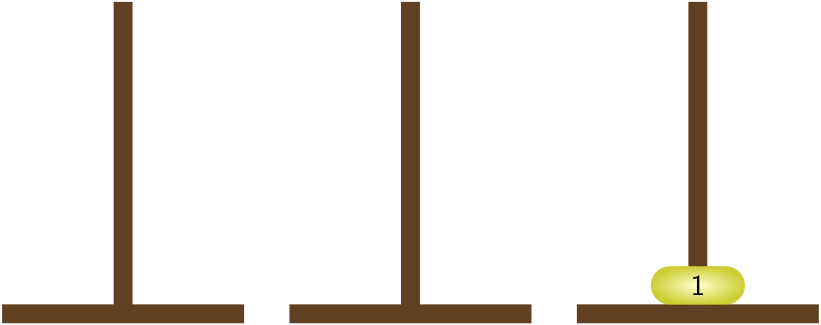
- Only one disk may be moved at a time.
- Each move consists of taking the upper disk from one of the stacks and placing it on top of another stack or on an empty rod.
- No disk may be placed on top of a disk that is smaller than it.

[Source: wikipedia]

Tower of Hanoi – 1 Disc
[created by M. Hofmann and B. Damman]

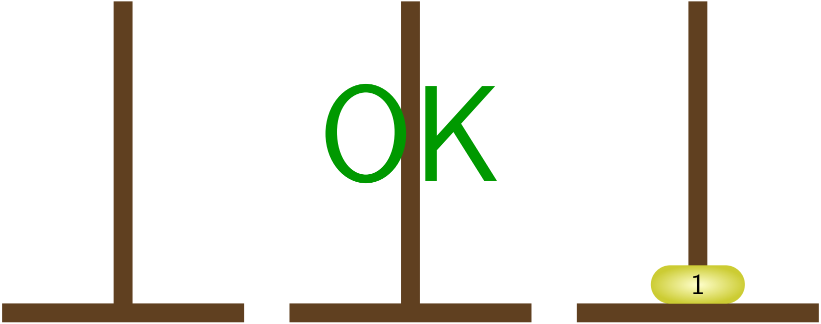


Tower of Hanoi – 1 Disc [created by M. Hofmann and B. Damman]

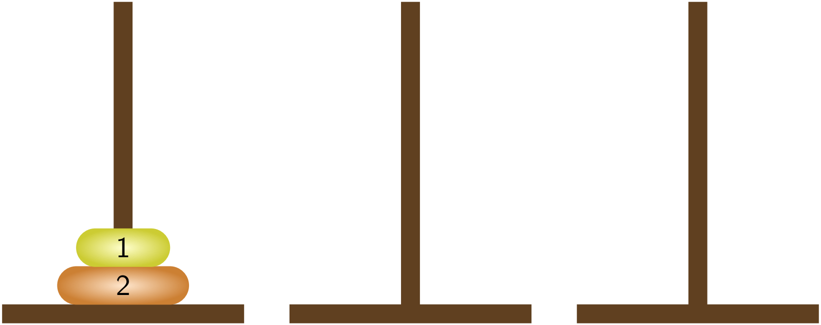


Moved disc from pole 1 to pole 3.

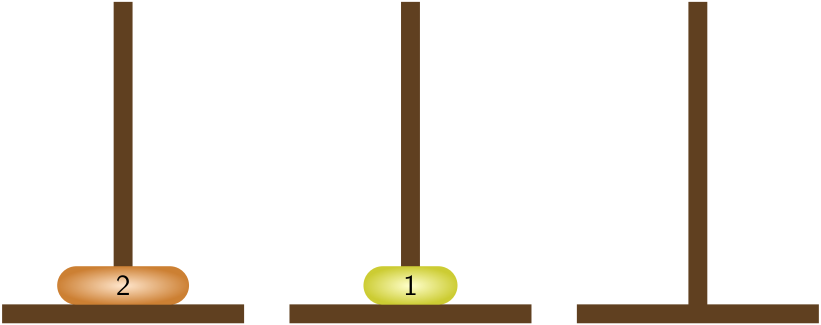
Tower of Hanoi – 1 Disc
[created by M. Hofmann and B. Damman]



Tower of Hanoi – 2 Discs [created by M. Hofmann and B. Damman]

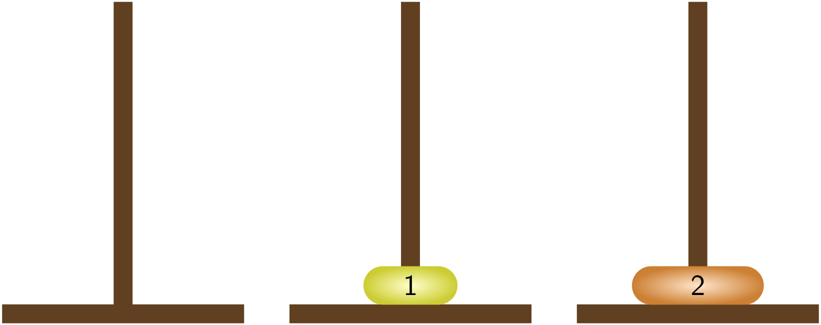


Tower of Hanoi – 2 Discs [created by M. Hofmann and B. Damman]



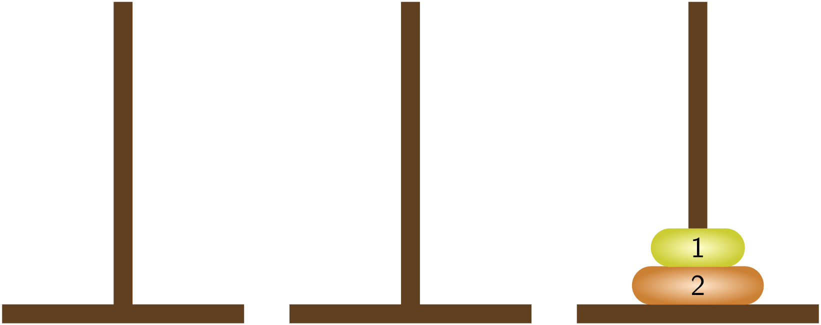
Moved disc from pole 1 to pole 2.

Tower of Hanoi – 2 Discs [created by M. Hofmann and B. Damman]



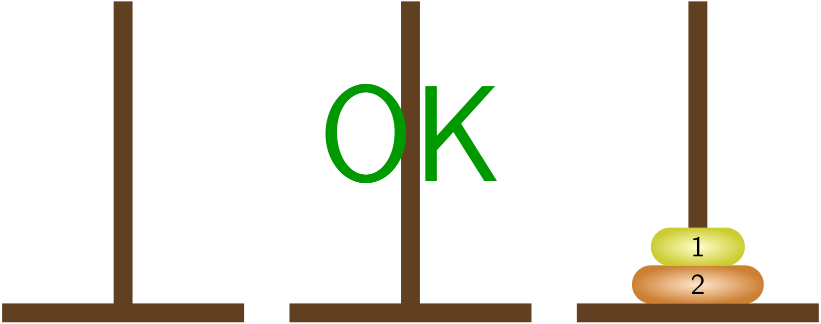
Moved disc from pole 1 to pole 3.

Tower of Hanoi – 2 Discs [created by M. Hofmann and B. Damman]

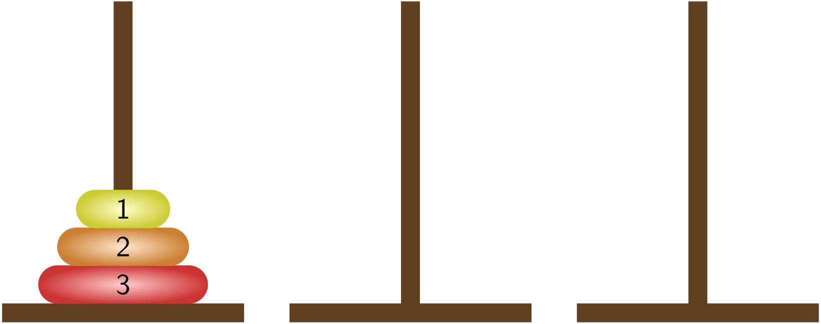


Moved disc from pole 2 to pole 3.

Tower of Hanoi – 2 Discs
[created by M. Hofmann and B. Damman]



Tower of Hanoi – 3 Discs [created by M. Hofmann and B. Damman]



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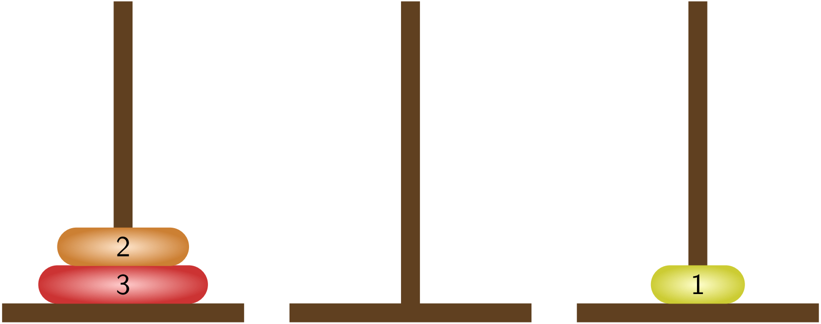
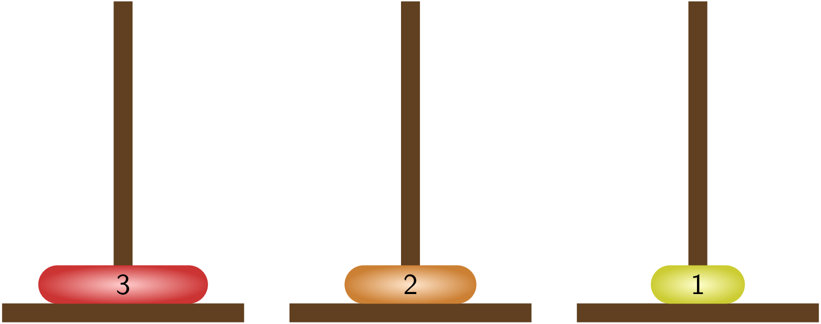


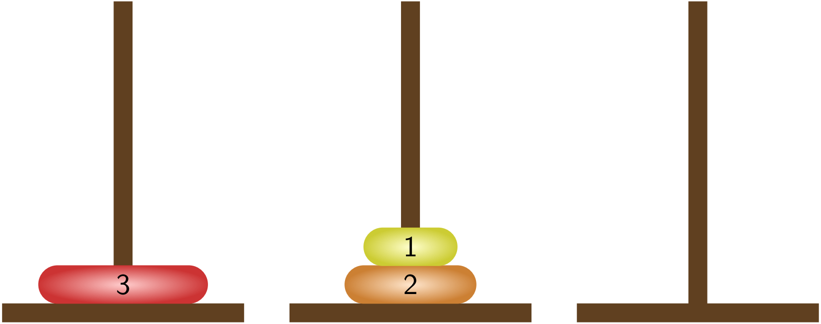
Moved disc from pole 1 to pole 3.

Tower of Hanoi – 3 Discs [created by M. Hofmann and B. Damman]



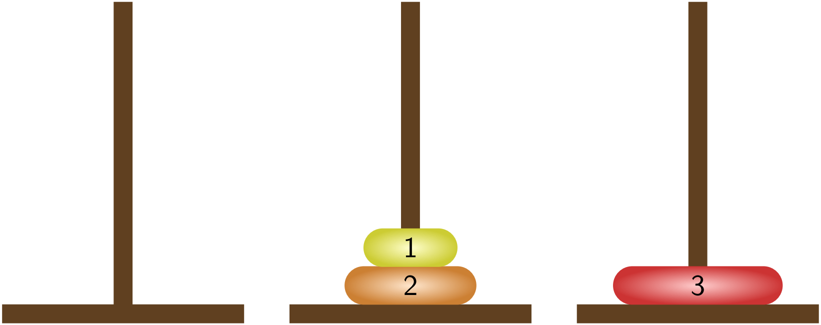
Moved disc from pole 1 to pole 2.

Tower of Hanoi – 3 Discs [created by M. Hofmann and B. Damman]



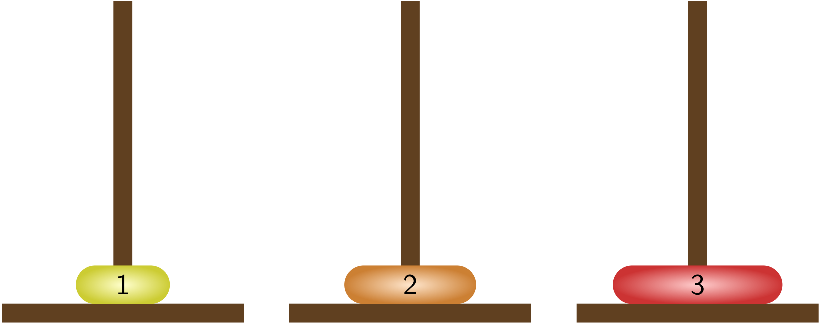
Moved disc from pole 3 to pole 2.

Tower of Hanoi – 3 Discs [created by M. Hofmann and B. Damman]



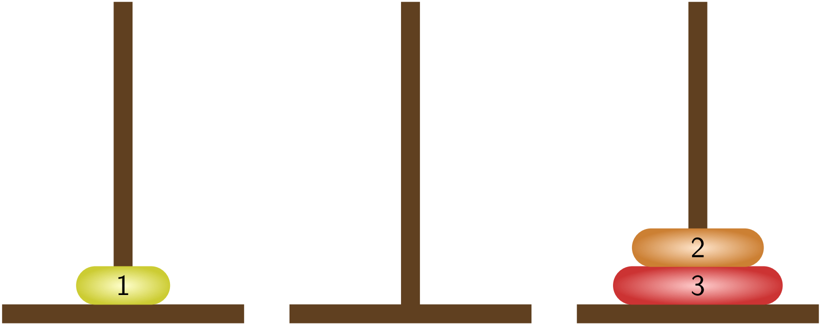
Moved disc from pole 1 to pole 3.

Tower of Hanoi – 3 Discs [created by M. Hofmann and B. Damman]



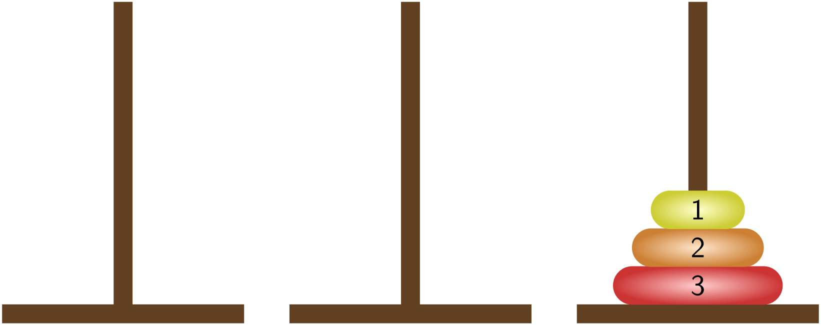
Moved disc from pole 2 to pole 1.

Tower of Hanoi – 3 Discs [created by M. Hofmann and B. Damman]



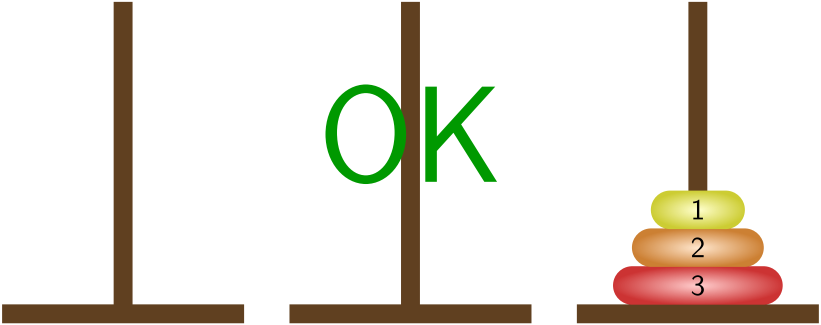
Moved disc from pole 2 to pole 3.

Tower of Hanoi – 3 Discs [created by M. Hofmann and B. Damman]

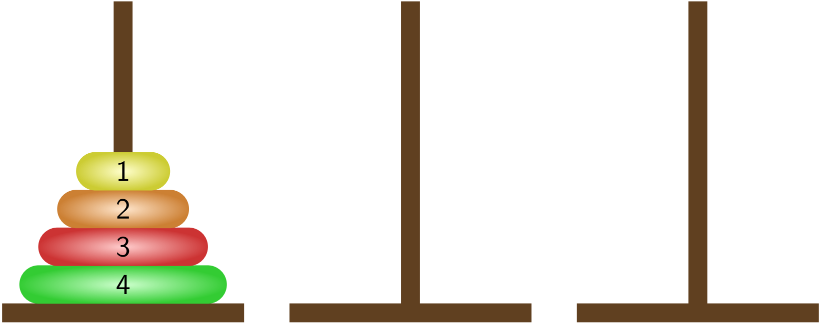


Moved disc from pole 1 to pole 3.

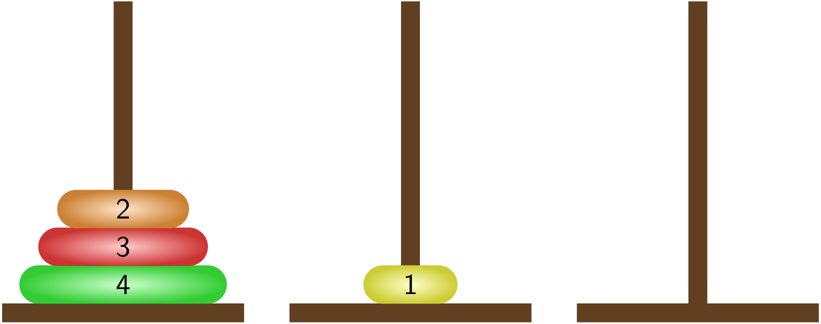
Tower of Hanoi – 3 Discs [created by M. Hofmann and B. Damman]



Tower of Hanoi – 4 Discs
[created by M. Hofmann and B. Damman]

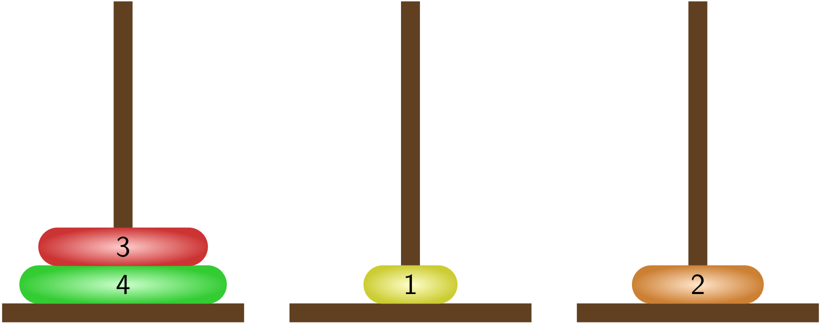


Tower of Hanoi – 4 Discs [created by M. Hofmann and B. Damman]



Moved disc from pole 1 to pole 2.

Tower of Hanoi – 4 Discs [created by M. Hofmann and B. Damman]



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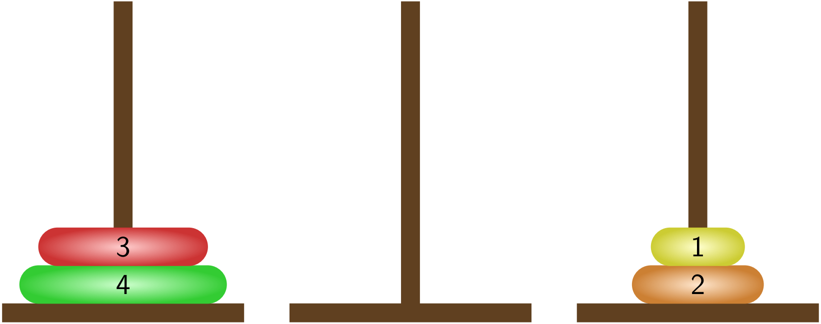
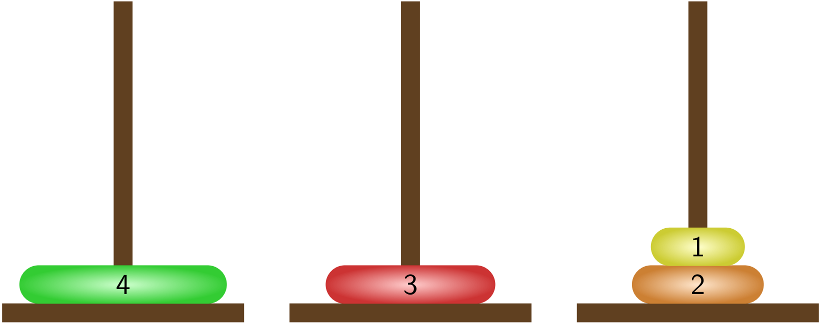
Tower of Hanoi – 4 Discs [created by M. Hofmann and B. Damman]



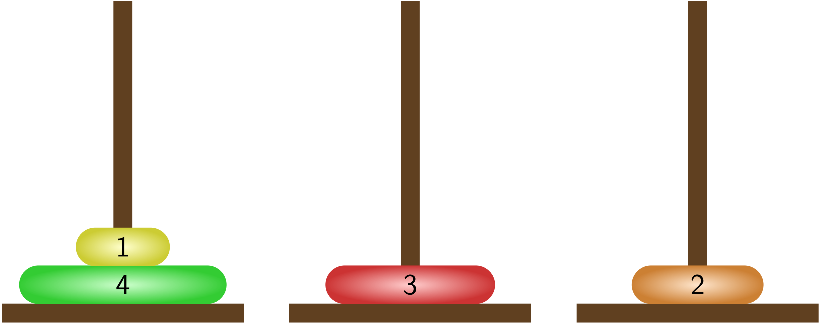
Moved disc from pole 2 to pole 3.

Tower of Hanoi – 4 Discs [created by M. Hofmann and B. Damman]



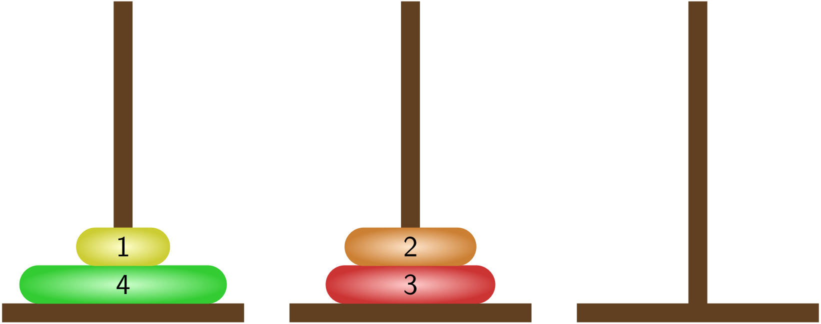
Moved disc from pole 1 to pole 2.

Tower of Hanoi – 4 Discs [created by M. Hofmann and B. Damman]



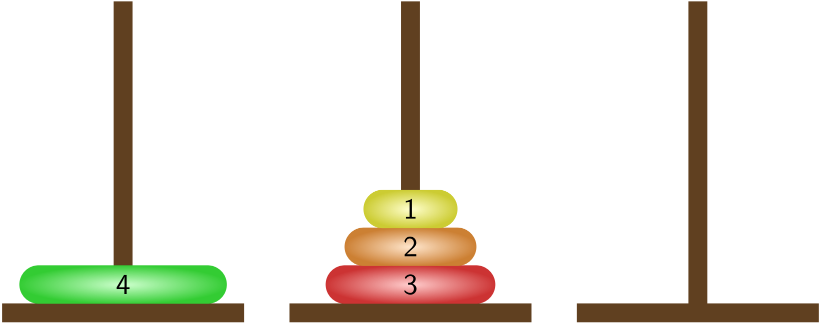
Moved disc from pole 3 to pole 1.

Tower of Hanoi – 4 Discs [created by M. Hofmann and B. Damman]



Moved disc from pole 3 to pole 2.

Tower of Hanoi – 4 Discs [created by M. Hofmann and B. Damman]



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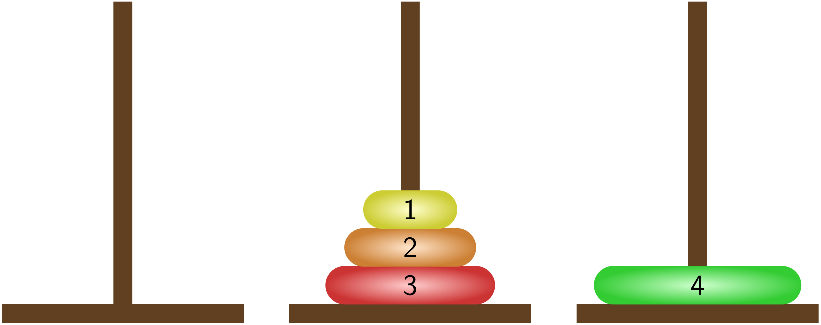
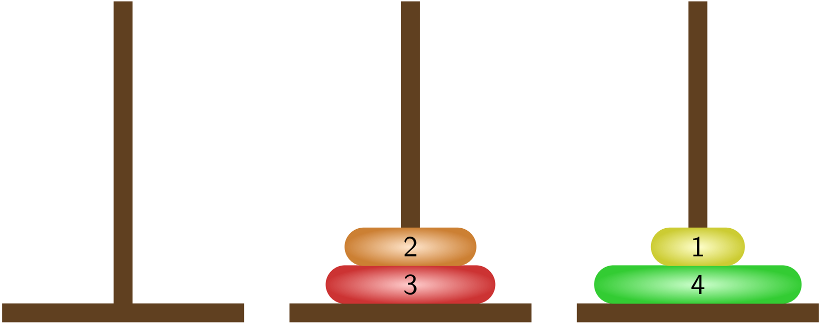


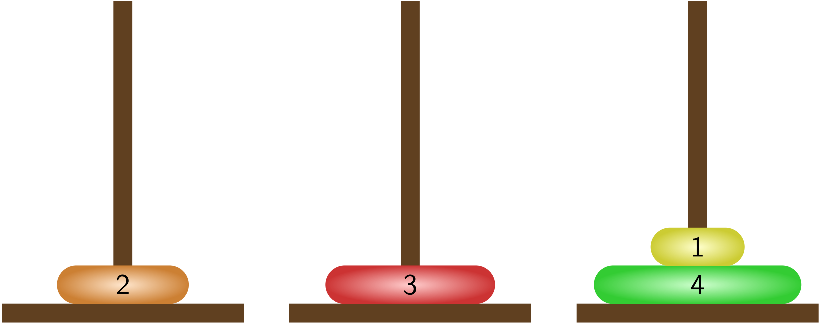
Moved disc from pole 1 to pole 3.

Tower of Hanoi – 4 Discs [created by M. Hofmann and B. Damman]



Moved disc from pole 2 to pole 3.

Tower of Hanoi – 4 Discs [created by M. Hofmann and B. Damman]



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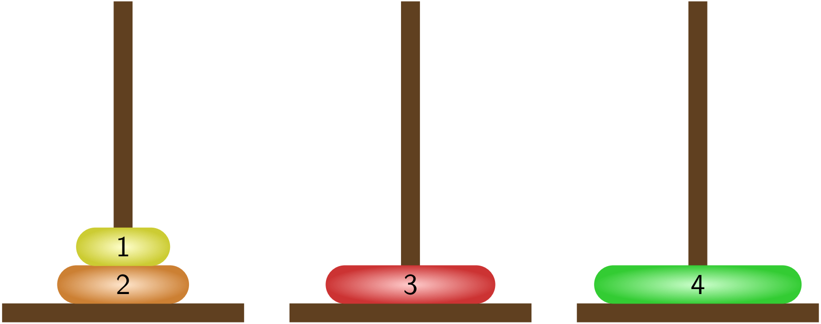
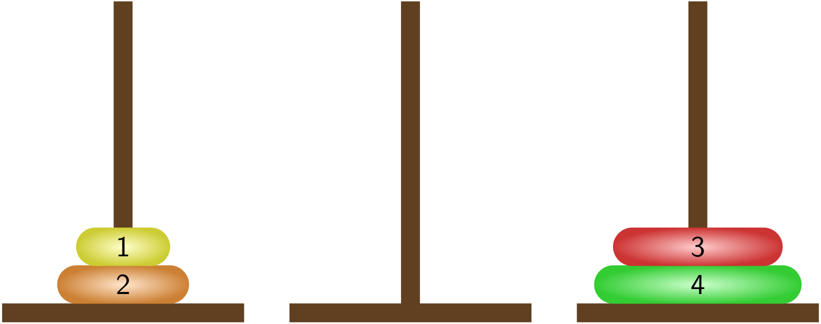


Moved disc from pole 3 to pole 1.

Tower of Hanoi – 4 Discs [created by M. Hofmann and B. Damman]



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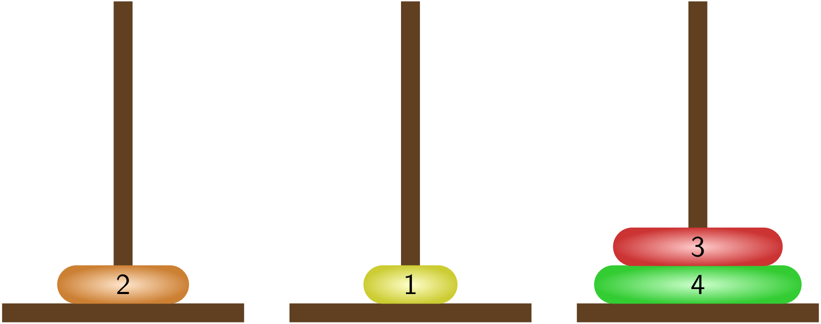

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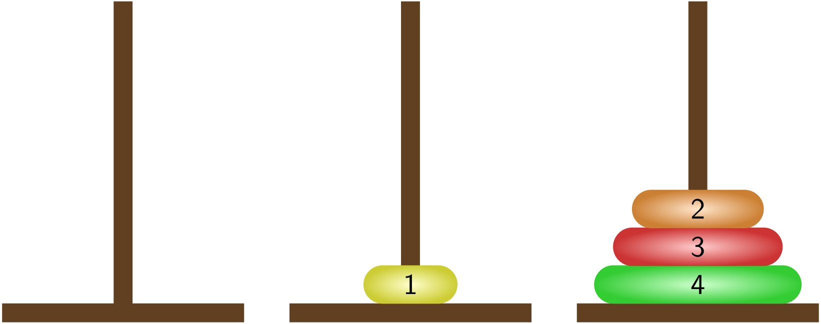

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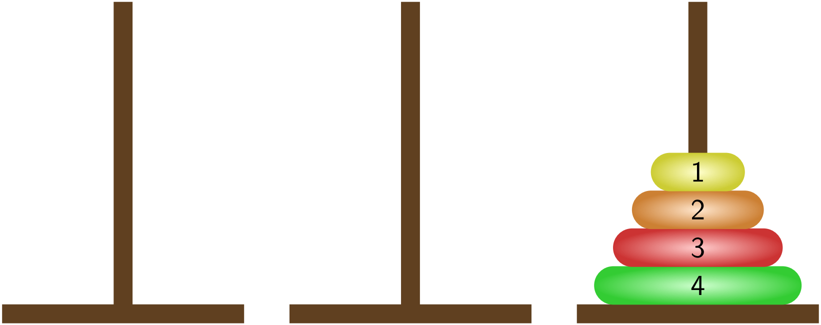
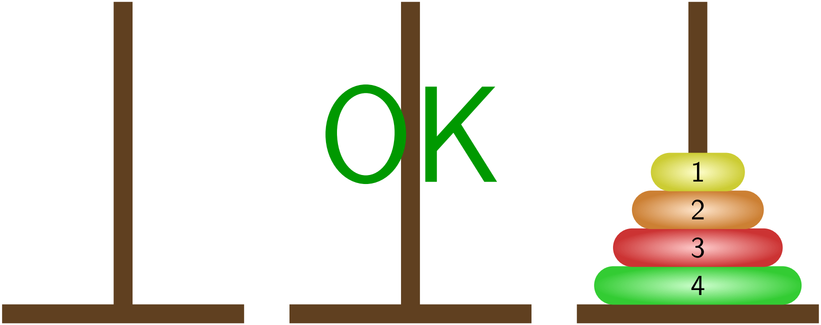
Tower of Hanoi – 4 Discs [created by M. Hofmann and B. Damman]

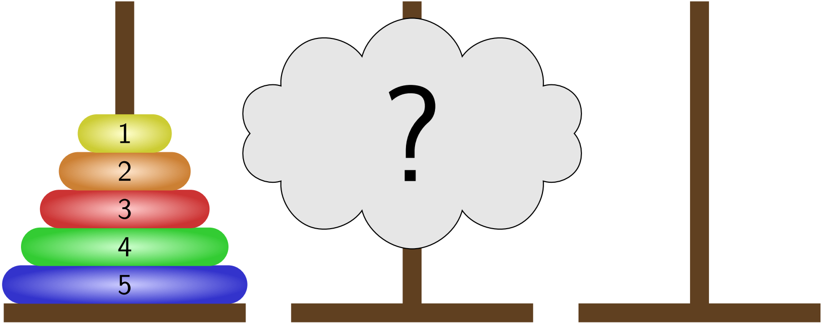


Moved disc from pole 2 to pole 3.

Tower of Hanoi – 4 Discs
[created by M. Hofmann and B. Damman]



Tower of Hanoi – 5 Disc
[created by M. Hofmann and B. Damman]

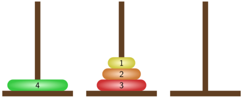


Tower of Hanoi – Inductive reasoning

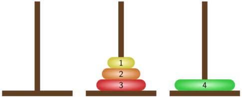
Original problem → Hanoi with n discs

Steps to solve it:

1. Hanoi with n - 1 discs:



2. Move disk:



3. Hanoi with n - 1 discs:

Tower of Hanoi – The code

```
// Pre: n >= 0
// Post: moves n disks from —> to using spare
void hanoi(int n, char from, char to, char spare) {
    if (n > 0) {
        hanoi(n - 1, from, spare, to);
        cout << from << " —> " << to << endl;
        hanoi(n - 1, spare, to, from);
    }
}

int main() {
    int n;
    cin >> n;
    hanoi(n, 'L', 'R', 'M');
}
```

Tower of Hanoi – Number of moves

$$moves(n) = \begin{cases} 1 + 2 \times moves(n-1) & , n > 0 \\ 0 & , n = 0 \end{cases}$$

n	moves(n)
1	1
2	3
3	7
4	15
5	31
6	63
...	...
n	$2^n - 1$

Tower of Hanoi – The myth

There is an Indian temple in Kashi Vishwanath containing a large room with three time-worn posts in it, surrounded by 64 golden disks. When the priests take the last move, the world ends.

Let us suppose the priests move one disk per second, then ...

n	time
1	1 s
5	31 s
10	17 m 3 s
15	9 h 6 m 7 s
20	12 d 3 h 16 m 15 s
...	...
60	$>$ 36,000,000,000 y
...	...
64	$\approx$ 585,000,000,000 y[1]

[1] 42 times the estimated current age of the universe