Generalising Discontinuity

Glyn Morrill and Josep-Maria Merenciano
Dept. de Llenguatges i Sistemes Informàtics
Universitat Politècnica de Catalunya
Pau Gargallo, 5
08028 Barcelona

morrill@lsi.upc.es, http://www-lsi.upc.es/~glyn/
and merenciano@lsi.upc.es

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Abstract

This paper makes two generalisations of categorial calculus of discontinuity. In the first we introduce unary modalities which mediate between continuous and discontinuous strings. In the second each of the modes of adjunction of the proposal to date, concatenation, juxtaposition and interpolation, are augmented with variants. Linguistic illustration and motivation is provided, and we show how adherence to a discipline of sorting renders the generalisations tractable within a particularly efficient logic programming paradigm.

Generalising Discontinuity

The present work continues in the line of others seeking to develop categorial type calculus of discontinuity and associated automated theorem proving/parsing (Moortgat 1988, 1990, 1991; Solias 1992; Morrill and Solias 1993; Morrill 1994, ch. 4, 1995a, 1995b, 1995c; Lloré and Morrill 1995, Calcagno 1995). In particular, it generalises the sorted discontinuity calculus outlined in the appendix of Morrill (1995b) and implemented in a linear clausal fragment by compilation as described in Morrill 1995c; familiarity with these two works is assumed in what follows.

We begin by summarising the point of departure for the present proposals. We then introduce two generalisations: unary "split" and "bridge" operators mediating between strings and split strings, and binary operators for staggered concatenation, and juxtaposition and interpolation adjunctions which inherit split points from their operands. We go on to show how these proposals fit into the linear logic programming paradigm for categorial parsing as deduction.

1 Sorted Discontinuity Calculus

The associative Lambek calculus (Lambek 1958) provides a logic of concatenation. Its types are specifications of concatenative comportment and by classifying words with respect to types, properties of strings are defined which are the deductive consequences. The non-associative Lambek calculus (Lambek 1961) is similarly a logic of juxtaposition, by which we mean putting side-by-side in a way which imposes grouping (concatenation, being associative, forgets grouping). But the existence of discontinuous phenomena in natural grammar guarantees that such logic of itself cannot be adequate. In discontinuity calculus, as presented for example in Morrill (1994, ch. 4, 1995b), it is sought to combine and extend logic of concatenation and juxtaposition with logic of interpolation. In one, unsorted, approach concatenation, juxtaposition and interpolation are each assumed to be total functions in a single abstract total algebra and the categorial types are formed from unsorted type-constructors without restriction.

The sorted discontinuity calculus is briefly introduced in the appendix of (Morrill 1995b). It is distinguished from the unsorted version in that instead of assuming all adjunctions to be total functions in an unsorted algebra, two sorts of object (string and split string) are assumed so that the adjunctions are sorted operations in a sorted algebra, and the categorial types come in a restricted form according to the sorted type-constructors. This formulation has particularly good computational properties; while the unsorted version has a logic programming implementation depending on matching under associativity and partial commutativity (Morrill 1995a), the sorted version has one depending on just unification of unstructured terms (i.e. constants and variables; Morrill 1995c).

The sorted discontinuity calculus is as follows. Let us assume a monoid $\langle L, +, \epsilon\rangle$ comprising the set of strings over some vocabulary, with $+$ the associative operation of concatenation (so that $s_1+(s_2+s_3) = (s_1+s_2)+s_3$), and with ϵ the empty string (so that $s+\epsilon = \epsilon+s = s$). The concatenation adjunction $+$ has functionality $L, L \rightarrow L$. We define a juxtaposition adjunction $(.,.)$ which is Cartesian product formation over L, of functionality $L, L \rightarrow L^2$; $(s_1, s_2) =_{df} \langle s_1, s_2\rangle$. And we further define an interpolation adjunction W of functionality $L^2, L \rightarrow L$; $\langle s_1, s_2\rangle W s =_{df} s_1+s+s_2$. Because these operations are sorted, the categorial types and type-constructors defined with respect to them are correspondingly sorted. We refer to sort L as sort string, and sort L^2 as sort split string.

The family of concatenation connectives $\{/, \backslash, \bullet\}$ are defined by "residuation" with respect to the concatenation adjunction $+$, which is of functionality $L, L \rightarrow L$. The existential conjunction (product) $A{\bullet}B$ (A product B) is the setwise sum of the concatenation adjunction over A and B; $A\backslash B$ (A under B) and B/A (B over A) are the universal directional implications (divisions).

(1)
$$\begin{array}{rcl} D(A\backslash B) & = & \{s|\ \forall s' \in D(A), s'{+}s \in D(B)\} \\ D(B/A) & = & \{s|\ \forall s' \in D(A), s{+}s' \in D(B)\} \\ D(A{\bullet}B) & = & \{s|\ \exists s_1, s_2, s = s_1{+}s_2\ \&\ s_1 \in D(A)\ \&\ s_2 \in D(B)\} \end{array}$$

Each of these type-constructors requires its operands to be of sort string and produces a composite type of sort string.

The family of juxtaposition connectives $\{<, >, \diamond\}$ are defined by residuation with respect to the juxtaposition adjunction $\langle .,. \rangle$, which is of functionality $L, L \rightarrow L^2$. The product $A{\diamond}B$ is the setwise sum of the juxtaposition adjunction over A and B; $A{>}B$ (B to A) and $B{<}A$ (B from A) are the directional divisions.

(2)
$$\begin{array}{lcl} D(A{>}B) & = & \{s \mid \forall s' \in D(A), \langle s', s\rangle \in D(B)\} \\ D(B{<}A) & = & \{s \mid \forall s' \in D(A), \langle s, s'\rangle \in D(B)\} \\ D(A{\diamond}B) & = & \{\langle s_1, s_2\rangle \mid s_1 \in D(A) \ \&\ s_2 \in D(B)\} \end{array}$$

Since juxtaposition combines two strings to form a split string, product types are of sort split string with sort string operands; and divisor types are of sort string and have the denominator type of sort string and the numerator type of sort split string.

The family of interpolation connectives $\{\uparrow, \downarrow, \odot\}$ are defined by residuation with respect to the interpolation adjunction W, which is of functionality $L^2, L \rightarrow L$. The product $A{\odot}B$ is the setwise sum of the interpolation adjunction over A and B; $A{\downarrow}B$ (A infix B) and $B{\uparrow}A$ (B extract A) are the divisions.

(3)
$$\begin{array}{lcl} D(A{\downarrow}B) & = & \{s \mid \forall \langle s_1, s_2\rangle \in D(A), s_1{+}s{+}s_2 \in D(B)\} \\ D(B{\uparrow}A) & = & \{\langle s_1, s_2\rangle \mid \forall s \in D(A), s_1{+}s{+}s_2 \in D(B)\} \\ D(A{\odot}B) & = & \{s \mid \exists s_1, s', s_2, s = s_1{+}s'{+}s_2 \ \&\ \langle s_1, s_2\rangle \in D(A) \ \&\ s' \in D(B)\} \end{array}$$

Sorting considerations apply in ways similar to those made before. In summary, let us assume that atomic formulas $\mathcal{A}$ are of sort string. The well-sorted category formulas (or: types) $\mathcal{F}$ of sort string and $\mathcal{G}$ of sort split string are defined by mutual recursion thus:

(4)
$$\begin{array}{lcl} \mathcal{F} & ::= & \mathcal{A} \mid \mathcal{F}/\mathcal{F} \mid \mathcal{F}\backslash\mathcal{F} \mid \mathcal{F}{\bullet}\mathcal{F} \mid \mathcal{G}{<}\mathcal{F} \mid \mathcal{F}{>}\mathcal{G} \mid \mathcal{G}{\downarrow}\mathcal{F} \mid \mathcal{G}{\odot}\mathcal{F} \\ \mathcal{G} & ::= & \mathcal{F}{\diamond}\mathcal{F} \mid \mathcal{F}{\uparrow}\mathcal{F} \end{array}$$

Each formula A of sort string has an interpretation $D(A) \subseteq L$ and each formula A of sort split string has an interpretation $D(A) \subseteq L^2$.

The system mixing the three families of connectives is sublinear in the space of logics arising from removing standard structural rules. Thus while linear logic (Girard 1987) results from removal of freely applying contraction and weakening, but not exchange, the present system lacks free commutativity also. This means that all theorems must be valid when reading divisions and products as the linear (multiplicative) implication and conjunction. Linear validity is a necessary condition for validity, though of course it is not sufficient because further sublinear structural conditions must be respected.

We present a tree-style natural deduction proof format in which linear logical resource-consciousness is reflected by closure of a unique assumption in conditionalisation. The sublinear conditions are expressed in labels (Gabbay 1991) reflecting the interpretation. We use boldface romans as constants naming elements of L, and we use $\alpha, \beta, \gamma, \ldots$ as variables over L term labels. A labelled formula of sort string has the form α: A and a labelled formula of sort split string is of the form (α, β): A. The labelled natural deduction rules can be seen as a restatement left-to-right and right-to-left of the bidirectional interpretation clauses rotated ninety degrees clockwise for the elimination (E) rules and anticlockwise for the introduction (I) rules,[1] with metavariables or Skolem constants according to quantifiers and the polarity of their context. We give only introduction rules for existentials since the elimination rules are both problematic, and apparently unmotivated linguistically.

(5)
$$\dfrac{\begin{array}{c}\vdots\\ \alpha\colon A\end{array} \qquad \begin{array}{c}\vdots\\ \gamma\colon A\backslash B\end{array}}{\alpha{+}\gamma\colon B}\backslash\mathrm{E} \qquad\qquad \dfrac{\begin{array}{c}\overline{\mathbf{a}\colon A}^{\,n}\\ \vdots\\ \mathbf{a}{+}\gamma\colon B\end{array}}{\gamma\colon A\backslash B}\backslash\mathrm{I}^n$$

[1] Cf. Ranta (1994), who attributes the observation to Martin-Löf (1987).

(6)
$$\dfrac{\begin{array}{c}\vdots\\ \gamma\colon B/A\end{array}\qquad\begin{array}{c}\vdots\\ \alpha\colon A\end{array}}{\gamma{+}\alpha\colon B}/\mathrm{E}\qquad\qquad \dfrac{\begin{array}{c}\overline{\mathbf{a}\colon A}^{\,n}\\ \vdots\\ \gamma{+}\mathbf{a}\colon B\end{array}}{\gamma\colon B/A}/\mathrm{I}^{n}$$

(7)
$$\dfrac{\begin{array}{c}\vdots\\ \alpha\colon A\end{array}\qquad\begin{array}{c}\vdots\\ \beta\colon B\end{array}}{\alpha{+}\beta\colon A{\bullet}B}\bullet\mathrm{I}$$

(8)
$$\dfrac{\begin{array}{c}\vdots\\ \alpha\colon A\end{array}\qquad\begin{array}{c}\vdots\\ \gamma\colon A{>}B\end{array}}{(\alpha,\gamma)\colon B}{>}\mathrm{E}\qquad\qquad \dfrac{\begin{array}{c}\overline{\mathbf{a}\colon A}^{\,n}\\ \vdots\\ (\mathbf{a},\gamma)\colon B\end{array}}{\gamma\colon A{>}B}{>}\mathrm{I}^{n}$$

(9)
$$\dfrac{\begin{array}{c}\vdots\\ \gamma\colon B{<}A\end{array}\qquad\begin{array}{c}\vdots\\ \alpha\colon A\end{array}}{(\gamma,\alpha)\colon B}{<}\mathrm{E}\qquad\qquad \dfrac{\begin{array}{c}\overline{\mathbf{a}\colon A}^{\,n}\\ \vdots\\ (\gamma,\mathbf{a})\colon B\end{array}}{\gamma\colon B{<}A}{<}\mathrm{I}^{n}$$

(10)
$$\dfrac{\begin{array}{c}\vdots\\ \alpha\colon A\end{array}\qquad\begin{array}{c}\vdots\\ \beta\colon B\end{array}}{(\alpha,\beta)\colon A{\diamond}B}\diamond\mathrm{I}$$

(11)
$$\dfrac{\begin{array}{c}\vdots\\ (\alpha_1,\alpha_2)\colon A\end{array}\qquad\begin{array}{c}\vdots\\ \gamma\colon A{\downarrow}B\end{array}}{\alpha_1{+}\gamma{+}\alpha_2\colon B}{\downarrow}\mathrm{E}\qquad\qquad \dfrac{\begin{array}{c}\overline{(\mathbf{a}_1,\mathbf{a}_2)\colon A}^{\,n}\\ \vdots\\ \mathbf{a}_1{+}\gamma{+}\mathbf{a}_2\colon B\end{array}}{\gamma\colon A{\downarrow}B}{\downarrow}\mathrm{I}^{n}$$

(12)
$$\dfrac{\begin{array}{c}\vdots\\ (\gamma_1,\gamma_2)\colon B{\uparrow}A\end{array}\qquad\begin{array}{c}\vdots\\ \alpha\colon A\end{array}}{\gamma_1{+}\alpha{+}\gamma_2\colon B}{\uparrow}\mathrm{E}\qquad\qquad \dfrac{\begin{array}{c}\overline{\mathbf{a}\colon A}^{\,n}\\ \vdots\\ \gamma_1{+}\mathbf{a}{+}\gamma_2\colon B\end{array}}{(\gamma_1,\gamma_2)\colon B{\uparrow}A}{\uparrow}\mathrm{I}^{n}$$

(13)
$$\dfrac{\begin{array}{c}\vdots\\ (\alpha_1,\alpha_2)\colon A\end{array}\qquad\begin{array}{c}\vdots\\ \beta\colon B\end{array}}{\alpha_1{+}\beta{+}\alpha_2\colon A{\odot}B}\odot\mathrm{I}$$

1.1 Examples

We assume the reader has some familiarity with semantic composition in type-logical grammar, and with linguistic applications such as reflexivisation and quantification treated in the referenced works. We illustrate here with respect to two constructions, VP Ellipsis and wide scope 'or' not yet mentioned in the literature.

1.1.1 VP Ellipsis

We describe our treatment by reference to examples from Dalrymple, Shieber and Pereira (1991), showing how a categorial type-driven treatment renders unnecessary a mechanism of higher-order unification. Consider the following.

(14) Dan likes golf and George does too.

Let us suppose the following to be given, either listed or derived in the lexicon, or derived categorially from more fundamental assignments.

(15) $$\begin{array}{lcl}(\textbf{and}, \textbf{does+too}) & - & \lambda x\lambda y\lambda z[(y\ z) \wedge (y\ x)] \\ & := & ((\mathrm{N}\backslash \mathrm{S})\backslash(\mathrm{N}\backslash \mathrm{S}))\uparrow \mathrm{N}\end{array}$$

Then there is the following derivation of 'likes golf and George does too' in N\S=VP.

(16) $$\dfrac{\textbf{Dan}: \mathrm{N} \qquad \dfrac{\textbf{likes+golf}: \mathrm{VP} \qquad \dfrac{(\textbf{and}, \textbf{does+too}): (\mathrm{VP}\backslash \mathrm{VP})\uparrow \mathrm{N} \qquad \textbf{George}: \mathrm{N}}{\textbf{and+George+does+too}: \mathrm{VP}\backslash \mathrm{VP}}\uparrow\mathrm{E}}{\textbf{likes+golf+and+George+does+too}: \mathrm{VP}}\backslash\mathrm{E}}{\textbf{Dan+likes+golf+and+George+does+too}: \mathrm{S}}\backslash\mathrm{E}$$

Semantics is derived by the standard Curry-Howard rendering of categorial deductions: functional application for rules of implicational use (elimination) and functional abstraction for rules of implicational proof (introduction). In the following we show the normalised semantic terms for each node of (16).

(17) $$\dfrac{\mathbf{d} \qquad \dfrac{\mathbf{lg} \qquad \dfrac{\lambda x\lambda y\lambda z[(y\ z) \wedge (y\ x)] \qquad \mathbf{g}}{\lambda y\lambda z[(y\ z) \wedge (y\ \mathbf{g})]}\uparrow\mathrm{E}}{\lambda z[(\mathbf{lg}\ z) \wedge (\mathbf{lg}\ \mathbf{g})]}\backslash\mathrm{E}}{[(\mathbf{lg}\ \mathbf{d}) \wedge (\mathbf{lg}\ \mathbf{g})]}\backslash\mathrm{E}$$

Thus (14) receives the required logical form $[(\mathbf{lg}\ \mathbf{d}) \wedge (\mathbf{lg}\ \mathbf{g})]$.

We leave it to the reader to check that, in the absence of other constraints, with 'himself' assigned semantics $\lambda x\lambda y((x\ y)\ y)$ in category (VP↑N)↓VP, 'John likes himself and Bill does too' is predicted to have both a sloppy and a strict reading. The point is that the *in situ* reflexive binder may take effect at either a verb phrase formed over 'likes himself', which gives rise to a sloppy reading, or one formed by 'likes himself and Bill does too', giving a strict reading. It may be similarly checked that with (18) and 'everyone' in category (S↑N)↓S, 'John greated everyone when Bill did' gets readings "When Bill greated everyone, John greated everyone" (with the quantifier scoping abstractly in 'greated everyone' of category N\S), and "for each person, when Bill greated him/her, so did John" (scoping over the entire sentence).

(18) $$\begin{array}{lcl}(\textbf{when}, \textbf{did}) & - & \lambda x\lambda y\lambda z((\textbf{when}\ (y\ z))\ (y\ x)) \\ & := & ((\mathrm{N}\backslash \mathrm{S})\backslash(\mathrm{N}\backslash \mathrm{S}))\uparrow \mathrm{N}\end{array}$$

1.1.2 Wide Scope "Or"

Under the phenomenon of wide-scope 'or' (Partee and Rooth 1983, Rooth and Partee 1982), a sentence such as 'John thinks Bill or Mary walks' is ambiguous between readings in which the disjunction scopes above or below the propositional attitude verb. The discontinuity apparatus allows a treatment analogous to that whereby 'John thinks someone walks' is ambiguous between specific and non-specific readings. The following lexical assignment to the disjunctive particle yields a coordinate structure in an *in situ* binder type which may operate at the subordinate or the superordinate level.

(19) **or** – $\lambda x \lambda y \lambda z[(z\ y) \vee (z\ x)]$
:= $(\mathrm{N}\backslash((\mathrm{S}{\uparrow}\mathrm{N}){\downarrow}\mathrm{S}))/\mathrm{N}$

Then the narrow scope and wide scope readings are delivered through the following two derivations in which the coordinate structure substitutes in at the subordinate and superordinate sentences respectively.

(20)
$$\dfrac{\dfrac{\dfrac{\overline{\mathbf{a}: \mathrm{N}}^{1} \quad \mathbf{walks}: \mathrm{N}\backslash\mathrm{S}}{\mathbf{a{+}walks}: \mathrm{S}}\backslash\mathrm{E}}{(\epsilon,\ \mathbf{walks}): \mathrm{S}{\uparrow}\mathrm{N}}{\uparrow}\mathrm{I}^1 \quad \dfrac{\mathbf{Bill}: \mathrm{N} \quad \dfrac{\mathbf{or}: (\mathrm{N}\backslash((\mathrm{S}{\uparrow}\mathrm{N}){\downarrow}\mathrm{S}))/\mathrm{N} \quad \mathbf{Mary}: \mathrm{N}}{\mathbf{or{+}Mary}: \mathrm{N}\backslash((\mathrm{S}{\uparrow}\mathrm{N}){\downarrow}\mathrm{S})}/\mathrm{E}}{\mathbf{Bill{+}or{+}Mary}: (\mathrm{S}{\uparrow}\mathrm{N}){\downarrow}\mathrm{S}}\backslash\mathrm{E}}{\mathbf{Bill{+}or{+}Mary{+}walks}: \mathrm{S}}{\downarrow}\mathrm{E}$$

(21)
$$\dfrac{\dfrac{\dfrac{\mathbf{John}: \mathrm{N} \quad \dfrac{\mathbf{thinks}: (\mathrm{N}\backslash\mathrm{S})/\mathrm{S} \quad \dfrac{\overline{\mathbf{a}: \mathrm{N}}^{1} \quad \mathbf{walks}: \mathrm{N}\backslash\mathrm{S}}{\mathbf{a{+}walks}: \mathrm{S}}\backslash\mathrm{E}}{\mathbf{thinks{+}a{+}walks}: \mathrm{N}\backslash\mathrm{S}}/\mathrm{E}}{\mathbf{John{+}thinks{+}a{+}walks}: \mathrm{S}}\backslash\mathrm{E}}{(\mathbf{John{+}thinks},\ \mathbf{walks}): \mathrm{S}{\uparrow}\mathrm{N}}{\uparrow}\mathrm{I}^1 \quad \mathbf{Bill{+}or{+}Mary}: (\mathrm{S}{\uparrow}\mathrm{N}){\downarrow}\mathrm{S}}{\mathbf{John{+}thinks{+}Bill{+}or{+}Mary{+}walks}: \mathrm{S}}{\downarrow}\mathrm{E}$$

The following two sections generalise the calculus that we have presented and illustrated.

2 Bridge and Split Operators

We propose here to enrich the discontinuity calculus of the previous section with two unary operators ^ (bridge) and $(split) which relate continuous and split strings. Bridging is to map split strings into strings and splitting is to map strings into split strings, so the formulas are extended as follows.

(22)
$$\begin{array}{lll} \mathcal{F} & ::= & \mathcal{A} \mid \mathcal{F}/\mathcal{F} \mid \mathcal{F}\backslash\mathcal{F} \mid \mathcal{F}{\bullet}\mathcal{F} \mid \mathcal{G}{<}\mathcal{F} \mid \mathcal{F}{>}\mathcal{G} \mid \mathcal{G}{\downarrow}\mathcal{F} \mid \mathcal{G}{\odot}\mathcal{F} \mid {}^\wedge\mathcal{G} \\ \mathcal{G} & ::= & \mathcal{F}{\diamond}\mathcal{F} \mid \mathcal{F}{\uparrow}\mathcal{F} \mid \$\mathcal{F} \end{array}$$

Interpretation is now made with respect to a *connection set* J which is a subset of L that contains ϵ. Intuitively, connections allow for hypothetical reasoning over discontinuous strings as if they were continuous, by supposing that they are connected; the connection set includes ϵ because the empty string always connects adjacent strings. The interpretations of the earlier binary operators are independent of the connection set and are as given before. However, ^ and $ behave as an existential and a universal respectively with respect to the connection set. Signs in $\$A$ are split strings which, joined by any connection, give a string in A; signs in ${}^\wedge A$ are the results of joining by some connection some split string in A.[2]

(23)
$$\begin{array}{lcl} D(\$A) & = & \{\langle s_1, s_2\rangle \mid \forall s \in J, s_1{+}s{+}s_2 \in D(A)\} \\ D({}^\wedge A) & = & \{s \mid \exists s_1, s_2, s', s = s_1{+}s'{+}s_2 \ \&\ s' \in J \ \&\ \langle s_1, s_2\rangle \in D(A)\} \end{array}$$

Labelled natural deduction rules are read off the interpretation clauses as before, and again the problematic existential elimination is excluded. Connections are represented $j(\alpha)$; because ϵ is a connection in all models, $j(\epsilon)$ may be assumed freely, but other connections assumed will have to be conditionalised in logical deductions.

(24)
$$\dfrac{\begin{matrix}\vdots & \vdots \\ (\gamma_1, \gamma_2)\colon \$A & j(\beta)\end{matrix}}{\gamma_1{+}\beta{+}\gamma_2\colon A}\,\$\mathrm{E} \qquad\qquad \dfrac{\begin{matrix}\overline{j(\mathbf{b})}^{\,n} \\ \vdots \\ \gamma_1{+}\mathbf{b}{+}\gamma_2\colon A\end{matrix}}{(\gamma_1, \gamma_2)\colon \$A}\,\$\mathrm{I}^n$$

(25)
$$\dfrac{\begin{matrix}\vdots & \vdots \\ (\gamma_1, \gamma_2)\colon A & j(\beta)\end{matrix}}{\gamma_1{+}\beta{+}\gamma_2\colon {}^\wedge A}\,{}^\wedge\mathrm{I}$$

The bridge and split type-constructors form a conjugate pair of respectively existential and universal modal operators (see Moortgat 1995) and thus satisfy laws such as $A \Rightarrow \$^\wedge A$:

(26)
$$\dfrac{\dfrac{(\alpha_1, \alpha_2)\colon A \qquad \overline{j(\mathbf{b})}^{\,1}}{\alpha_1{+}\mathbf{b}{+}\alpha_2\colon {}^\wedge A}\,{}^\wedge\mathrm{I}}{(\alpha_1, \alpha_2)\colon \$^\wedge A}\,\$\mathrm{I}^1$$

By way of linguistic illustration, we observe that assignment of a type R/^(S↑N) to a relative pronoun allows medial relativisation such as 'that Mary sent to John' to be generated as a continuous string:

(27)
$$\dfrac{\mathbf{that}\colon \mathrm{R}/{}^\wedge(\mathrm{S}{\uparrow}\mathrm{N}) \qquad \dfrac{\dfrac{\dfrac{\mathbf{Mary}\colon \mathrm{N} \qquad \dfrac{\dfrac{\mathbf{sent}\colon ((\mathrm{N}\backslash\mathrm{S})/\mathrm{PP})/\mathrm{N} \qquad \overline{\mathbf{b}\colon \mathrm{N}}^{\,1}}{\mathbf{sent{+}b}\colon (\mathrm{N}\backslash\mathrm{S})/\mathrm{PP}}\,/\mathrm{E} \qquad \mathbf{to{+}John}\colon \mathrm{PP}}{\mathbf{sent{+}b{+}to{+}John}\colon \mathrm{N}\backslash\mathrm{S}}\,/\mathrm{E}}{\mathbf{Mary{+}sent{+}b{+}to{+}John}\colon \mathrm{S}}\,\backslash\mathrm{E}}{\mathbf{(Mary{+}sent,\ to{+}John)}\colon \mathrm{S}{\uparrow}\mathrm{N}}\,{\uparrow}\mathrm{I}^1 \qquad j(\epsilon)}{\mathbf{Mary{+}sent{+}to{+}John}\colon {}^\wedge(\mathrm{S}{\uparrow}\mathrm{N})}\,{}^\wedge\mathrm{I}}{\mathbf{that{+}Mary{+}sent{+}to{+}John}\colon \mathrm{R}}\,/\mathrm{E}$$

The subsequent section introduces the second generalisation, relating to binary operators.

[2] The operators are identity mappings with respect to the semantic dimension of signs. Note that as a logical basic type J would obey $A \Rightarrow A{\bullet}J$ (and $A \Rightarrow J{\bullet}A$), but not $A{\bullet}J \Rightarrow A$ (or $J{\bullet}A \Rightarrow A$), whereas a true product unit I with $D(I) = \{\epsilon\}$ would obey all four such laws. With an I-type connection set the interpretations become:

$$\begin{array}{lcl} D(\$A) & = & \{\langle s_1, s_2\rangle \mid s_1{+}s_2 \in D(A)\} \\ D({}^\wedge A) & = & \{s \mid \exists s_1, s_2, s = s_1{+}s_2 \ \&\ \langle s_1, s_2\rangle \in D(A)\} \end{array}$$

We emphasise the J-version because the commutativity of the neutral element ($s{+}\epsilon = \epsilon{+}s$) would undermine certain intended applications. Observe that so far as prosodic dimensions of signs are concerned we could define $\$A$ as $A{\uparrow}J$ and ${}^\wedge A$ as $A{\odot}J$, but semantically it is unclear how to give meanings to unit types, or a meaning in $A{\uparrow}J$ which, applied to that of J returns itself as value. For these reasons we propose unary modalities and not unit types.

3 Generalised Sorted Discontinuity Calculus

The concatenation, juxtaposition, and interpolation adjunctions of the discontinuity calculus can be illustrated as follows:

(28)
$$\boxed{\alpha}+\boxed{\beta} \quad = \quad \boxed{\alpha}\boxed{\beta}$$
$$(\boxed{\alpha}, \boxed{\beta}) \quad = \quad \boxed{\alpha}\cdot\cdot\boxed{\beta}$$
$$\boxed{\alpha}\cdot\cdot\boxed{\gamma}\ W\ \boxed{\beta} \quad = \quad \boxed{\alpha}\boxed{\beta}\boxed{\gamma}$$

These are natural operations in the realm of strings and split strings, but others are imaginable, and linguistically motivated. In particular we consider here a generalisation of the discontinuity calculus in which each initial adjunction is augmented with variants (though these do not exhaust the conceivable options). Juxtaposition and interpolation have left and right variants according to positions of split points; concatenation has a single staggered variant because of the absence of a split point.

(29)

Discont. Calculus	Generalised Discont. Calculus
$+$: concatenation	$+_2$: staggered concatenation
$(., .)$: juxtaposition	$(.,_l\ .)$: juxtaposition inheriting split on the left $(.,_r\ .)$: juxtaposition inheriting split on the right
W: interpolation	W_l: interpolation at the left interior W_r: interpolation at the right interior

The staggered concatenation adjunction is of functionality $L^2, L^2 \to L$; left and right inheriting juxtaposition are of functionality $L^2, L \to L^2$ and $L, L^2 \to L^2$ respectively; the left and right interior interpolations are of functionality $L^2, L \to L^2$. The variants perform the same basic role as their mother operations: concatenation outputs strings, juxtaposition outputs split strings, preserving operand order; interpolation performs a wrapping of the first operand around the second. The definitions of the new adjunctions are as follows.

(30)

$\langle\alpha,\beta\rangle+_2\langle\gamma,\delta\rangle =_{df} \alpha+\gamma+\beta+\delta$					
$(\langle\alpha,\beta\rangle,_l\gamma)$	$=_{df}$	$\langle\alpha,\beta+\gamma\rangle$	$(\alpha,_r\langle\beta,\gamma\rangle)$	$=_{df}$	$\langle\alpha+\beta,\gamma\rangle$
$\langle\alpha,\gamma\rangle W_l\beta$	$=_{df}$	$\langle\alpha+\beta,\gamma\rangle$	$\langle\alpha,\gamma\rangle W_r\beta$	$=_{df}$	$\langle\alpha,\beta+\gamma\rangle$

More graphically, we have (31).

(31)
$$\boxed{\alpha}\cdot\cdot\boxed{\beta} +_2 \boxed{\gamma}\cdot\cdot\boxed{\delta} = \boxed{\alpha}\boxed{\gamma}\boxed{\beta}\boxed{\delta}$$
$$(\boxed{\alpha}\cdot\cdot\boxed{\beta},_l \boxed{\gamma}) \quad = \quad \boxed{\alpha}\cdot\cdot\boxed{\beta}\boxed{\gamma} \qquad (\boxed{\alpha},_r \boxed{\beta}\cdot\cdot\boxed{\gamma}) \quad = \quad \boxed{\alpha}\boxed{\beta}\cdot\cdot\boxed{\gamma}$$
$$(\boxed{\alpha}\cdot\cdot\boxed{\gamma}\ W_l\ \boxed{\beta}) \quad = \quad \boxed{\alpha}\boxed{\beta}\cdot\cdot\boxed{\gamma} \qquad (\boxed{\alpha}\cdot\cdot\boxed{\gamma}\ W_r\ \boxed{\beta}) \quad = \quad \boxed{\alpha}\cdot\cdot\boxed{\beta}\boxed{\gamma}$$

The community of discontinuity connectives becomes generalised to (32) where $m \in \{l, r, 0\}$, $n \in \{2, 0\}$, and the zero variants, those we already had, will continue to be written without explicit subscript.

(32)

$+_n$	$\backslash_n$	under (staggered)	$/_n$	over (staggered)	$\bullet_n$
$(.,_m\ .)$	$>_m$	to (left, right)	$<_m$	from (left, right)	$\diamond_m$
W_m	$\uparrow_m$	extract (left, right)	$\downarrow_m$	infix (left, right)	$\odot_m$

Interpretation is made by residuation in just the same way as has been seen earlier. By way of example, for the staggered concatenation family we have:

(33)
$$\begin{array}{lcl} D(A\backslash_2 B) & = & \{\langle s_3, s_4\rangle |\ \forall\langle s_1, s_2\rangle \in D(A), s_1{+}s_3{+}s_2{+}s_4 \in D(B)\} \\ D(B/_2 A) & = & \{\langle s_1, s_2\rangle |\ \forall\langle s_3, s_4\rangle \in D(A), s_1{+}s_3{+}s_2{+}s_4 \in D(B)\} \\ D(A\bullet_2 B) & = & \{s_1{+}s_3{+}s_2{+}s_4 |\ \langle s_1, s_2\rangle \in D(A)\ \&\ \langle s_3, s_4\rangle \in D(B)\} \end{array}$$

We do not list labelled deduction rules since these are entirely predictable, being obtained in just the same way as those for the original discontinuity calculus. To mention a single case, introduction of staggered product is (34).

(34)
$$\dfrac{\begin{matrix} \vdots & \qquad & \vdots \\ (\alpha_1, \alpha_2)\colon A & \qquad & (\beta_1, \beta_2)\colon B \end{matrix}}{\alpha_1{+}\beta_1{+}\alpha_2{+}\beta_2\colon A\bullet_2 B}\bullet_2\mathrm{I}$$

3.1 Examples

3.1.1 Gapping as Like-Category Coordination

We present a characterisation of gapping as almost like category coordination. We take our inspiration from Hendriks (1995), but our generalisation of the discontinuity calculus is different from hers; in particular our generalised discontinuity calculus has the computationally convenient sorted formulation which the generalisation of Hendriks necessarily lacks,[3] but our analysis can be said to be borrowed. We treat the example 'John studies logic and Charles phonetics' by assigning 'and' the almost like category coordinator type (X$>_l$X)/$^\wedge$X where X is S$\uparrow$TV and TV is (N\S)/N, with semantics $\lambda x\lambda y\lambda z[(y\ z) \wedge (x\ z)]$.

(35)
$$\dfrac{(\mathbf{John}, \mathbf{logic})\colon \mathrm{S}\uparrow\mathrm{TV} \qquad \dfrac{\mathrm{and} \qquad \dfrac{\dfrac{\dfrac{\mathbf{Charles}\colon \mathrm{N} \quad \overline{\mathbf{a}\colon \mathrm{TV}}^{\,1} \quad \mathbf{phn}\colon \mathrm{N}}{\mathbf{Charles{+}a{+}phn}\colon \mathrm{S}}}{(\mathbf{Charles}, \mathbf{phn})\colon \mathrm{S}\uparrow\mathrm{TV}}\uparrow\mathrm{I}^1 \qquad j(\epsilon)}{\mathbf{Charles{+}phn}\colon {}^\wedge(\mathrm{S}\uparrow\mathrm{TV})}{}^\wedge\mathrm{I}}{\mathbf{and{+}Charles{+}phn}\colon \mathrm{X}{>_l}\mathrm{X}}/\mathrm{E}}{(\mathbf{John}, \mathbf{logic{+}and{+}Charles{+}phn})\colon \mathrm{S}\uparrow\mathrm{TV}}{>_r}\mathrm{E}$$

Starting at the top right hand corner, 'Charles **a** phonetics' is derived straightforwardly as a sentence from the hypothetical transitive verb **a**. The hypothetical can be withdrawn to yield a split form which wants to wrap around a transitive verb to form a sentence. This is mapped by $^\wedge$I which fuses the right hand conjunct to a string of the right type for the coordinator to consume by over elimination, which prefixes the coordinator. The left hand conjunct 'John logic' is also derivable as S$\uparrow$TV, in just the same way as 'Charles phonetics'; when the coordinator combines with this conjunct, by to left elimination, the split marking of *this* conjunct is inherited by the result, again in type S$\uparrow$TV. So this will wrap around the transitive verb interpolating it in the first conjunct, and distributing its semantics over the conjuncts. The semantics is spelled out in (36).

[3] The problem is that the new interaction principle [MA], p.113 requires Δ_2 to be of split string sort *qua* the split operand of wrap (her notation is swapped relative to ours); but then since the new g-mode gets a string sort left operand in the top line, it cannot take first operand Δ_2, of split string sort, in the second line.

(36)

$$\dfrac{\lambda x((x\ \mathbf{logic})\ \mathbf{j}) \qquad \dfrac{\lambda x\lambda y\lambda z[(y\ z)\wedge(x\ z)] \qquad \dfrac{\dfrac{\dfrac{\mathbf{c}\quad \overset{-1}{x}\quad \mathbf{phn}}{((x\ \mathbf{phn})\ \mathbf{c})}}{\lambda x((x\ \mathbf{phn})\ \mathbf{c})}\uparrow\mathrm{I}^1}{\lambda x((x\ \mathbf{phn})\ \mathbf{c})}{}^\wedge\mathrm{I}}{\lambda y\lambda z[(y\ z)\wedge((z\ \mathbf{phn})\ \mathbf{c})]}/\mathrm{E}}{\lambda z[((z\ \mathbf{logic})\ \mathbf{j})\wedge((z\ \mathbf{phn})\ \mathbf{c})]}>_r\mathrm{E}$$

The last step illustrates inference with a inheriting juxtaposition. Our next example will illustrate staggered concatenation.

3.1.2 Comparative Subdeletion

We make the second illustration of generalised discontinuity with reference to comparative subdeletion. Again the treatment is inspired by Hendriks (1995), but it uses the present sorted calculus, and the analysis assumes that 'more ...than' in examples such as the following has a unitary meaning.

(37) a. More sheep ran than fish swam.
b. John ate more bagels than Mary ate donuts.

Our analytical perspective is that 'more ... than' combines with two sentences each lacking one quantifier; 'more' occupies the determiner gap in the first, and the two sentences are conjoined with 'than'. Semantically there is a comparison, in the case of (37b) for example, between the cardinality of the set of bagels that John ate, and the cardinality of the set of donuts that Mary ate. The construction is triggered by the following lexical assignment, where Q abbreviates the quantifier type $((S{\uparrow}N){\downarrow}S)/CN$.

(38) (**more**, **than**) $-$ $\lambda x\lambda y[\lambda z(x\ \lambda p\lambda q[(p\ z)\wedge(q\ z)]) > \lambda z(y\ \lambda p\lambda q[(p\ z)\wedge(q\ z)])]$
$:=$ $(S{\uparrow}Q)\backslash_2(S/{}^\wedge(S{\uparrow}Q))$

Then there is the following derivation of (37b), where TV again abbreviates $(N\backslash S)/N$.

(39)

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathbf{John}\text{: N}\quad \mathbf{ate}\text{: TV}\quad \overline{\mathbf{a}\text{: Q}}^{1}\quad \mathbf{bagels}\text{: CN}}{\mathbf{John{+}ate{+}a{+}bagels}\text{: S}}}{(\mathbf{John{+}ate},\ \mathbf{bagels})\text{: S}{\uparrow}\text{Q}}\uparrow\mathrm{I}^1 \qquad \text{more than}}{\mathbf{John{+}ate{+}more{+}bagels{+}than}\text{: S}/{}^\wedge(\text{S}{\uparrow}\text{Q})}\backslash_2\mathrm{E} \qquad \dfrac{(\mathbf{Mary{+}ate},\ \mathbf{donuts})\text{: S}{\uparrow}\text{Q}\quad j(\epsilon)}{\mathbf{Mary{+}ate{+}donuts}\text{: }{}^\wedge(\text{S}{\uparrow}\text{Q})}{}^\wedge\mathrm{I}}{\mathbf{John{+}ate{+}more{+}bagels{+}than{+}Mary{+}ate{+}donuts}\text{: S}}/\mathrm{E}$$

Observe in particular the staggered concatenation inference step $\backslash_2$E with combines (**John+ate**, **bagels**) with (**more**, **than**) to yield **John+ate+more +bagels+than**. The semantics of (39) is as follows.

(40)

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathbf{ate}\quad \overset{-1}{w}\quad \mathbf{bagel}}{((w\ \mathbf{bagel})\ \lambda u((\mathbf{ate}\ u)\ \mathbf{j}))}}{\lambda w((w\ \mathbf{bagel})\ \lambda u((\mathbf{ate}\ u)\ \mathbf{j}))}\uparrow\mathrm{I}^1 \qquad \text{more than}}{\lambda y[\lambda z[(\mathbf{bagel}\ z)\wedge((\mathbf{ate}\ z)\ \mathbf{j})] > \lambda z(y\ \lambda p\lambda q[(p\ z)\wedge(q\ z)])]}\backslash_2 \qquad \dfrac{\lambda w((w\ \mathbf{donut})\ \lambda u((\mathbf{ate}\ u)\ \mathbf{m}))}{\lambda w((w\ \mathbf{donut})\ \lambda u((\mathbf{ate}\ u)\ \mathbf{m}))}{}^\wedge\mathrm{I}}{[\lambda z[(\mathbf{bagel}\ z)\wedge((\mathbf{ate}\ z)\ \mathbf{j})] > \lambda z[(\mathbf{donut}\ z)\wedge((\mathbf{ate}\ z)\ \mathbf{m})]}/\mathrm{E}$$

The relevant comparison of cardinalities is indeed made.

Rather than continue here with linguistic illustration of interior edge interpolation we pass on directly to consider computational aspects.

4 Computation

The current section shows how the generalised discontinuity proposals fit into the paradigm for logic programming of categorial deduction developed in Morrill (1995a, 1995c) and Lloré and Morrill (1995). The general proposal is to compile categorial assignments into clauses of linear logic. The compilation is performed systematically, according to the interpretations of categorial type-constructors; the target formalism of a linear logic programming fragment (Hodas and Miller 1994) is suitable because it is the most specific level of propositional logic embracing all the sublinear categorial calculi with their discontinuity, partial commutativity, and so forth, and because in structuring resources as bags rather than lists, we eliminate the need to conjecture partition points of ordered sequents, a source of inefficient don't know non-determinism indigenous to Lambek sequent deduction.

It is possible to work just with algebraic interpretation, as in Morrill (1995a), but in Morrill (1995c) and Lloré and Morrill (1995) it is observed that by exploiting the binary relational models (van Benthem 1991) of associative Lambek calculus one can avoid computation of matching under associativity, and instead propagate constraints under associativity by methods analogous to the use of string positions/difference lists in the logic programming of DCGs.

Both the original sorted discontinuity calculus and its generalisation here can be interpreted and implemented according to just binary relational models. However, because linguistically we wish to be able to represent not only precedence relations, but also dominance relations (e.g. using bracket operators; Morrill 1992, 1994 ch. 7) the algebraic dimension is needed to induce hierarchical structure that cannot be captured in binary relations. For this reason, we deal here with the more general problem of interpretation and computation according to combined algebraic and relational models. In this general setting matching under associativity is not altogether avoided. However we combine algebraic and relational style models into multidimensional hybrid models which allow us to exploit constraint propagation and adopt a lazy approach to computation of matching under associativity, by only attempting to check the algebraic conditions once satisfaction of the binary relational conditions have been confirmed.

4.1 Hybrid Models

We begin by reviewing the hybrid models for the sorted version of the original discontinuity calculus. Interpretation takes place relative to a monoid $\langle L, +, \epsilon\rangle$ and a set V. Each formula A of sort string has an interpretation $D(A) \subseteq V^2 \times L$ and each formula A of sort split string has an interpretation $D(A) \subseteq V^4 \times L^2$.

The family of connectives $\{/, \backslash, \bullet\}$ are defined by residuation with respect to a concatenation adjunction of functionality $V^2 \times L, V^2 \times L \to V^2 \times L$. The adjunction is a partial operation, defined on $\langle v_1, v_2, s_1\rangle$ and $\langle v_3, v_4, s_2\rangle$ (respectively) just in case $v_2 = v_3$, in which case its value is $\langle v_1, v_4, s_1{+}s_2\rangle$.

$$
\begin{array}{llcl}
(41) & D(A\backslash B) & = & \{\langle v_2, v_3, s\rangle |\ \forall\langle v_1, v_2, s'\rangle \in D(A), \\
 & & & \qquad\qquad \langle v_1, v_3, s'{+}s\rangle \in D(B)\} \\
 & D(B/A) & = & \{\langle v_1, v_2, s\rangle |\ \forall\langle v_2, v_3, s'\rangle \in D(A), \\
 & & & \qquad\qquad \langle v_1, v_3, s{+}s'\rangle \in D(B)\} \\
 & D(A{\bullet}B) & = & \{\langle v_1, v_3, s\rangle |\ \exists v_2, s_1, s_2, s = s_1{+}s_2 \\
 & & & \qquad\qquad \&\ \langle v_1, v_2, s_1\rangle \in D(A) \\
 & & & \qquad\qquad \&\ \langle v_2, v_3, s_2\rangle \in D(B)\}
\end{array}
$$

The family of connectives $\{<, >, \circ\}$ are defined by residuation with respect to a juxtaposition adjunction of functionality $V^2 \times L, V^2 \times L \to V^4 \times L^2$. It is defined as Cartesian product formation:

applied to $\langle v_1, v_2, s_1\rangle$ and $\langle v_3, v_4, s_2\rangle$ (respectively) its value is $\langle v_1, v_2, v_3, v_4, s_1, s_2\rangle$.

(42)
$$\begin{array}{lcl}
D(A{>}B) & = & \{\langle v_3, v_4, s\rangle |\ \forall\langle v_1, v_2, s'\rangle \in D(A),\\
 & & \qquad \langle v_1, v_2, v_3, v_4, s', s\rangle \in D(B)\}\\
D(B{<}A) & = & \{\langle v_1, v_2, s\rangle |\ \forall\langle v_3, v_4, s'\rangle \in D(A),\\
 & & \qquad \langle v_1, v_2, v_3, v_4, s, s'\rangle \in D(B)\}\\
D(A{\diamond}B) & = & \{\langle v_1, v_2, v_3, v_4, s_1, s_2\rangle |\\
 & & \qquad \langle v_1, v_2, s_1\rangle \in D(A)\\
 & & \qquad \&\ \langle v_3, v_4, s_2\rangle \in D(B)\}
\end{array}$$

The family of connectives $\{\uparrow, \downarrow, \odot\}$ are defined by residuation with respect to an interpolation adjunction of functionality $V^4 \times L^2, V^2 \times L \to V^2 \times L$. It is a partial operation, defined on $\langle v_1, v_2, v_3, v_4, s_1, s_2\rangle$ and $\langle v_5, v_6, s\rangle$ (respectively) just in case $v_2 = v_5$ and $v_3 = v_6$, in which case its value is $\langle v_1, v_4, s_1{+}s{+}s_2\rangle$.

(43)
$$\begin{array}{lcl}
D(A{\downarrow}B) & = & \{\langle v_2, v_3, s\rangle |\ \forall\langle v_1, v_2, v_3, v_4, s_1, s_2\rangle \in D(A),\\
 & & \qquad \langle v_1, v_4, s_1{+}s{+}s_2\rangle \in D(B)\}\\
D(B{\uparrow}A) & = & \{\langle v_1, v_2, v_3, v_4, s_1, s_2\rangle |\ \forall\langle v_2, v_3, s\rangle \in D(A),\\
 & & \qquad \langle v_1, v_4, s_1{+}s{+}s_2\rangle \in D(B)\}\\
D(A{\odot}B) & = & \{\langle v_1, v_4, s\rangle |\ \exists v_2, v_3, s_1, s', s_2, s = s_1{+}s'{+}s_2\\
 & & \qquad \&\ \langle v_1, v_2, v_3, v_4, s_1, s_2\rangle \in D(A)\\
 & & \qquad \&\ \langle v_2, v_3, s'\rangle \in D(B)\}
\end{array}$$

Exactly as before we can present a tree-style linear natural deduction proof format with the sublinear conditions expressed by labels. We use naturals as labels naming elements of V; we use $I, J, K, \ldots$ in italic as variables over V labels, in boldface for uniquely occuring V constants. A string label is of the form $I - J - \alpha$ and a split string label is of the form $(I - J, K - L) - (\alpha, \beta)$. Again, the labelled natural deduction rules can be seen as a restatement left-to-right and right-to-left of the bidirectional interpretation clauses rotated ninety degrees clockwise for the elimination (E) rules and anticlockwise the introduction (I) rules, with metavariables or Skolem constants according to quantifiers and the polarity of their context.

(44)
$$\dfrac{\begin{array}{c}\vdots\\ I - J - \alpha\colon A\end{array}\qquad \begin{array}{c}\vdots\\ J - K - \gamma\colon A\backslash B\end{array}}{I - K - \alpha{+}\gamma\colon B}\backslash\mathrm{E}
\qquad\qquad
\dfrac{\begin{array}{c}\overline{\mathbf{I} - J - \mathbf{a}\colon A}^{\,n}\\ \vdots\\ \mathbf{I} - K - \mathbf{a}{+}\gamma\colon B\end{array}}{J - K - \gamma\colon A\backslash B}\backslash\mathrm{I}^n$$

(45)
$$\dfrac{\begin{array}{c}\vdots\\ I - J - \gamma\colon B/A\end{array}\qquad \begin{array}{c}\vdots\\ J - K - \alpha\colon A\end{array}}{I - K - \gamma{+}\alpha\colon B}/\mathrm{E}
\qquad\qquad
\dfrac{\begin{array}{c}\overline{J - \mathbf{K} - \mathbf{a}\colon A}^{\,n}\\ \vdots\\ I - \mathbf{K} - \gamma{+}\mathbf{a}\colon B\end{array}}{I - J - \gamma\colon B/A}/\mathrm{I}^n$$

(46)
$$\dfrac{\begin{array}{c}\vdots\\ I - J - \alpha\colon A\end{array}\qquad \begin{array}{c}\vdots\\ J - K - \beta\colon B\end{array}}{I - K - \alpha{+}\beta\colon A{\bullet}B}\bullet\mathrm{I}$$

(47)
$$\dfrac{\begin{array}{c}\vdots\\ I - J - \alpha\colon A\end{array}\qquad \begin{array}{c}\vdots\\ K - L - \gamma\colon A{>}B\end{array}}{(I - J, K - L) - (\alpha, \gamma)\colon B}{>}\mathrm{E}
\qquad\qquad
\dfrac{\begin{array}{c}\overline{\mathbf{I} - \mathbf{J} - \mathbf{a}\colon A}^{\,n}\\ \vdots\\ (\mathbf{I} - \mathbf{J}, K - L) - (\mathbf{a}, \gamma)\colon B\end{array}}{K - L - \gamma\colon A{>}B}{>}\mathrm{I}^n$$

(48)
$$\dfrac{\begin{array}{c}\vdots\\ I - J - \gamma\colon B{<}A\end{array} \quad \begin{array}{c}\vdots\\ K - L - \alpha\colon A\end{array}}{(I - J, K - L) - (\gamma, \alpha)\colon B}{<}\mathrm{E} \qquad \begin{array}{c}\overline{\mathbf{J} - \mathbf{K} - \mathbf{a}\colon A}^{n}\\ \vdots\\ \dfrac{(I - J, \mathbf{J} - \mathbf{K}) - (\gamma, \mathbf{a})\colon B}{I - J - \gamma\colon B{<}A}{<}\mathrm{I}^{n}\end{array}$$

(49)
$$\dfrac{\begin{array}{c}\vdots\\ I - J - \alpha\colon A\end{array} \quad \begin{array}{c}\vdots\\ K - L - \beta\colon B\end{array}}{(I - J, K - L) - (\alpha, \beta)\colon A{\diamond}B}{\diamond}\mathrm{I}$$

(50)
$$\dfrac{\begin{array}{c}\vdots\\ (I - J, K - L) - (\alpha_1, \alpha_2)\colon A\end{array} \quad \begin{array}{c}\vdots\\ J - K - \gamma\colon A{\downarrow}B\end{array}}{I - L - \alpha_1{+}\gamma{+}\alpha_2\colon B}{\downarrow}\mathrm{E} \qquad \begin{array}{c}\overline{(\mathbf{I} - J, K - \mathbf{L}) - (\mathbf{a}_1, \mathbf{a}_2)\colon A}^{n}\\ \vdots\\ \dfrac{\mathbf{I} - \mathbf{L} - \mathbf{a}_1{+}\gamma{+}\mathbf{a}_2\colon B}{J - K - \gamma\colon A{\downarrow}B}{\downarrow}\mathrm{I}^{n}\end{array}$$

(51)
$$\dfrac{\begin{array}{c}\vdots\\ (I - J, K - L) - (\gamma_1, \gamma_2)\colon B{\uparrow}A\end{array} \quad \begin{array}{c}\vdots\\ J - K - \alpha\colon A\end{array}}{I - L - \gamma_1{+}\alpha{+}\gamma_2\colon B}{\uparrow}\mathrm{E} \qquad \begin{array}{c}\overline{J - K - \mathbf{a}\colon A}^{n}\\ \vdots\\ \dfrac{I - L - \gamma_1{+}\mathbf{a}{+}\gamma_2\colon B}{(I - J, K - L) - (\gamma_1, \gamma_2)\colon B{\uparrow}A}{\uparrow}\mathrm{I}^{n}\end{array}$$

(52)
$$\dfrac{\begin{array}{c}\vdots\\ (I - J, K - L) - (\alpha_1, \alpha_2)\colon A\end{array} \quad \begin{array}{c}\vdots\\ J - K - \beta\colon B\end{array}}{I - L - \alpha_1{+}\beta{+}\alpha_2\colon A{\odot}B}{\odot}\mathrm{I}$$

Such labelled natural deduction for the generalisations of discontinuity are obtained directly. In (52) we give the example of just staggered product introduction.

(53)
$$\dfrac{\begin{array}{c}\vdots\\ (I - J, K - L) - (\alpha_1, \alpha_2)\colon A\end{array} \quad \begin{array}{c}\vdots\\ (J - K, L - M) - (\beta_1, \beta_2)\colon B\end{array}}{I - M - \alpha_1{+}\beta_1{+}\alpha_2{+}\beta_2\colon A{\bullet_2}B}{\bullet_2}\mathrm{I}$$

Before considering the logic programming, we repeat some earlier derivations, now with the hybrid model labelling. With respect to the VP Ellipsis example, we begin analysis according to occurrences with string positions named as shown in (54).

(54) $_0$ Dan $_1$ likes $_2$ golf $_3$ and $_4$ George $_5$ does $_6$ too $_7$

Then there is the following derivation of ‘likes golf and George does too’ in N\S=VP.

(55)
$$\dfrac{1 - 3 - \textbf{likes+golf}\colon \mathrm{VP} \quad \dfrac{(3 - 4,\ 5 - 7) - (\textbf{and},\ \textbf{does+too})\colon (\mathrm{VP}\backslash\mathrm{VP}){\uparrow}\mathrm{N} \quad 4 - 5 - \textbf{George}\colon \mathrm{N}}{3 - 7 - \textbf{and+George+does+too}\colon \mathrm{VP}\backslash\mathrm{VP}}{\uparrow}\mathrm{E}}{1 - 7 - \textbf{likes+golf+and+George+does+too}\colon \mathrm{VP}}\backslash\mathrm{E}$$

Similarly, for the wide scope “or” case assume the string positions (56).

(56) $_0$ John $_1$ thinks $_2$ Bill $_3$ or $_4$ Mary $_5$ walks $_6$

Then the narrow scope and wide scope readings are delivered by the following two derivations in which the coordinate structure substitutes in below and above ‘thinks’ respectively.

(57)

$$\dfrac{\dfrac{\dfrac{\overline{2-5-\mathbf{a}\colon \mathrm{N}}^{1} \quad 5-6-\mathbf{walks}\colon \mathrm{N}\backslash\mathrm{S}}{2-6-\mathbf{a{+}walks}\colon \mathrm{S}}\backslash E}{(2-2,\ 5-6)-(\epsilon,\ \mathbf{walks})\colon \mathrm{S}{\uparrow}\mathrm{N}}{\uparrow}I^{1} \quad \dfrac{2-3-\mathbf{Bill}\colon \mathrm{N} \quad \dfrac{3-4-\mathbf{or}\colon (\mathrm{N}\backslash((\mathrm{S}{\uparrow}\mathrm{N}){\downarrow}\mathrm{S}))/\mathrm{N} \quad 4-5-\mathbf{Mary}\colon \mathrm{N}}{3-5-\mathbf{or{+}Mary}\colon \mathrm{N}\backslash((\mathrm{S}{\uparrow}\mathrm{N}){\downarrow}\mathrm{S})}/E}{2-5-\mathbf{Bill{+}or{+}Mary}\colon (\mathrm{S}{\uparrow}\mathrm{N}){\downarrow}\mathrm{S}}\backslash E}{2-5-\mathbf{Bill{+}or{+}Mary{+}walks}\colon \mathrm{S}}{\downarrow}E$$

(58)

$$\dfrac{\dfrac{\dfrac{0-1-\mathbf{John}\colon \mathrm{N} \quad \dfrac{1-2-\mathbf{thinks}\colon (\mathrm{N}\backslash\mathrm{S})/\mathrm{S} \quad \dfrac{\overline{2-5-\mathbf{a}\colon \mathrm{N}}^{1} \quad 5-6-\mathbf{walks}\colon \mathrm{N}\backslash\mathrm{S}}{2-6-\mathbf{a{+}walks}\colon \mathrm{S}}\backslash E}{1-6-\mathbf{thinks{+}a{+}walks}\colon \mathrm{N}\backslash\mathrm{S}}/E}{0-6-\mathbf{John{+}thinks{+}a{+}walks}\colon \mathrm{S}}\backslash E}{(\mathrm{O}-2,\ 5-6)-(\mathbf{John{+}thinks},\ \mathbf{walks})\colon \mathrm{S}{\uparrow}\mathrm{N}}{\uparrow}I^{1} \quad 6-7-\mathbf{Bill{+}or{+}Mary}\colon (\mathrm{S}{\uparrow}\mathrm{N}){\downarrow}\mathrm{S}}{0-7-\mathbf{John{+}thinks{+}Bill{+}or{+}Mary{+}walks}\colon \mathrm{S}}{\downarrow}E$$

In the hybrid models for the unary bridge and split modalities interpretation is made with respect to a connection set which is a subset of $V^2 \times L$ that is a superset of $\{\langle v, v, \epsilon\rangle |\ v \in V\}$; that is, the family of possible connection sets is (59).

(59) $\{J \subseteq V \times V \times L |\ \forall v, \langle v, v, \epsilon\rangle \in J\}$

The operators $^\wedge$ and $\$$ behave as an existential and a universal respectively with respect to the connection set J. Signs in $\$A$ are split strings which, joined by any connection give a string in A; signs in $^\wedge A$ are the results of joining by some connection some split string in A.

(60)
$$\begin{array}{rcl} D(\$A) & = & \{\langle v_1, v_2, v_3, v_4, s_1, s_2\rangle | \forall \langle v_2, v_3, s\rangle \in J, \\ & & \qquad \langle v_1, v_4, s_1{+}s{+}s_2\rangle \in D(A)\} \\ D(^\wedge A) & = & \{\langle v_1, v_4, s\rangle | \exists s_1, s_2, \langle v_2, v_3, s'\rangle \in J, s = s_1{+}s'{+}s_2 \\ & & \qquad \&\ \langle v_1, v_2, v_3, v_4, s_1, s_2\rangle \in D(A)\} \end{array}$$

Labelled natural deduction rules are read off the interpretation clauses as before. Connections are represented $j(I, J, \alpha)$ and instances of $j(I, I, \epsilon)$ may be introduced at any point as logical axioms.

(61)
$$\dfrac{\begin{matrix}\vdots\\ (I-J, K-L)-(\gamma_1, \gamma_2)\colon \$A\end{matrix} \qquad \begin{matrix}\vdots\\ j(J, K, \beta)\end{matrix}}{I-L-\gamma_1{+}\beta{+}\gamma_2\colon A}\$E \qquad \dfrac{\begin{matrix}\overline{j(J, K, \mathbf{b})}^{n}\\ \vdots\\ I-L-\gamma_1{+}\mathbf{b}{+}\gamma_2\colon A\end{matrix}}{(I-J, K-L)-(\gamma_1, \gamma_2)\colon \$A}\$I^{n}$$

(62)
$$\dfrac{\begin{matrix}\vdots\\ (I-J, K-L)-(\gamma_1, \gamma_2)\colon A\end{matrix} \qquad \begin{matrix}\vdots\\ j(J, K, \beta)\end{matrix}}{I-L-\gamma_1{+}\beta{+}\gamma_2\colon {}^\wedge A}{}^\wedge I$$

The hybrid derivation (63) shows the conjugate property $\sigma\colon A \Rightarrow \sigma\colon \$^\wedge A$ seen before for the purely relational labelling.

(63)
$$\dfrac{\dfrac{(I-J, K-L)-(\alpha_1, \alpha_2)\colon A \qquad \overline{j(J, K, \mathbf{b})}^{1}}{I-L-\alpha_1{+}\mathbf{b}{+}\alpha_2\colon {}^\wedge A}{}^\wedge I}{(I-J, K-L)-(\alpha_1, \alpha_2)\colon \$^\wedge A}\$I^{1}$$

The following derivation shows the hybrid version of medial extraction which was also been seen before in the non-hybrid case.

(64) $_0$ that $_1$ Mary $_2$ sent $_3$ to $_4$ John $_5$

(65)

$$\dfrac{\dfrac{\dfrac{1-2-\mathbf{Mary}\colon \mathrm{N} \qquad \dfrac{\dfrac{2-3-\mathbf{sent}\colon ((\mathrm{N}\backslash \mathrm{S})/\mathrm{PP})/\mathrm{N} \qquad \dfrac{}{3-3-\mathbf{b}\colon \mathrm{N}}1}{2-3-\mathbf{sent{+}b}\colon (\mathrm{N}\backslash \mathrm{S})/\mathrm{PP}}/\mathrm{E} \qquad 3-4-\mathbf{to{+}John}}{2-4-\mathbf{sent{+}b{+}to{+}John}\colon \mathrm{N}\backslash \mathrm{S}}/\mathrm{E}}{1-4-\mathbf{Mary{+}sent{+}b{+}to{+}John}\colon \mathrm{S}}\backslash \mathrm{E}}{(1-3,\, 3-4)-(\mathbf{Mary{+}sent},\, \mathbf{to{+}John})\colon \mathrm{S}{\uparrow}\mathrm{N}}{\uparrow}\mathrm{I}^1 \qquad j(3,3,\epsilon)}{1-4-\mathbf{Mary{+}sent{+}to{+}John}\colon {}^\wedge(\mathrm{S}{\uparrow}\mathrm{N})}{}^\wedge\mathrm{I}$$

Finally, we give hybrid version of the generalised discontinuity gapping treatment. It should be clear that the methods applying to yield hybrid models and labelled natural deduction for the generalised discontinuity are exactly the same as those for the basic discontinuity.

(66) $_0$ John $_1$ studies $_2$ logic $_3$ and $_4$ Charles $_5$ phonetics $_6$

(67)

$$\dfrac{(0-1,\, 2-3)-(\mathbf{John},\, \mathbf{logic})\colon \mathrm{S}{\uparrow}\mathrm{TV} \qquad \dfrac{\text{and} \qquad \dfrac{\dfrac{\dfrac{4-5-\mathbf{Charles}\colon \mathrm{N} \qquad \dfrac{}{5-5-\mathbf{a}\colon \mathrm{TV}}1 \qquad 5-6-\mathbf{phn}\colon \mathrm{N}}{4-6-\mathbf{Charles{+}a{+}phn}\colon \mathrm{S}}}{(4-5, 5-6)-(\mathbf{Charles}, \mathbf{phn})\colon \mathrm{S}{\uparrow}\mathrm{TV}}{\uparrow}\mathrm{I}^1 \qquad j(5,5,\epsilon)}{4-6-\mathbf{Charles{+}phn}\colon {}^\wedge\mathrm{S}{\uparrow}\mathrm{TV}}{}^\wedge\mathrm{I}}{3-6-\mathbf{and{+}Charles{+}phn}\colon \mathrm{X}{>_l}\mathrm{X}}/\mathrm{E}}{(0-1, 2-6)-(\mathbf{John}, \mathbf{logic{+}and{+}Charles{+}phn})\colon \mathrm{S}{\uparrow}\mathrm{TV}}{>_r}\mathrm{E}$$

The next section shows how the hybrid models we have introduced and illustrated support a clausal compilation and automated proof search.

4.2 Linear clausal fragment

Logic programming is a paradigm of computation as proof search. We represent a program as a list of program clauses Γ and the task of deciding whether a query A follows is the task of determining whether the sequent $\Gamma \Rightarrow A$ is derivable (output takes the form of computing values for the variables in A such that $\Gamma \Rightarrow A$). We assume, following the works referenced above, the linear clausal fragment (68).

(68)
$$\begin{array}{lcl} \mathcal{PCLS} & ::= & \mathcal{ATOM} \mathrel{\circ\!\!-} \mathcal{AGENDA} \\ \mathcal{AGENDA} & ::= & \mathbf{1} \mid \mathcal{GOAL} \otimes \mathcal{AGENDA} \\ \mathcal{GOAL} & ::= & \mathcal{ATOM} \mid (\mathcal{AGENDA} \mathrel{\circ\!\!-} \mathcal{PCLS}) \end{array}$$

Thus a program clause $\mathcal{PCLS}$ comprises an atomic head which is the postcondition of a linear implication, and a body which is a (possibly empty) list of goals which is the precondition of the implication. Goals may be atoms or may themselves be implications from a program clause to a list of goals. Such implicational goals are the only extension to ordinary Horn clauses exploited here.

The propositional linear logic programming rules are as follows. The termination condition requires all the program clauses to have been consumed so that the empty agenda only follows from the empty program database:

(69) $\Rightarrow \mathbf{1}$

The resolution rule uses up and therefore removes from the program database the program clause against which resolution is performed:

(70)
$$\frac{\Gamma \Rightarrow B_1 \otimes \ldots \otimes B_n \otimes C}{A \multimapinv B_1 \otimes \ldots \otimes B_n \otimes \mathbf{1}, \Gamma \Rightarrow A \otimes C}\text{RES}$$

The deduction theorem rule distributes the program clauses between its two premises:

(71)
$$\frac{A, \Gamma \Rightarrow B \qquad \Delta \Rightarrow C}{\Gamma, \Delta \Rightarrow (B \multimapinv A) \otimes C}\text{DT}$$

The rule requires us to show that $B \multimapinv A$ follows from some of the premises, Γ, while the remainder Δ of the premises yield the remainder C of the agenda. By the deduction theorem, Γ yields $A \multimapinv B$ if and only if $A, \Gamma \Rightarrow B$. In practice a lazy approach to the partitioning can be adopted whereby the whole bag of conclusion premises, plus A, are made available to try to prove B and, after checking that A has been consumed, those not used are supplied to try to prove C (Lloré and Morrill 1995).

Compilation takes place according to an immediate correspondence with interpretation. In the case of the associative implications, for example, we have the following, which is just a restatement of the hybrid interpretation clauses.

(72) a.
$$\frac{\forall I, \alpha (I - K - \alpha{+}\gamma\colon B \quad \multimapinv \quad I - J - \alpha\colon A)}{J - K - \gamma\colon A\backslash B}$$

b.
$$\frac{\forall K, \alpha (I - K - \gamma{+}\alpha\colon B \quad \multimapinv \quad J - K - \alpha\colon A)}{I - J - \gamma\colon B/A}$$

Categorial type assignments are translated into quantifier-free linear clauses by polar translation functions; the polarity is used to indicate whether new symbols introduced for quantified variables in the interpretation clauses are metavariables or Skolem constants. They are identity functions on atomic assignments; on complex category predicates they are defined mutually as follows (for related unfolding see Roorda 1991, Moortgat 1992, Hendriks 1993 and Oehrle 1994); $\overline{p}$ indicates the polarity complementary to p:

(73) a.
$$\frac{I - K - \alpha{+}\gamma\colon B^p \quad \multimapinv \quad I - J - \alpha\colon A^{\overline{p}}}{J - K - \gamma\colon A\backslash B^p} I, \alpha \text{ new variable/constant as } p +/-$$

b.
$$\frac{I - K - \gamma{+}\alpha\colon B^p \quad \multimapinv \quad J - K - \alpha\colon A^{\overline{p}}}{I - J - \gamma\colon B/A^p} I, \alpha \text{ new variable/constant as } p +/-$$

The program clauses and agenda are read directly off the unfoldings, with the only manipulation being a flattening of positive implications into uncurried form:

(74) $((X^+ \multimapinv Y_1^-) \multimapinv \ldots) \multimapinv Y_n^- \ \triangleright\ X^+ \multimapinv Y_1^- \otimes \ldots \otimes Y_n^-$

We shall allow an agenda $X_1 \otimes \ldots \otimes X_n \otimes \mathbf{1}$ to be written $X_1 \otimes \ldots \otimes X_n$ and we shall also allow unit program clauses $X \multimapinv \mathbf{1}$ to be abbreviated X.

The unfolding for the remaining connectives of the basic discontinuity calculus is as follows. Unfolding is provided for negative (succedent occurrences) of products, but not for positive (antecedent occurrences), the compilations of which would fall outside of our linear logic programming fragment.

(75)
$$\frac{I-J-\alpha\colon A^{-} \qquad \otimes \qquad J-K-\beta\colon B^{-}}{I-K-\gamma\colon A{\bullet}B^{-}}\,J,\alpha,\beta \text{ new variables; } \gamma=\alpha{+}\beta$$

(76)
$$\frac{(I-J,K-L)-(\alpha,\gamma)\colon B^{p} \qquad \circ\!\!- \qquad I-J-\alpha\colon A^{\overline{p}}}{K-L-\gamma\colon A{>}B^{p}}\,I,J,\alpha \text{ new variables/constants as } p\ +/-$$

$$\frac{(I-J,K-L)-(\gamma,\alpha)\colon B^{p} \qquad \circ\!\!- \qquad K-L-\alpha\colon A^{\overline{p}}}{I-J-\gamma\colon B{<}A^{p}}\,K,L,\alpha \text{ new variables/constants as } p\ +/-$$

$$\frac{I-J-\alpha\colon A^{-} \qquad \otimes \qquad K-L-\beta\colon B^{-}}{(I-J,K-L)-\gamma\colon A{\diamond}B^{-}}\,\alpha,\beta \text{ new variables; } \gamma=(\alpha,\beta)$$

(77)
$$\frac{I-L-\alpha_1{+}\gamma{+}\alpha_2\colon B^{p} \qquad \circ\!\!- \qquad I-J,K-L-(\alpha_1,\alpha_2)\colon A^{\overline{p}}}{J-K-\gamma\colon A{\downarrow}B^{p}}\,I,L,\alpha_1,\alpha_2 \text{ new variables/constants as } p\ +/-$$

$$\frac{I-L-\gamma_1{+}\alpha{+}\gamma_2\colon B^{p} \qquad \circ\!\!- \qquad J-K-\alpha\colon A^{\overline{p}}}{(I-J,K-L)-(\gamma_1,\gamma_2)\colon B{\uparrow}A^{p}}\,\alpha \text{ new variable/constant as } p\ +/-$$

$$\frac{(I-J,K-L)-(\alpha_1,\alpha_2)\colon A^{-} \qquad \otimes \qquad J-K-\beta\colon B^{-}}{I-L-\gamma\colon A{\odot}B^{-}}\,J,K,\alpha_1,\alpha_2,\beta \text{ new variables; } \gamma=\alpha_1{+}\beta{+}\alpha_2$$

Evidently the generalised discontinuity connectives will be unfolded in the corresponding systematic manner, therefore for binary adjunctions we again give just the case of staggered concatenation product.

(78)

$$\frac{(I-J,K-L)-(\alpha_1,\alpha_2)\colon A^{-} \qquad \otimes \qquad (J-K,L-M)-(\beta_1,\beta2)\colon B^{-}}{I-M-\gamma\colon A{\bullet_2}B^{-}}\,J,K,L,\alpha_1,\alpha_2,\beta_1,\beta_2 \text{ new variables; } \gamma=\alpha_1{+}\beta_1{+}\alpha_2{+}\beta_2$$

We do however show the unary split and bridge unfolding explicitly in (79).

(79)
$$\frac{I-L-\gamma_1{+}\beta{+}\gamma_2\colon A^{p} \qquad \circ\!\!- \qquad j(J,K,\beta)^{\overline{p}}}{(I-J,K-L)-(\gamma_1,\gamma_2)\colon \$A^{p}}\,\beta \text{ new variable/constant as } p\ +/-$$

$$\frac{(I-J,K-L)-(\alpha_1,\alpha_2)\colon A^{-} \qquad \otimes \qquad j(J,K,\beta)^{-}}{I-L-\gamma\colon {}^\wedge A^{-}}\,J,K,\alpha_1,\alpha_2,\beta \text{ new variables; } \gamma=\alpha_1{+}\beta{+}\alpha_2$$

We consider in detail the logic programming analysis of medial extraction. Assume the string positions named in (80).

(80) $_0$ that $_1$ Mary $_2$ sent $_3$ to $_4$ John $_5$

The relative pronoun type is unfolded as follows.

(81)

$$\frac{0-I-\mathbf{that}{+}a\colon \mathrm{R} \qquad \circ\!\!- \qquad \dfrac{\dfrac{1-I-a_1{+}\mathbf{b}{+}a_2\colon \mathrm{S} \qquad \circ\!\!- \qquad J-K-\mathbf{b}\colon \mathrm{N}}{(1-J,K-I)-(a_1,a_2)\colon \mathrm{S}{\uparrow}\mathrm{N}^{-}} \qquad \otimes \qquad j(J,K,f)}{1-I-a\colon {}^\wedge(\mathrm{S}{\uparrow}\mathrm{N})^{-}}\,a=a_1{+}f{+}a_2}{0-1-\mathbf{that}\colon \mathrm{R}/{}^\wedge(\mathrm{S}{\uparrow}\mathrm{N})^{+}}$$

And the prepositional ditransitive as shown in (82).

(82)

$$\cfrac{\cfrac{\cfrac{P - N - e+\mathbf{sent}+c+d\colon \mathrm{S} \quad\multimapinv\quad P - 2 - e\colon \mathrm{N}}{2 - N - \mathbf{sent}+c+d\colon \mathrm{N}\backslash\mathrm{S}} \quad\multimapinv\quad M - N - d\colon \mathrm{PP}}{2 - M - \mathbf{sent}+c\colon (\mathrm{N}\backslash\mathrm{S})/\mathrm{PP}} \quad\multimapinv\quad 3 - M - c\colon \mathrm{N}}{2 - 3 - \mathbf{sent}\colon ((\mathrm{N}\backslash\mathrm{S})/\mathrm{PP})/\mathrm{N}}$$

Initially we have the linear program clauses compiled from the word tokens in the string marked by parentheses, as well as the universal axiom $j(Q,Q,\epsilon)$ marked by braces signifying optionality and iterability of use. Agendas are marked by bare numerals. The initial agenda is to show that the entire span of the string is an R.

(83)

$$\begin{array}{ll}
\{\} & j(Q,Q,\epsilon)\\
() & 0 - I - \mathbf{that}+a_1+f+a_2\colon \mathrm{R} \multimapinv (1 - I - a_1+\mathbf{b}+a_2\colon \mathrm{S} \multimapinv J - K - \mathbf{b}\colon \mathrm{N}) \otimes j(J,K,f)\\
() & 1 - 2 - \mathbf{Mary}\colon \mathrm{N}\\
() & P - N - e+\mathbf{sent}+c+d\colon \mathrm{S} \multimapinv P - 2 - e\colon \mathrm{N} \otimes M - N - d\colon \mathrm{PP} \otimes 3 - M - c\colon \mathrm{N}\\
() & 3 - 4 - \mathbf{to}+\mathbf{John}\colon \mathrm{PP}\\
1. & 0 - 4 - g\colon \mathrm{R}
\end{array}$$

This unit agenda is resolved against the relative pronoun clause; the use of the clause is indicated by coindexing with the agenda against which it is resolved. The string position unifications (notated term/variable) are made and a new agenda obtained from the program clause subgoals accordingly, but the monoid term unification (notated term=term) is postponed.

(84)

$$\begin{array}{lll}
\{\} & j(Q,Q,\epsilon) & \\
(1) & 0 - I - \mathbf{that}+a_1+f+a_2\colon \mathrm{R} \multimapinv (1 - I - a_1+\mathbf{b}+a_2\colon \mathrm{S} \multimapinv J - K - \mathbf{b}\colon \mathrm{N}) \otimes j(J,K,f) & \\
() & 1 - 2 - \mathbf{Mary}\colon \mathrm{N} & \\
() & P - N - e+\mathbf{sent}+c+d\colon \mathrm{S} \multimapinv P - 2 - e\colon \mathrm{N} \otimes M - N - d\colon \mathrm{PP} \otimes 3 - M - c\colon \mathrm{N} & \\
() & 3 - 4 - \mathbf{to}+\mathbf{John}\colon \mathrm{PP} & \\
1. & 0 - 4 - g\colon \mathrm{R} & 4/I, g = \mathbf{that}+a_1+f+a_2\\
2. & (1 - 4 - a_1+\mathbf{b}+a_2\colon \mathrm{S} \multimapinv J - K - \mathbf{b}\colon \mathrm{N}) \otimes j(J,K,f) &
\end{array}$$

The top goal on the agenda is implicational. Its precondition is added as an additional premise for a subproof of the postcondition. This premise is required to have been used before exiting the subproof and continuing with the rest of the old agenda.

(85)

$$\begin{array}{lll}
\{\} & j(Q,Q,\epsilon) & \\
(1) & 0 - I - \mathbf{that}+a_1+f+a_2\colon \mathrm{R} \multimapinv (1 - I - a_1+\mathbf{b}+a_2\colon \mathrm{S} \multimapinv J - K - \mathbf{b}\colon \mathrm{N}) \otimes j(J,K,f) & \\
() & 1 - 2 - \mathbf{Mary}\colon \mathrm{N} & \\
() & P - N - e+\mathbf{sent}+c+d\colon \mathrm{S} \multimapinv P - 2 - e\colon \mathrm{N} \otimes M - N - d\colon \mathrm{PP} \otimes 3 - M - c\colon \mathrm{N} & \\
() & 3 - 4 - \mathbf{to}+\mathbf{John}\colon \mathrm{PP} & \\
1. & 0 - 4 - g\colon \mathrm{R} & 4/I, g = \mathbf{that}+a_1+f+a_2\\
2. & (1 - 4 - a_1+\mathbf{b}+a_2\colon \mathrm{S} \multimapinv J - K - \mathbf{b}\colon \mathrm{N}) \otimes j(J,K,f) & \\
() & \begin{array}{|l} J - K - \mathbf{b}\colon \mathrm{N} \\ \hline 1 - 4 - a_1+\mathbf{b}+a_2\colon \mathrm{S} \end{array} & \\
3. & &
\end{array}$$

The top goal on the subagenda is resolved against the prepositional ditransitive clause. The string position unification is performed; the monoid term unification postponed.

(86)

$$\begin{array}{lll}
\{\} & j(Q,Q,\epsilon) & \\
(1) & 0 - I - \mathbf{that}{+}a_1{+}f{+}a_2\colon \mathrm{R} \mathrel{\circ\!\!-} (1 - I - a_1{+}\mathbf{b}{+}a_2\colon \mathrm{S} \mathrel{\circ\!\!-} J - K - \mathbf{b}\colon \mathrm{N}) \otimes j(J,K,f) & \\
() & 1 - 2 - \mathbf{Mary}\colon \mathrm{N} & \\
(3) & P - N - e{+}\mathbf{sent}{+}c{+}d\colon \mathrm{S} \mathrel{\circ\!\!-} P - 2 - e\colon \mathrm{N} \otimes M - N - d\colon \mathrm{PP} \otimes 3 - M - c\colon \mathrm{N} & \\
() & 3 - 4 - \mathbf{to}{+}\mathbf{John}\colon \mathrm{PP} & \\
1. & 0 - 4 - g\colon \mathrm{R} & 4/I, g = \mathbf{that}{+}a_1{+}f{+}a_2 \\
2. & (1 - 4 - a_1{+}\mathbf{b}{+}a_2\colon \mathrm{S} \mathrel{\circ\!\!-} J - K - \mathbf{b}\colon \mathrm{N}) \otimes j(J,K,f) & \\
() & \vert\ \underline{J - K - \mathbf{b}\colon \mathrm{N}} & \\
3. & \vert\ 1 - 4 - a_1{+}\mathbf{b}{+}a_2\colon \mathrm{S} & 1/P, 4/N, a_1{+}\mathbf{b}{+}a_2 = e{+}\mathbf{sent}{+}c{+}d \\
4. & \vert\ 1 - 2 - e\colon \mathrm{N} \otimes M - 4 - d\colon \mathrm{PP} \otimes 3 - M - c\colon \mathrm{N} &
\end{array}$$

The current goal is resolved against the subject unit program clause.

(87)

$$\begin{array}{lll}
\{\} & j(Q,Q,\epsilon) & \\
(1) & 0 - I - \mathbf{that}{+}a_1{+}f{+}a_2\colon \mathrm{R} \mathrel{\circ\!\!-} (1 - I - a_1{+}\mathbf{b}{+}a_2\colon \mathrm{S} \mathrel{\circ\!\!-} J - K - \mathbf{b}\colon \mathrm{N}) \otimes j(J,K,f) & \\
(4) & 1 - 2 - \mathbf{Mary}\colon \mathrm{N} & \\
(3) & P - N - e{+}\mathbf{sent}{+}c{+}d\colon \mathrm{S} \mathrel{\circ\!\!-} P - 2 - e\colon \mathrm{N} \otimes M - N - d\colon \mathrm{PP} \otimes 3 - M - c\colon \mathrm{N} & \\
() & 3 - 4 - \mathbf{to}{+}\mathbf{John}\colon \mathrm{PP} & \\
1. & 0 - 4 - g\colon \mathrm{R} & 4/I, g = \mathbf{that}{+}a_1{+}f{+}a_2 \\
2. & (1 - 4 - a_1{+}\mathbf{b}{+}a_2\colon \mathrm{S} \mathrel{\circ\!\!-} J - K - \mathbf{b}\colon \mathrm{N}) \otimes j(J,K,f) & \\
() & \vert\ \underline{J - K - \mathbf{b}\colon \mathrm{N}} & \\
3. & \vert\ 1 - 4 - a_1{+}\mathbf{b}{+}a_2\colon \mathrm{S} & 1/P, 4/N, a_1{+}\mathbf{b}{+}a_2 = e{+}\mathbf{sent}{+}c{+}d \\
4. & \vert\ 1 - 2 - e\colon \mathrm{N} \otimes M - 4 - d\colon \mathrm{PP} \otimes 3 - M - c\colon \mathrm{N} & e = \mathbf{Mary} \\
5. & \vert\ M - 4 - d\colon \mathrm{PP} \otimes 3 - M - c\colon \mathrm{N} &
\end{array}$$

The next goal on the agenda is resolved against the prepositional phrase unit program clause.

(88)

$$\begin{array}{lll}
\{\} & j(Q,Q,\epsilon) & \\
(1) & 0 - I - \mathbf{that}{+}a_1{+}f{+}a_2\colon \mathrm{R} \mathrel{\circ\!\!-} (1 - I - a_1{+}\mathbf{b}{+}a_2\colon \mathrm{S} \mathrel{\circ\!\!-} J - K - \mathbf{b}\colon \mathrm{N}) \otimes j(J,K,f) & \\
(4) & 1 - 2 - \mathbf{Mary}\colon \mathrm{N} & \\
(3) & P - N - e{+}\mathbf{sent}{+}c{+}d\colon \mathrm{S} \mathrel{\circ\!\!-} P - 2 - e\colon \mathrm{N} \otimes M - N - d\colon \mathrm{PP} \otimes 3 - M - c\colon \mathrm{N} & \\
(5) & 3 - 4 - \mathbf{to}{+}\mathbf{John}\colon \mathrm{PP} & \\
1. & 0 - 4 - g\colon \mathrm{R} & 4/I, g = \mathbf{that}{+}a_1{+}f{+}a_2 \\
2. & (1 - 4 - a_1{+}\mathbf{b}{+}a_2\colon \mathrm{S} \mathrel{\circ\!\!-} J - K - \mathbf{b}\colon \mathrm{N}) \otimes j(J,K,f) & \\
() & \vert\ \underline{J - K - \mathbf{b}\colon \mathrm{N}} & \\
3. & \vert\ 1 - 4 - a_1{+}\mathbf{b}{+}a_2\colon \mathrm{S} & 1/P, 4/N, a_1{+}\mathbf{b}{+}a_2 = e{+}\mathbf{sent}{+}c{+}d \\
4. & \vert\ 1 - 2 - e\colon \mathrm{N} \otimes M - 4 - d\colon \mathrm{PP} \otimes 3 - M - c\colon \mathrm{N} & e = \mathbf{Mary} \\
5. & \vert\ M - 4 - d\colon \mathrm{PP} \otimes 3 - M - c\colon \mathrm{N} & 3/M, d = \mathbf{to}{+}\mathbf{John} \\
6. & \vert\ 3 - 3 - c\colon \mathrm{N} &
\end{array}$$

The current goal is now resolved against the subproof hypothesis. The subproof postcondition has been shown, and the subproof precondition has been used, so we exit and pursue the remainder of the agenda after the implicational top goal.

(89)

$$\begin{array}{lll}
\{\} & j(Q,Q,\epsilon) & \\
(1) & 0-I-\mathbf{that}{+}a_1{+}f{+}a_2\colon \mathrm{R} \mathbin{\circ\!-} (1-I-a_1{+}\mathbf{b}{+}a_2\colon \mathrm{S} \mathbin{\circ\!-} J-K-\mathbf{b}\colon \mathrm{N}) \otimes j(J,K,f) & \\
(4) & 1-2-\mathbf{Mary}\colon \mathrm{N} & \\
(3) & P-N-e{+}\mathbf{sent}{+}c{+}d\colon \mathrm{S} \mathbin{\circ\!-} P-2-e\colon \mathrm{N} \otimes M-N-d\colon \mathrm{PP} \otimes 3-M-c\colon \mathrm{N} & \\
(5) & 3-4-\mathbf{to}{+}\mathbf{John}\colon \mathrm{PP} & \\
1. & 0-4-g\colon \mathrm{R} & 4/I, g=\mathbf{that}{+}a_1{+}f{+}a_2 \\
2. & (1-4-a_1{+}\mathbf{b}{+}a_2\colon \mathrm{S} \mathbin{\circ\!-} J-K-\mathbf{b}\colon \mathrm{N}) \otimes j(J,K,f) & \\
(6) & \big|\; \underline{J-K-\mathbf{b}\colon \mathrm{N}} & \\
3. & \big|\; 1-4-a_1{+}\mathbf{b}{+}a_2\colon \mathrm{S} & 1/P, 4/N, a_1{+}\mathbf{b}{+}a_2 = e{+}\mathbf{sent}{+}c{+}d \\
4. & \big|\; 1-2-e\colon \mathrm{N} \otimes M-4-d\colon \mathrm{PP} \otimes 3-M-c\colon \mathrm{N} & e=\mathbf{Mary} \\
5. & \big|\; M-4-d\colon \mathrm{PP} \otimes 3-M-c\colon \mathrm{N} & 3/M, d=\mathbf{to}{+}\mathbf{John} \\
6. & \big|\; 3-3-c\colon \mathrm{N} & 3/J, 3/K, c=\mathbf{b} \\
7. & j(3,3,f) &
\end{array}$$

This last goal is satisfied by resolution against the universal connection axiom.

(90)

$$\begin{array}{lll}
\{7\} & j(Q,Q,\epsilon) & \\
(1) & 0-I-\mathbf{that}{+}a_1{+}f{+}a_2\colon \mathrm{R} \mathbin{\circ\!-} (1-I-a_1{+}\mathbf{b}{+}a_2\colon \mathrm{S} \mathbin{\circ\!-} J-K-\mathbf{b}\colon \mathrm{N}) \otimes j(J,K,f) & \\
(4) & 1-2-\mathbf{Mary}\colon \mathrm{N} & \\
(3) & P-N-e{+}\mathbf{sent}{+}c{+}d\colon \mathrm{S} \mathbin{\circ\!-} P-2-e\colon \mathrm{N} \otimes M-N-d\colon \mathrm{PP} \otimes 3-M-c\colon \mathrm{N} & \\
(5) & 3-4-\mathbf{to}{+}\mathbf{John}\colon \mathrm{PP} & \\
1. & 0-4-g\colon \mathrm{R} & 4/I, g=\mathbf{that}{+}a_1{+}f{+}a_2 \\
2. & (1-4-a_1{+}\mathbf{b}{+}a_2\colon \mathrm{S} \mathbin{\circ\!-} J-K-\mathbf{b}\colon \mathrm{N}) \otimes j(J,K,f) & \\
(6) & \big|\; \underline{J-K-\mathbf{b}\colon \mathrm{N}} & \\
3. & \big|\; 1-4-a_1{+}\mathbf{b}{+}a_2\colon \mathrm{S} \otimes j(J,K,f) & 1/P, 4/N, a_1{+}\mathbf{b}{+}a_2 = e{+}\mathbf{sent}{+}c{+}d \\
4. & \big|\; 1-2-e\colon \mathrm{N} \otimes M-4-d\colon \mathrm{PP} \otimes 3-M-c\colon \mathrm{N} & e=\mathbf{Mary} \\
5. & \big|\; M-4-d\colon \mathrm{PP} \otimes 3-M-c\colon \mathrm{N} & 3/M, d=\mathbf{to}{+}\mathbf{John} \\
6. & \big|\; 3-3-c\colon \mathrm{N} & 3/J, 3/K, c=\mathbf{b} \\
7. & j(3,3,f) & f=\epsilon
\end{array}$$

Only at this point are the monoid matchings chased back:

(91)

$$\begin{array}{ll}
7. & f=\epsilon \\
6. & c=\mathbf{b} \\
5. & d=\mathbf{to}{+}\mathbf{John} \\
4. & e=\mathbf{Mary} \\
3. & a_1=\mathbf{Mary}{+}\mathbf{sent}, a_2=\mathbf{to}{+}\mathbf{John} \\
1. & g=\mathbf{that}{+}\mathbf{Mary}{+}\mathbf{sent}{+}\mathbf{to}{+}\mathbf{John}
\end{array}$$

That is, the parsing as deduction process requires just unification of unstructured terms (the constants and variables of the binary relational labelling), and algebraic matching (one-way unification) under associativity only after the conditions on word order have been verified. (In the absence of hierarchical structure the algebraic matching succeeds if and only if the relational labelling does, so algebraic labelling is only needed when some non-associativity is present.) These properties are common to the basic discontinuity calculus, and the two generalisations of it made here.

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