Nondeterministic Discontinuous Lambek Calculus*

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Abstract

The search for a full treatment of wrapping in type logical grammar (TLG) has been a task of long-standing. In this paper we present the (nondeterministic generalized) discontinuous Lambek calculus **DL**, which we believe constitutes a general and natural extension of the Lambek calculus. Like the Lambek calculus it is a sequence logic without structural rules. Although there is only space to illustrate some of them here, there are linguistic applications to medial extraction, discontinuous idioms, parentheticals, gapping, complement alternation, particle shift, VP ellipsis, reflexivization, quantification, pied-piping, appositive relativisation, comparative subdeletion and cross-serial dependencies.

'Wrapping' was introduced in Yngve (1960, p.448)[18]. Here we develop the wrapping approach to discontinuity, 'syntax-semantics mismatch', in TLG. As basic TLG we define **L**, by which we mean one of the slightly differing varieties of the systems of Lambek (1958, 1988)[4][3].

(1) **Definition** (*basic prosodic algebra*). A *basic prosodic algebra* is an algebra $(L, +, 0)$ of arity $(2, 0)$ which is a free monoid. I.e. L is a set, $0 \in L$, and $+$ is a binary operation on L such that for all $s_1, s_2, s_3, s \in L$,

$$\begin{array}{rcll} s_1+(s_2+s_3) & = & (s_1+s_2)+s_3 & \text{associativity} \\ 0+s = & s & = s+0 & \text{identity} \end{array}$$

Furthermore, up to associativity every element of L has a unique factorization into primes (freeness).[1]

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[1] Factors of an element s are elements $s_1, \ldots, s_n$ such that $s = s_1 + \cdots + s_n$; a prime is an element which has no factors other than itself and 0.

(2) **Definition** (*types of* **L**). The set $\mathcal{F}$ of *types* of **L** is defined on the basis of a set $\mathcal{P}$ of primitive basic types as follows:

$$\mathcal{F} ::= \mathcal{P} \mid \mathcal{F}\bullet\mathcal{F} \mid \mathcal{F}\backslash\mathcal{F} \mid \mathcal{F}/\mathcal{F}$$

(3) **Definition** (*prosodic interpretation of* **L**). A *prosodic interpretation of* **L** is a function $[\![\cdot]\!]$ mapping each type $A \in \mathcal{F}$ into a subset of L such that:

$$\begin{array}{rcl} [\![A\backslash C]\!] & = & \{s_2 \mid \forall s_1 \in [\![A]\!], s_1+s_2 \in [\![C]\!]\} \\ [\![C/B]\!] & = & \{s_1 \mid \forall s_2 \in [\![B]\!], s_1+s_2 \in [\![C]\!]\} \\ [\![A\bullet B]\!] & = & \{s_1+s_2 \mid s_1 \in [\![A]\!] \ \&\ s_2 \in [\![B]\!]\} \end{array}$$

Observe that $(\backslash, \bullet, /; \subseteq)$ constitutes a residuated triple, i.e.

$$B \subseteq A\backslash C \text{ iff } A\bullet B \subseteq C \text{ iff } A \subseteq C/B$$

(4) **Definition** (*configurations and sequents of* **L**). The set $\mathcal{O}$ of *configurations* of **L** is defined as follows, where Λ is the (metalinguistic) empty string:

$$\mathcal{O} ::= \Lambda \mid \mathcal{F}, \mathcal{O}$$

The set Σ of *sequents* of **L** is defined as follows:

$$\Sigma ::= \mathcal{O} \Rightarrow \mathcal{F}$$

(5) **Definition** (*prosodic interpretation of configurations and validity of sequents in* **L**). We extend the interpretation of types to include configurations as follows:

$$\begin{array}{rcl} [\![\Lambda]\!] & = & \{0\} \\ [\![A,\Gamma]\!] & = & \{s_1+s_2 \mid s_1 \in [\![A]\!] \ \&\ s_2 \in [\![\Gamma]\!]\} \end{array}$$

A sequent $\Gamma \Rightarrow A$ is *valid* iff $[\![\Gamma]\!] \subseteq [\![A]\!]$ in every interpretation.

The sequent calculus for **L** is defined in figure 1.

(6) **Definition** (*semantic types*). The set $\mathcal{T}$ of semantic types is defined on the basis of a set δ of primitive semantic types by:

$$\mathcal{T} ::= \delta \mid \mathcal{T}\&\mathcal{T} \mid \mathcal{T} \to \mathcal{T}$$

(7) **Definition** (*semantic frame*). A *semantic frame* is a $\mathcal{T}$-indexed family of sets $\{D_\tau\}_{\tau\in\mathcal{T}}$ such that:

$$\begin{array}{rcl} D_{\tau_1\&\tau_2} & = & D_{\tau_1} \times D_{\tau_2} \\ D_{\tau_1\to\tau_2} & = & D_{\tau_2}^{D_{\tau_1}} \end{array}$$

$$\frac{}{A \Rightarrow A}\, id \qquad \frac{\Gamma \Rightarrow A \qquad \Delta(A) \Rightarrow B}{\Delta(\Gamma) \Rightarrow B}\, Cut$$

$$\frac{\Gamma \Rightarrow A \qquad \Delta(C) \Rightarrow D}{\Delta(\Gamma, A\backslash C) \Rightarrow D}\backslash L \qquad \frac{A, \Gamma \Rightarrow C}{\Gamma \Rightarrow A\backslash C}\backslash R \qquad \frac{\Gamma \Rightarrow B \qquad \Delta(C) \Rightarrow D}{\Delta(C/B, \Gamma) \Rightarrow D}/L \qquad \frac{\Gamma, B \Rightarrow C}{\Gamma \Rightarrow C/B}/R$$

$$\frac{\Delta(A, B) \Rightarrow D}{\Delta(A\bullet B) \Rightarrow D}\bullet L \qquad \frac{\Gamma \Rightarrow A \qquad \Delta \Rightarrow B}{\Gamma, \Delta \Rightarrow A\bullet B}\bullet R$$

Figure 1: The *sequent calculus* for **L**, where $\Delta(\Gamma)$ indicates a configuration Δ with a distinguished subconfiguration Γ.

(8) **Definition** (*semantic terms*). The sets Φ_τ of *semantic terms* of type τ for each type τ are defined on the basis of a set C_τ of constants of type τ and an enumerably infinite set V_τ of variables of type τ for each type τ as follows:[2]

$$\begin{array}{rcl}
\Phi_\tau & ::= & C_\tau \mid V_\tau \mid (\Phi_{\tau' \to \tau}\ \Phi_{\tau'}) \mid \pi_1 \Phi_{\tau \& \tau'} \mid \pi_2 \Phi_{\tau' \& \tau} \\
\Phi_{\tau \to \tau'} & ::= & \lambda V_\tau \Phi_{\tau'} \\
\Phi_{\tau \& \tau'} & ::= & (\Phi_\tau, \Phi_{\tau'})
\end{array}$$

(9) **Definition** (*semantic type map for* **L**). The *semantic type map* for **L** is the following homomorphism T from syntactic types $\mathcal{F}$ to semantic types $\mathcal{T}$:

$$\begin{array}{rcl}
T(A\bullet B) & = & T(A)\&T(B) \\
T(A\backslash C) & = & T(A) \to T(C) \\
T(C/B) & = & T(B) \to T(C)
\end{array}$$

Categorial semantics, Curry-Howard type-logical semantics, works because under such a type map categorial derivations are homomorphically sent to intuitionistic proofs, i.e. pure terms of the typed lambda calculus. These compose lexical semantics expressed as terms of higher-order logic into meanings in higher-order logic of projected expressions.

The first type-logical formulation of discontinuity, i.e. with an interpretation of types and with a sequent calculus, appeared in Moortgat (1988)[5]. Moortgat defined discontinuous types as follows (we modify Moortgat's notations):

(10) $$\begin{array}{rcl}
[\![A \downarrow_\forall C]\!] & = & \{s \mid \forall s_1{+}s_2 \in [\![A]\!], s_1{+}s{+}s_2 \in [\![C]\!]\} \\
[\![C \uparrow_\exists B]\!] & = & \{s \mid \exists s_1, s_2, s = s_1{+}s_2, \forall s' \in [\![B]\!], s_1{+}s'{+}s_2 \in [\![C]\!]\}
\end{array}$$

[2] We allow ourselves to abbreviate $((\phi\ \psi)\ \chi)$ as $(\phi\ \psi\ \chi)$.

The following sequent rules were given:

$$(11)\ \frac{\Gamma \Rightarrow A \qquad \Delta(C) \Rightarrow D}{\Delta(\Gamma(A \downarrow_{\forall} C)) \Rightarrow D} \downarrow_{\forall} L \qquad \frac{\Gamma, B, \Delta \Rightarrow C}{\Gamma, \Delta \Rightarrow C \uparrow_{\exists} B} \uparrow_{\exists} R$$

Thus e.g. medial extraction, not otherwise derivable in the Lambek calculus, is obtained from a relative pronoun type R/(S$\uparrow_{\exists}$N). And S(neg)$\downarrow_{\forall}$S(pos) would be the type of a freely floating negation particle, if there really were such an element. However, the other sequent rules cannot be formulated, so the logic is incomplete.

Moortgat (1991)[6][3] defined a three-place in-situ binder type-constructor Q for e.g. quantifier phrases, Q(S, N, S), and subject-oriented reflexives, Q(N\S, N, N\S). The left sequent rule is:

$$(12)\ \frac{\Gamma(A) \Rightarrow B \qquad \Delta(C) \Rightarrow D}{\Delta(\Gamma(Q(B, A, C))) \Rightarrow D} QL$$

However the best that came be managed on the right is:

$$(13)\ \frac{\Gamma \Rightarrow A}{\Gamma \Rightarrow Q(B, A, B)} QR$$

This is insufficient to derive e.g. Q(S, N, S) $\Rightarrow$ Q(N\S, N, N\S) (that a quantifier phrase can occur in a verb phrase conjunct, H. Hendriks, p.c.) so the logic is incomplete again. Moortgat indicated that something like $Q(B, A, C)$ might be decomposed $(B{\uparrow}A){\downarrow}C$, but he did not have a calculus ensuring that the two points of discontinuity would be the same.

Versmissen (1991)[17] observed that we want in some way to mark points of discontinuity. Solias (1992)[16] and Morrill and Solias (1993)[13] had tupling to do so and had wrapping as a derived operation: $sWt =_{df} \pi_1 s{+}t{+}\pi_2 s$. But then projection/wrapping is undefined on prosodic objects built by concatenation (and on primes) and there is no control for when this arises. Thus Morrill (1994, ch. 4; 1995)[14][7] took wrapping to be primitive (and everwhere defined), and so did not require such 'tupling' to be projective, but only to satisfy the 'split-wrap' law of interaction: $(s_1, s_3)Ws_2 = s_1{+}s_2{+}s_3$.

[3] Moortgat (1991) also proposed a substring product:

(i) $[\![A \odot B]\!] = \{s_1{+}s_2{+}s_3 |\ s_1{+}s_3 \in [\![A]\!]\ \&\ s_2 \in [\![B]\!]\}$

$$\text{(ii)}\ \frac{\Gamma_1, \Gamma_2 \Rightarrow A \qquad \Delta \Rightarrow B}{\Gamma_1, \Delta, \Gamma_2 \Rightarrow A \odot B} \odot R$$

But again a left rule cannot be given.

In both cases however, the represention of discontinuous expressions as tuples or something like them in an (unsorted) algebra introduces (infinitely many) prosodic objects in which points of discontinuity, because embedded under concatenation, can never wrap, e.g. $s_1+(s_2, s_3)$, so we are positing much useless junk.

This is remedied by the sorting of Morrill and Merenciano (1996)[12] which restricts points of discontinuity to be only ever nonembedded (i.e. principle): external to concatenation, and so always potentially useful to undergo wrap. But in the generalized case (i.e. with no upper bound on the number of points of discontinuity), the infinite number of arities of tupling would require both pairing and the empty tuple for list-construction. Here we reduce the machinary to a single operator of arity zero (i.e. a constant) and still get the generalized case. The key to our treatment of discontinuity is the notion of a "separator" (Morrill 2002)[10]:

(14) **Definition** (*graded prosodic algebra*). A *graded prosodic algebra* is a free algebra $(L, +, 0, 1)$ of arity $(2, 0, 0)$ such that $(L, +, 0)$ is a monoid and 1 is a prime.

The constant 1 is called the *separator.*

(15) **Definition** (*sorts and sort domains*). The *sorts* of a graded prosodic algebra $(L, +, 0, 1)$ are the naturals $0, 1, \ldots$. The sort of an element s is defined by the morphism of monoides σ to the additive monoid of naturals thus:

$$\begin{array}{rcll} \sigma(1) & = & 1 & \\ \sigma(a) & = & 0 & \text{for a prime } a \neq 1 \\ \sigma(s_1+s_2) & = & \sigma(s_1)+\sigma(s_2) & \end{array}$$

The *sort domains* L_i of sort i are defined as follows:

$$L_i \quad = \quad \{s|\sigma(s) = i\}, i \geq 0$$

I.e. the sort of a prosodic element is simply the number of separators it contains; we require the separator 1 to be a prime in order that the induction be well-defined. The fact that there is a homomorphism from a graded prosodic algebra to the additive monoid of naturals means that a graded prosodic algebra is an instance of what is known as a *graded monoid*.

Morrill (1997)[8] introduced sequent calculus for (sorted) discontinuity in which a discontinuous type has multiple occurrences at the loci of its segments, punctuated by surds. This is called hypersequent calculus in the appendix of Morrill (2003)[11], though in a usage of the term distinct from that of A. Avron. The spirit is to maintain everything in 'evaluated/spelt-out' linearized form. The surd notation is meant to be suggestive of the (commutative) arithmetic law $\sqrt{A} \cdot \sqrt{A} = A$ (for us, non-commutatively, $\sqrt[0]{A}\bullet\{1\}\bullet\sqrt[1]{A} = A$).

(16) **Definition** (*discontinuous prosodic structure*). The *discontinuous prosodic structure* defined by a graded prosodic algebra $(L, +, 0, 1)$ is the ω-sorted structure $(L_0, L_1, L_2, \ldots, +, 0, 1; \times)$ of arity $(2, 0, 0; 2)$ such that:

$$\begin{array}{lll} + & : & L_i \times L_j \to L_{i+j} \text{ as in the graded prosodic algebra} \\ \times & : & L_{i+1} \times L_j \times L_{i+j} \text{ is the smallest relation such that} \\ & & \forall s_1{+}1{+}s_3 \in L_{i+1}, s_2 \in L_j, \times(s_1{+}1{+}s_3, s_2, s_1{+}s_2{+}s_3) \end{array}$$

I.e. the third argument of $\times$ is the result of replacing by the second argument any separator in the first argument. The system **DL** of (nondeterministic generalized) discontinuous Lambek calculus is as follows.

(17) **Definition** (*sorted types of* **DL**). The sets $\mathcal{F}_i$ of *types of sort i of* **DL** for each sort i are defined by mutual recursion as follows on the basis of sets $\mathcal{P}_i$ of primitive types of sort i for each sort i.[4]

$$\begin{array}{rcl} \mathcal{F}_i & ::= & \mathcal{P}_i \\ \mathcal{F}_{i+j} & ::= & \mathcal{F}_i\bullet\mathcal{F}_j \\ \mathcal{F}_j & ::= & \mathcal{F}_i\backslash\mathcal{F}_{i+j} \\ \mathcal{F}_i & ::= & \mathcal{F}_{i+j}/\mathcal{F}_j \\ \mathcal{F}_i & ::= & {}^\wedge\mathcal{F}_{i+1} \\ \mathcal{F}_{i+1} & ::= & {}^\vee\mathcal{F}_i \\ \mathcal{F}_{i+j} & ::= & \mathcal{F}_{i+1}\odot\mathcal{F}_j \\ \mathcal{F}_j & ::= & \mathcal{F}_{i+1}\downarrow\mathcal{F}_{i+j} \\ \mathcal{F}_{i+1} & ::= & \mathcal{F}_{i+j}\uparrow\mathcal{F}_j \end{array}$$

(18) **Definition** (*prosodic interpretation of* **DL** *types*). A *prosodic interpretation of* **DL** *types* is a function $[\![\cdot]\!]$ mapping each type $A_i \in \mathcal{F}_i$

[4] Consequently there is the following *prosodic sort map* S sending **DL** types to their sorts:

$$\begin{array}{rcl} S(A\bullet B) & = & S(A) + S(B) \\ S(A\backslash C) & = & S(C) - S(A) \\ S(C/B) & = & S(C) - S(B) \\ S({}^\wedge A) & = & S(A) - 1 \\ S({}^\vee B) & = & S(B) + 1 \\ S(A\odot B) & = & S(A) + S(B) - 1 \\ S(A\downarrow C) & = & 1 + S(C) - S(A) \\ S(C\uparrow B) & = & 1 + S(C) - S(B) \end{array}$$

into a subset of L_i such that:

$$\begin{array}{rcl}
[\![A\backslash C]\!] & = & \{s_2 |\ \forall s_1 \in [\![A]\!], s_1{+}s_2 \in [\![C]\!]\} \\
[\![C/B]\!] & = & \{s_1 |\ \forall s_2 \in [\![B]\!], s_1{+}s_2 \in [\![C]\!]\} \\
[\![A\bullet B]\!] & = & \{s_1{+}s_2 |\ s_1 \in [\![A]\!] \ \&\ s_2 \in [\![B]\!]\} \\
[\![{}^{\vee}B]\!] & = & \{s |\ \forall s', \times(s,0,s') \Rightarrow s' \in [\![B]\!]\} \\
[\![{}^{\wedge}A]\!] & = & \{s' |\ \exists s \in [\![A]\!], \times(s,0,s')\} \\
[\![A{\downarrow}C]\!] & = & \{s_2 |\ \forall s_1 \in [\![A]\!], \times(s_1,s_2,s) \Rightarrow s \in [\![C]\!]\} \\
[\![C{\uparrow}B]\!] & = & \{s_1 |\ \forall s_2 \in [\![B]\!], \times(s_1,s_2,s) \Rightarrow s \in [\![C]\!]\} \\
[\![A\odot B]\!] & = & \{s |\ \exists s_1 \in [\![A]\!], s_2 \in [\![B]\!], \times(s_1,s_2,s)\}
\end{array}$$

Observe that modulo sorting, $(\downarrow, \odot, \uparrow; \subseteq)$, like$(\backslash, \bullet, /; \subseteq)$, is a residuated triple:

$$B \subseteq A{\downarrow}C \text{ iff } A\odot B \subseteq C \text{ iff } A \subseteq C{\uparrow}B$$

And $({}^{\vee}, {}^{\wedge}; \subseteq)$ is a residuated pair:

$$A \subseteq {}^{\vee}B \text{ iff } {}^{\wedge}A \subseteq B$$

(19) **Definition** (*figures, configurations and sequents of* **DL**). In **DL**, the *figures* $\mathcal{Q}_i$ of sort i for each sort i are defined as follows:

$$\begin{array}{rcll}
\mathcal{Q}_0 & ::= & A & \text{for } S(A)=0 \\
\mathcal{Q}_{S(A)} & ::= & \sqrt[0]{A}, [], \ldots, [], \sqrt[S(A)]{A} & \text{for } S(A)>0
\end{array}$$

By the vectorial notation $\overrightarrow{A}$ we mean the figure of sorted type A, i.e. A if $S(A)=0$ and $\sqrt[0]{A}, [], \sqrt[1]{A}, \ldots, \sqrt[S(A)-1]{A}, [], \sqrt[S(A)]{A}$ if $S(A)>0$.

The *configurations* $\mathcal{O}_i$ of sort i for each sort i are defined unambiguously by mutual recursion as follows:

$$\begin{array}{rcll}
\mathcal{O}_0 & ::= & \Lambda & \\
\mathcal{O}_i & ::= & A, \mathcal{O}_i & \text{for } S(A)=0 \\
\mathcal{O}_{i+1} & ::= & [], \mathcal{O}_i & \\
\mathcal{O}_{\Sigma_{k=0}^{S(A)} j_k} & ::= & \sqrt[0]{A}, \mathcal{O}_{j_0}, \ldots, \mathcal{O}_{j_{S(A)-1}}, \sqrt[S(A)]{A}, \mathcal{O}_{j_{S(A)}} & \text{for } S(A)>0
\end{array}$$

The *sequents* Σ_i of sort i for each sort i are defined as follows:

$$\Sigma_i \ ::= \ \mathcal{O}_i \Rightarrow \mathcal{Q}_i$$

Observe that the segments of discontinuous types are well-nested in configurations, i.e. there are no crossing discontinuities.

(20) **Definition** (*prosodic interpretation of configurations and validity of sequents in* **DL**). We extend the interpretation of types to include configurations, as follows:

$$\begin{array}{rcl}
[\![\Lambda]\!] & = & \{0\} \\
[\![A, \Gamma]\!] & = & \{s_1{+}s_2 |\ s_1 \in [\![A]\!] \ \&\ s_2 \in [\![\Gamma]\!]\} \\
[\![[], \Gamma]\!] & = & \{1{+}s |\ s \in [\![\Gamma]\!]\} \\
[\![\sqrt[0]{A}, \Gamma_0, \ldots, \Gamma_{S(A)-1}, \sqrt[S(A)]{A}, \Gamma_{S(A)}]\!] & = & \{s_0{+}t_0{+}\cdots{+}t_{S(A)-1}{+}s_{S(A)}{+}t_{S(A)} | \\
& & \ s_0{+}1{+}\cdots{+}1{+}s_{S(A)} \in [\![A]\!] \\
& & \ \&\ t_j \in [\![\Gamma_j]\!], 0 \leq j \leq S(A)\}
\end{array}$$

A sequent $\Gamma \Rightarrow X$ is *valid* iff in every interpretation, $[\![\Gamma]\!] \subseteq [\![X]\!]$.

The (hyper)sequent calculus for **DL** is defined in figure 2.

(21) **Definition** (*semantic type map for* **DL**). The *semantic type map* T for **DL** is as follows:

$$\begin{array}{rcl}
T(A{\bullet}B) & = & T(A)\&T(B) \\
T(A\backslash C) & = & T(A) \to T(C) \\
T(C/B) & = & T(B) \to T(C) \\
T({}^{\wedge}A) & = & T(A) \\
T({}^{\vee}B) & = & T(B) \\
T(A{\odot}B) & = & T(A)\&T(B) \\
T(A{\downarrow}C) & = & T(A) \to T(C) \\
T(C{\uparrow}B) & = & T(B) \to T(C)
\end{array}$$

The semantic type map sends derivations into intuitionistic proofs so the usual Curry-Howard categorial type-logical semantics comes for free for **DL**.

We can present type-logical calculi in a labelled deductive system (LDS) of natural deduction in which prosodic terms α and semantic terms ϕ label types A: $\alpha{-}\phi{:}A$. The natural deduction LDS for **DL** is given in figure 3; the vectorial notation $\overrightarrow{a}$ means $a_0{+}1{+}\cdots{+}1{+}a_n$ where n is the sort of a.

We turn now to linguistic applications. Parentheticals are adsentential modifiers such as *fortunately* which, to a very rough first approximation, can appear anywhere in the sentence they modify:

(22) a. Fortunately, John has perseverence.

b. John, fortunately, has perseverence.

c. John has, fortunately, perseverence.

d. John has perseverence, fortunately.

Such a distribution is captured by the following type assignment, cf. Morrill and Merenciano (1996)[12].

(23) **fortunately** – *fortunately*
:= ${}^{\vee}\mathrm{S}{\downarrow}\mathrm{S}$

For example, (22c) is derived in figure 4.[5]

Particle shift is the alternation in the order of a particle verb's object and its particle:[6]

[5] Of course, parentheticals cannot really occur *any*where, e.g. **The, fortunately, man left.* In the end there will have to be some kinds of prosodic domains which they cannot penetrate.

[6] We have no explanation of why the pronoun must be stressed in *John called up HER.*

$$\frac{}{\overrightarrow{A} \Rightarrow \overrightarrow{A}}\,id \qquad \frac{\Gamma \Rightarrow \overrightarrow{A} \qquad \Delta(\overrightarrow{A}) \Rightarrow \overrightarrow{B}}{\Delta(\Gamma) \Rightarrow \overrightarrow{B}}\,Cut$$

$$\frac{\Gamma \Rightarrow \overrightarrow{A} \qquad \Delta(\overrightarrow{C}) \Rightarrow \overrightarrow{D}}{\Delta(\Gamma, \overrightarrow{A\backslash C}) \Rightarrow \overrightarrow{D}}\backslash L \qquad \frac{\overrightarrow{A}, \Gamma \Rightarrow \overrightarrow{C}}{\Gamma \Rightarrow \overrightarrow{A\backslash C}}\backslash R \qquad \frac{\Gamma \Rightarrow \overrightarrow{B} \qquad \Delta(\overrightarrow{C}) \Rightarrow \overrightarrow{D}}{\Delta(\overrightarrow{C/B}, \Gamma) \Rightarrow \overrightarrow{D}}/L \qquad \frac{\Gamma, \overrightarrow{B} \Rightarrow \overrightarrow{C}}{\Gamma \Rightarrow \overrightarrow{C/B}}/R$$

$$\frac{\Delta(\overrightarrow{A}, \overrightarrow{B}) \Rightarrow \overrightarrow{D}}{\Delta(\overrightarrow{A \bullet B}) \Rightarrow \overrightarrow{D}}\bullet L \qquad \frac{\Gamma_1 \Rightarrow \overrightarrow{A} \qquad \Gamma_2 \Rightarrow \overrightarrow{B}}{\Gamma_1, \Gamma_2 \Rightarrow \overrightarrow{A \bullet B}}\bullet R$$

$$\frac{\Delta(\overrightarrow{B}) \Rightarrow \overrightarrow{D}}{\Delta(\overrightarrow{{}^{\vee}B}|_i\Lambda) \Rightarrow \overrightarrow{D}}{}^{\vee}L \qquad \frac{\Delta|_1\Lambda \Rightarrow \overrightarrow{B} \qquad \cdots \qquad \Delta|_{S(B)}\Lambda \Rightarrow \overrightarrow{B}}{\Delta \Rightarrow \overrightarrow{{}^{\vee}B}}{}^{\vee}R$$

$$\frac{\Delta(\overrightarrow{A}|_1\Lambda) \Rightarrow \overrightarrow{D} \qquad \cdots \qquad \Delta(\overrightarrow{A}|_{S(A)}\Lambda) \Rightarrow \overrightarrow{D}}{\Delta(\overrightarrow{{}^{\wedge}A}) \Rightarrow \overrightarrow{D}}{}^{\wedge}L \qquad \frac{\Delta \Rightarrow \overrightarrow{A}}{\Delta|_i\Lambda \Rightarrow \overrightarrow{{}^{\wedge}A}}{}^{\wedge}R$$

$$\frac{\Gamma \Rightarrow \overrightarrow{A} \qquad \Delta(\overrightarrow{C}) \Rightarrow \overrightarrow{D}}{\Delta(\Gamma|_i\overrightarrow{A{\downarrow}C}) \Rightarrow \overrightarrow{D}}{\downarrow}L \qquad \frac{\overrightarrow{A}|_1\Gamma \Rightarrow \overrightarrow{C} \qquad \cdots \qquad \overrightarrow{A}|_{S(A)}\Gamma \Rightarrow \overrightarrow{C}}{\Gamma \Rightarrow \overrightarrow{A{\downarrow}C}}{\downarrow}R$$

$$\frac{\Gamma \Rightarrow \overrightarrow{B} \qquad \Delta(\overrightarrow{C}) \Rightarrow \overrightarrow{D}}{\Delta(\overrightarrow{C{\uparrow}B}|_i\Gamma) \Rightarrow \overrightarrow{D}}{\uparrow}L \qquad \frac{\Gamma|_1\overrightarrow{B} \Rightarrow \overrightarrow{C} \qquad \cdots \qquad \Gamma|_{S(C)}\overrightarrow{B} \Rightarrow \overrightarrow{C}}{\Gamma \Rightarrow \overrightarrow{C{\uparrow}B}}{\uparrow}R$$

$$\frac{\Delta(\overrightarrow{A}|_1\overrightarrow{B}) \Rightarrow \overrightarrow{D} \qquad \cdots \qquad \Delta(\overrightarrow{A}|_{S(A)}\overrightarrow{B}) \Rightarrow \overrightarrow{D}}{\Delta(\overrightarrow{A \odot B}) \Rightarrow \overrightarrow{D}}\odot L \qquad \frac{\Gamma_1 \Rightarrow \overrightarrow{A} \qquad \Gamma_2 \Rightarrow \overrightarrow{B}}{\Gamma_1|_i\Gamma_2 \Rightarrow \overrightarrow{A \odot B}}\odot R$$

Figure 2: The *hypersequent calculus* for **DL**, where $\Delta(\Gamma)$ means that in some distinguished positions in Δ, the segments of Γ appear in the given order, and $\Delta|_i\Gamma$ is the result of replacing the i-th separator in Δ by Γ.

$$\frac{\begin{array}{c}\vdots\\ \alpha - \phi{:}A\end{array} \quad \begin{array}{c}\vdots\\ \gamma - \chi{:}A\backslash C\end{array}}{\alpha+\gamma - (\chi\ \phi){:}C}\, E\backslash \qquad \frac{\begin{array}{c} a - x{:}A^i \\ \vdots \\ a+\gamma - \chi{:}C\end{array}}{\gamma - \lambda x\chi{:}A\backslash C}\, I\backslash^i \qquad \frac{\begin{array}{c}\vdots\\ \gamma - \chi{:}C/B\end{array} \quad \begin{array}{c}\vdots\\ \beta - \psi{:}B\end{array}}{\gamma+\beta - (\chi\ \beta){:}C}\, E/ \qquad \frac{\begin{array}{c} b - y{:}B^i \\ \vdots \\ \gamma+a - \chi{:}C\end{array}}{\gamma - \lambda y\chi{:}C/B}\, I/^i$$

$$\frac{\begin{array}{c}\vdots\\ \gamma - \chi{:}A\bullet B\end{array} \quad \begin{array}{c} a - x{:}A^i \quad b - y{:}B^i \\ \vdots \\ \delta(a+b) - \omega(x,y){:}D\end{array}}{\delta(\gamma) - \omega(\pi_1\chi, \pi_2\chi){:}D}\, E\bullet^i \qquad \frac{\begin{array}{c}\vdots\\ \alpha - \phi{:}A\end{array} \quad \begin{array}{c}\vdots\\ \beta - \psi{:}B\end{array}}{\alpha+\beta - (\phi, \psi){:}A\bullet B}\, I\bullet$$

$$\frac{\begin{array}{c}\vdots\\ \beta - \psi{:}^{\vee}B\end{array}}{\beta|_i 0 - \psi{:}B}\, E^{\vee} \qquad \frac{\begin{array}{c}\vdots\\ \beta|_1 0 - \psi{:}B\end{array} \quad \cdots \quad \begin{array}{c}\vdots\\ \beta|_{S(B)} 0 - \psi{:}B\end{array}}{\beta - \psi{:}^{\vee}B}\, I^{\vee}$$

$$\frac{\begin{array}{c}\vdots\\ \gamma - \chi{:}^{\wedge}A\end{array} \quad \begin{array}{c} \alpha - x{:}A^i \\ \vdots \\ \delta(\alpha|_1 0) - \omega(x){:}D\end{array} \quad \cdots \quad \begin{array}{c} \alpha - x{:}A^i \\ \vdots \\ \delta(\alpha|_{S(A)} 0) - \omega(x){:}D\end{array}}{\delta(\gamma) - \omega(\chi){:}D}\, E^{\wedge i} \qquad \frac{\begin{array}{c}\vdots\\ \alpha - \phi{:}A\end{array}}{\alpha|_i 0 - \phi{:}^{\wedge}A}\, I^{\wedge}$$

$$\frac{\begin{array}{c}\vdots\\ \alpha - \phi{:}A\end{array} \quad \begin{array}{c}\vdots\\ \gamma - \chi{:}A{\downarrow}C\end{array}}{\alpha|_i\gamma - (\chi\ \phi){:}C}\, E{\downarrow} \qquad \frac{\begin{array}{c} \vec{a} - x{:}A^i \\ \vdots \\ \vec{a}|_1\gamma - \chi{:}C\end{array} \quad \cdots \quad \begin{array}{c} \vec{a} - x{:}A^i \\ \vdots \\ \vec{a}|_{S(A)}\gamma - \chi{:}C\end{array}}{\gamma - \lambda x\chi{:}A{\downarrow}C}\, I{\downarrow}^i$$

$$\frac{\begin{array}{c}\vdots\\ \gamma - \chi{:}C{\uparrow}B\end{array} \quad \begin{array}{c}\vdots\\ \beta - \psi{:}B\end{array}}{\gamma|_i\beta - (\chi\ \beta){:}C}\, E{\uparrow} \qquad \frac{\begin{array}{c} b - y{:}B^i \\ \vdots \\ \gamma|_1 b - \chi{:}C\end{array} \quad \cdots \quad \begin{array}{c} b - y{:}B^i \\ \vdots \\ \gamma|_{S(C)} b - \chi{:}C\end{array}}{\gamma - \lambda x\chi{:}C{\uparrow}B}\, I{\uparrow}^i$$

$$\frac{\begin{array}{c}\vdots\\ \gamma - \chi{:}A\odot B\end{array} \quad \begin{array}{c} \vec{a} - x{:}A^i \quad b - y{:}B^i \\ \vdots \\ \delta(\vec{a}|_1 b) - \omega(x,y){:}D\end{array} \quad \cdots \quad \begin{array}{c} \vec{a} - x{:}A^i \quad b - y{:}B^i \\ \vdots \\ \delta(\vec{a}|_{S(C)} b) - \omega(x,y){:}D\end{array}}{\delta(\gamma) - \omega(\pi_1\chi, \pi_2\chi){:}D}\, E\odot^i$$

Figure 3: Natural deduction LDS for **DL**, where $\alpha|_i\beta$ is the result of replacing the i-th separator in α by β.

$$
\dfrac{
  \dfrac{
    \dfrac{
      \dfrac{\text{John}}{\textbf{John}-j{:}\mathrm{N}}
      \quad
      \dfrac{
        \dfrac{\text{has}}{\textbf{has}-\mathit{have}{:}(\mathrm{N}\backslash\mathrm{S})/\mathrm{N}}
        \quad
        \dfrac{\text{perseverence}}{\textbf{perseverence}-\mathit{perseverence}{:}\mathrm{N}}
      }{\textbf{has+perseverence}-(\mathit{have\ perseverence}){:}\mathrm{N}\backslash\mathrm{S}}\,E/
    }{\textbf{John+has+perseverence}-(\mathit{have\ perseverence}\ j){:}\mathrm{S}}\,E\backslash
  }{\textbf{John+has+}1\textbf{+perseverence}-(\mathit{have\ perseverence}\ j){:}{}^{\vee}\mathrm{S}}\,I^{\vee}
  \quad
  \dfrac{\text{fortunately}}{\textbf{fortunately}-\mathit{fortunately}{:}{}^{\vee}\mathrm{S}{\downarrow}\mathrm{S}}
}{\textbf{John+has+fortunately+perseverence}-(\mathit{fortunately}\ (\mathit{have\ perseverence}\ j)){:}\mathrm{S}}\,E{\downarrow}
$$

Figure 4: Derivation of parentesisization.

$$
\dfrac{
  \dfrac{
    \dfrac{\text{called up}}{\textbf{called}+1+\textbf{up}+1-\mathit{phone}{:}{}^{\vee}(\mathrm{N}\backslash\mathrm{S})\uparrow\mathrm{N}}
    \quad
    \dfrac{\text{Mary}}{\textbf{Mary}-m{:}\mathrm{N}}
  }{\textbf{called}+1+\textbf{up}+\textbf{Mary}-(\mathit{phone}\ m){:}{}^{\vee}(\mathrm{N}\backslash\mathrm{S})}\,E{\uparrow}
}{\textbf{called+up+Mary}-(\mathit{phone}\ m){:}\mathrm{N}\backslash\mathrm{S}}\,E^{\vee}
$$

$$
\dfrac{
  \dfrac{
    \dfrac{\text{called up}}{\textbf{called}+1+\textbf{up}+1-\mathit{phone}{:}{}^{\vee}(\mathrm{N}\backslash\mathrm{S})\uparrow\mathrm{N}}
    \quad
    \dfrac{\text{Mary}}{\textbf{Mary}-m{:}\mathrm{N}}
  }{\textbf{called+Mary+up}+1-(\mathit{phone}\ m){:}{}^{\vee}(\mathrm{N}\backslash\mathrm{S})}\,E{\uparrow}
}{\textbf{called+Mary+up}-(\mathit{phone}\ m){:}\mathrm{N}\backslash\mathrm{S}}\,E^{\vee}
$$

Figure 5: Derivations of particle shift.

(24) a. John called up Mary.

b. John called Mary up.

The alternation is generated by the following single lexical assignment, cf. Morrill (2002)[10]:

(25) $\textbf{called}+1+\textbf{up}+1 \quad - \quad \mathit{phone}$
$\qquad\qquad\qquad\qquad\quad := \quad {}^{\vee}(\mathrm{N}\backslash\mathrm{S})\uparrow\mathrm{N}$

The derivations of (24a, b) are given in figure 5.

Chomsky (1957)[1] argued informally that even if natural languages were context-free, context-free grammar could not give a scientifically satisfactory characterisation of English. Huybregts (1985)[2] and Shieber (1985)[15] formally proved that at least one natural language, Swiss-German, is not context-free. The relevant feature is cross-serial dependency accompanied by morphological matching between verbs and their dependants. Dutch subor-

$$
\dfrac{\dfrac{\dfrac{\dfrac{\text{boeken}}{\textbf{boeken}-\mathit{books}\mathrm{:N}} \quad \dfrac{\text{lezen}}{1+\textbf{lezen}-\mathit{read}\mathrm{:N\backslash(N\backslash Sinf)}}}{\textbf{boeken}+1+\textbf{lezen}-(\mathit{read\ books})\mathrm{:N\backslash Sinf}}\,E\backslash \quad \dfrac{\text{kunnen}}{1+\textbf{kunnen}-\mathit{be\text{-}able}\mathrm{:(N\backslash Sinf)\!\downarrow\!(N\backslash Sinf)}}}{\textbf{boeken}+1+\textbf{kunnen}+\textbf{lezen}-(\mathit{be\text{-}able}\ (\mathit{read\ books}))\mathrm{:N\backslash Sinf}}\,E\!\downarrow \quad \dfrac{\text{wil}}{\textbf{wil}-\mathit{want}\mathrm{:(N\backslash Sinf)\!\downarrow\!(N\backslash S)}}}{\textbf{boeken}+\textbf{wil}+\textbf{kunnen}+\textbf{lezen}-(\mathit{want}\ (\mathit{be\text{-}able}\ (\mathit{read\ books})))\mathrm{:N\backslash Sinf}}\,E\!\downarrow
$$

Figure 6: Derivation of verb raising.

dinate clauses exhibit the same semantic cross-serial dependencies; consider for example the verb raising:

(26) ... dat Jan boeken wil kunnen lezen
that Jan books wants be-able read
that Jan wants to be able to read books

The idea of our analysis (Morrill 2000)[9] is to mark the left edge of the infinitival subordinate clause verb cluster with a separator, and to have successive verb raising triggers infixing at this point and inserting another separator to their own left (if they are infinitive) or closing off the discontinuity (if they are finite). Let there be verb type assigments as follows, where Sinf is of sort 1:

$$
(27)\qquad
\begin{array}{rcl}
\textbf{kan} & - & \mathit{be\text{-}able} \\
 & := & (\mathrm{N\backslash Sinf})\!\downarrow\!(\mathrm{N\backslash S}) \\
1+\textbf{kunnen} & - & \mathit{be\text{-}able} \\
 & := & (\mathrm{N\backslash Sinf})\!\downarrow\!(\mathrm{N\backslash Sinf}) \\
\textbf{las} & - & \mathit{read} \\
 & := & \mathrm{N\backslash(N\backslash S)} \\
1+\textbf{lezen} & - & \mathit{read} \\
 & := & \mathrm{N\backslash(N\backslash Sinf)} \\
\textbf{wil} & - & \mathit{want} \\
 & := & (\mathrm{N\backslash Sinf})\!\downarrow\!(\mathrm{N\backslash S})
\end{array}
$$

Then (26) has the derivation in figure 6.

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