

Computational Coverage of Type Logical Grammar: The Montague Test

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Abstract It is nearly half a century since Montague made his contributions to the field of logical semantics. In this time, computational linguistics has taken an almost entirely statistical turn and mainstream linguistics has adopted an almost entirely non-formal methodology. But in a minority approach reaching back before the linguistic revolution, and to the origins of computing, type logical grammar (TLG) has continued championing the flags of symbolic computation and logical rigor in discrete grammar. In this paper, we aim to concretise a measure of progress for computational grammar in the form of the *Montague Test*. This is the challenge of providing a computational cover grammar of the Montague fragment. We formulate this Montague Test and show how the challenge is met by the type logical parser/theorem-prover CatLog2.

Keywords Montague semantics · Montague grammar · categorial grammar · type logical grammar · computational grammar · semantic parsing · parsing as deduction · parsing/theorem-proving

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1 Introduction

Perhaps nobody does Montague semantics anymore, or perhaps everybody does Montague semantics now and it has become a part of the scenery. Around 1970, Richard Montague wrote three papers, “Universal grammar” (Montague 1970b), “English as a formal language” (Montague 1970a), and “The proper treatment of quantification in ordinary English” (Montague 1973), which overturned the prevailing view that natural language semantics was too ephemeral to be formalised. The third paper, especially, introduced lambda calculus and higher-order intensional logic for semantic representation by presenting a formal fragment of English with a translation into logic.

Montague’s approach was first popularised in the textbook Dowty et al. 1981. Since then, linguistics has become infused with Montague semantics starting with journals such as *Linguistics and Philosophy* and conferences such as the Amsterdam Colloquium, and spreading out in such a way that today there is an extensive interdisciplinary field of formal semantics based on lambda calculus and type logic. It is not that nobody does Montague semantics anymore, it is that now Montague semantics is taken for granted by many.

If you don’t know where you have come from, you don’t know where you are going. How can we be sure we are making progress? Here, in relation to Montague semantics, we propose as an exercise of intermediate difficulty, as a health check on approaches, the *Montague Test*, which is to provide a computational cover grammar of the Montague fragment as represented by the example sentences of Dowty et al. 1981:chap. 7.

Our broad concern is whether linguistics, rather than building on the achievements of the past and consolidating them, is rather in danger of drifting from trend to trend or lurching from fashion to fashion, in an aleatory or even cyclic fashion. Linguistics has its scholarly roots in the arts and humanities and from such origins a certain tendency to fantasia and self-proclamation persists. Perhaps this headiness partially explains why linguistics has remained a novice science while, for example, biology and computational biology have gone from strength to strength. Our plea here is that before a linguistic approach is deamed the new revolution, it proves its credentials by providing a computational cover grammar of the 50 years old Montague fragment.

In providing a computational cover grammar, we semantically parse the sentences provided with analysis trees in Dowty et al. 1981:chap. 7, assigning them logical translations “corresponding” to those given there, and distinguishing the same readings with comparable truth conditions. This minicorpus, which includes quantification, intensionality and some coordination and anaphora, is as follows:¹

(7-7) **John walks** $walk'(j)$

¹The reference numbers are taken directly from Dowty et al. 1981:chap. 7. Observe that the minicorpus preserves Montague’s practice of assigning raised types to extensional verbs for uniformity with intensional verbs.

(7-16) **every man talks** $\forall x[man'(x) \rightarrow talk'(x)]$

(7-19) **the fish walks** $\exists y[\forall x[fish'(x) \leftrightarrow x = y] \wedge walk'(y)]$

(7-32) **every man walks or talks** $\forall y[man'(y) \rightarrow [walk'(y) \vee talk'(y)]]$

(7-34) **every man walks or every man talks**
 $[\forall x[man'(x) \rightarrow walk'(x)] \vee \forall x[man'(x) \rightarrow talk'(x)]]$

(7-39) **a woman walks and she talks**
 $\exists x[woman'(x) \wedge [walk'(x) \wedge talk'(x)]]$

(7-43, 45) **John believes that a fish walks**
 $believe'(j, \wedge \exists x[fish'(x) \wedge walk'(x)])$
 $\exists x[fish'(x) \wedge believe'(j, \wedge [walk'(x)])]$

(7-48, 49, 52) **every man believes that a fish walks**
 $\exists x[fish'(x) \wedge \forall y[man'(y) \rightarrow believe'(y, \wedge [walk'(x)])]]$
 $\forall y[man'(y) \rightarrow \exists x[fish'(x) \wedge believe'(y, \wedge [walk'(x)])]]$
 $\forall y[man'(y) \rightarrow believe'(y, \wedge [\exists x[fish'(x) \wedge walk'(x)])]]$

(7-57) **every fish such that it walks talks**
 $\forall x[[fish'(x) \wedge walk'(x)] \rightarrow talk'(x)]$

(7-60, 62) **John seeks a unicorn**
 $try'(j, \wedge [find'(\wedge \lambda P \exists x[unicorn'(x) \wedge [{}^V P](x)])])$
 $try'(j, \wedge \lambda z[\exists x[unicorn'(x) \wedge [find'(\wedge \lambda P[{}^V P](z))](j)]]])$

(7-73) **John is Bill** $j = b$

(7-76) **John is a man** $man'(j)$

(7-83) **necessarily John walks** $\square[walk'(j)]$

(7-86) **John walks slowly** $slowly'(\wedge walk')(j)$

(7-91) **John tries to walk** $try'(\wedge walk')(j)$

(7-94) **John tries to catch a fish and eat it**
 $try'(j, \wedge \lambda y \exists x[fish'(x) \wedge$
 $[catch'(\wedge \lambda P[{}^V P](y))(x)) \wedge eat'(\wedge \lambda P[{}^V P](y))(x)]]])$

	cont. mult.	disc. mult.	add.	qu.	norm. mod.	brack. mod.	exp.	limited contr. & weak.
primary	/ • I	↑ ⊙ J	& ⊕	∧ ∨	□ ◇	[] ⁻¹ ⟨ ⟩	! ?	 W
sem. inactive variants	•—○ ○—•	↑↓ ↓↑	⊓ ⊔	∩ ∪	■ ◆			
det.	◁ ⁻¹ ▷ ⁻¹	∨						diff.
synth.	◁ ▷	∧						
nondet.	÷	≙ ≚						
synth.	○	◇						—

Table 1 Categorical connectives**(7-98) John finds a unicorn**

$$\exists x[\text{unicorn}'(x) \wedge [\text{find}'(\wedge \lambda P[[\vee P](x)])(j)]]$$

(7-105) every man such that he loves a woman loses her

$$\exists y[\text{woman}'(y) \wedge \forall x[[\text{man}'(x) \wedge \text{love}'(\wedge \lambda P([\vee P](y)])(x)) \rightarrow \text{lose}'(\wedge \lambda P([\vee P](y)])(x)]]$$

(7-110) John walks in a park

$$\exists x[\text{park}'(x) \wedge \text{in}'(\wedge \lambda P([\vee P](x)))(\wedge \text{walk}'(j))]$$

(7-116, 118) every man doesn't walk

$$\neg \forall x[\text{man}'(x) \rightarrow \text{walk}'(x)]$$

$$\forall x[\text{man}'(x) \rightarrow \neg \text{walk}'(x)]$$

2 Type Logical Grammar

Type logical grammar (TLG) is a categorial theory of syntax and semantics in which words and expressions are classified by logical types. TLG

is expounded in Moortgat 1988, 1997, Morrill 1994, 2011, Carpenter 1997, Jäger 2005, Moot & Retoré 2012. The logical types form an intuitionistic sublinear logic and their rules are universal; a grammar comprises just a lexicon classifying basic expressions. TLG is thus a purely lexical formalism.

A sign $\alpha: A: \phi$ consists of a *prosodic form* α , a *syntactic type* A , and a *semantic form* ϕ . A *prosodic sort map* s maps syntactic types to prosodic sorts which are the number of points of discontinuity of expressions of that type; a *semantic type map* T maps syntactic types to semantic types which are essentially formulas of intuitionistic propositional logic/types of lambda calculus under the Curry-Howard correspondence. In a sign $\alpha: A: \phi$, α must be of prosodic sort $s(A)$ and ϕ must be of semantic type $T(A)$.

The categorial connectives of our type logical grammar are as shown in table 1. They comprise the primary connectives, in the first row, semantically inactive variants, in the second row, and deterministic (unary) and nondeterministic (binary) defined connectives in the third and fourth rows.

Regarding the primary connectives, the displacement connectives (Morrill et al. 2011) are made up of the continuous (Lambek) and discontinuous multiplicatives. Then there are additives (Morrill 1990a), quantifiers (Morrill 1994), normal modalities (Morrill 1990b, Moortgat 1997), bracket modalities (Morrill 1992, Moortgat 1996), exponentials (Morrill & Valentín 2015a), limited contraction (Jäger 2005) and limited weakening (Morrill & Valentín 2014b).

The semantically inactive secondary connectives are made up of semantically inactive multiplicatives (Morrill & Valentín 2014b), additives (Morrill 1994), quantifiers (Morrill 1994), and normal modalities (Hepple 1990, Moortgat 1997). The deterministic secondary connectives are made up of the unary connectives projection and injection (Morrill et al. 2009) and split and bridge (Morrill & Merenciano 1996), and the nondeterministic secondary connectives are made up of concatenative binary connectives of division and product and discontinuous binary connectives of extraction, infixation and product (Morrill et al. 2011). At the bottom right is a meta-logical (“negation as failure”) connective of difference (Morrill & Valentín 2014a).

A lexicon consists of a set of (lexical) signs. Our lexicon for the Montague fragment is as follows; rules for connectives used in the fragment are given in the Appendix:

a : $\blacksquare \forall g(\forall f((Sf \uparrow \blacksquare Nt(s(g))) \downarrow Sf) / CNs(g)) : \lambda A \lambda B \exists C[(A C) \wedge (B C)]$

and : $\blacksquare \forall f((\blacksquare ? Sf \setminus []^{-1} []^{-1} Sf) / \blacksquare Sf) : (\Phi^{n+} \text{ o and})$

and : $\blacksquare \forall a \forall f((\blacksquare ? (\langle \rangle Na \setminus Sf) \setminus []^{-1} []^{-1} (\langle \rangle Na \setminus Sf)) / \blacksquare (\langle \rangle Na \setminus Sf)) : (\Phi^{n+} (s \text{ o}) \text{ and})$

believes : $\square((\langle \rangle \exists g Nt(s(g)) \setminus Sf) / (CP\text{that} \sqcup \square Sf)) : \wedge \lambda A \lambda B ((\sim \text{believe } A) B)$

bill : $\blacksquare Nt(s(m)) : b$

catch : $\square((\langle \rangle \exists a Na \setminus Sb) / \exists a Na) : \wedge \lambda A \lambda B ((\sim \text{catch } A) B)$

doesnt : $\blacksquare \forall g \forall a((Sg \uparrow ((\langle \rangle Na \setminus Sf) / (\langle \rangle Na \setminus Sb))) \downarrow Sg) : \lambda A \neg (A \lambda B \lambda C (B C))$

eat : $\square((\langle \rangle \exists a Na \setminus Sb) / \exists a Na) : \wedge \lambda A \lambda B ((\sim \text{eat } A) B)$

every : $\blacksquare \forall g(\forall f((Sf \uparrow Nt(s(g))) \downarrow Sf) / CNs(g)) : \lambda A \lambda B \forall C[(A C) \rightarrow (B C)]$

finds : $\square((\langle \rangle \exists g Nt(s(g)) \setminus Sf) / \exists a Na) : \wedge \lambda A \lambda B ((\sim \text{find } A) B)$

fish : $\square CNs(n) : \text{fish}$

he : $\blacksquare []^{-1} \forall g((\blacksquare Sg \mid \blacksquare Nt(s(m))) / (\langle \rangle Nt(s(m)) \setminus Sg)) : \lambda AA$

her : $\blacksquare \forall g \forall a(((\langle \rangle Na \setminus Sg) \uparrow \blacksquare Nt(s(f))) \downarrow (\blacksquare (\langle \rangle Na \setminus Sg) \mid \blacksquare Nt(s(f)))) : \lambda AA$

in : $\square(\forall a \forall f((\langle \rangle Na \setminus Sf) \setminus (\langle \rangle Na \setminus Sf)) / \exists a Na) : \wedge \lambda A \lambda B \lambda C ((\sim \text{in } A) (B C))$

is : $\blacksquare((\langle \rangle \exists g Nt(s(g)) \setminus Sf) / (\exists a Na \oplus (\exists g((CNg / CNg) \sqcup (CNg \setminus CNg)) - I))) : \lambda A \lambda B (A \rightarrow C.[B = C]; D.((D \lambda E [E = B]) B))$

it : $\blacksquare \forall f \forall a(((\langle \rangle Na \setminus Sf) \uparrow \blacksquare Nt(s(n))) \downarrow (\blacksquare (\langle \rangle Na \setminus Sf) \mid \blacksquare Nt(s(n)))) : \lambda AA$

it : $\blacksquare []^{-1} \forall f((\blacksquare Sf \mid \blacksquare Nt(s(n))) / (\langle \rangle Nt(s(n)) \setminus Sf)) : \lambda AA$

john : $\blacksquare Nt(s(m)) : j$

loses : $\square((\langle \rangle \exists g Nt(s(g)) \setminus Sf) / \exists a Na) : \wedge \lambda A \lambda B ((\sim \text{lose } A) B)$

loves : $\square((\langle \rangle \exists g Nt(s(g)) \setminus Sf) / \exists a Na) : \wedge \lambda A \lambda B ((\sim \text{love } A) B)$

man : $\square CNs(m) : \text{man}$

necessarily : $\blacksquare (SA / \square SA) : \text{Nec}$

or : $\blacksquare \forall f((\blacksquare ? Sf \setminus []^{-1} []^{-1} Sf) / \blacksquare Sf) : (\Phi^{n+} \text{ o or})$

or : $\blacksquare \forall a \forall f((\blacksquare ? (\langle \rangle Na \setminus Sf) \setminus []^{-1} []^{-1} (\langle \rangle Na \setminus Sf)) / \blacksquare (\langle \rangle Na \setminus Sf)) : (\Phi^{n+} (s \text{ o}) \text{ or})$

or : $\blacksquare \forall f((\blacksquare ? (Sf / (\langle \rangle \exists g Nt(s(g)) \setminus Sf)) \setminus []^{-1} []^{-1} (Sf / (\langle \rangle \exists g Nt(s(g)) \setminus Sf))) / \blacksquare (Sf / (\langle \rangle \exists g Nt(s(g)) \setminus Sf))) : (\Phi^{n+} (s \text{ o}) \text{ or})$

park : $\square CNs(n) : \text{park}$

seeks : $\square((\langle \rangle \exists g Nt(s(g)) \setminus Sf) / \square \forall a \forall f(((Na \setminus Sf) / \exists b Nb) \setminus (Na \setminus Sf))) : \wedge \lambda A \lambda B ((\sim \text{try } \wedge ((\sim A \sim \text{find } B)) B))$

she : $\blacksquare []^{-1} \forall g((\blacksquare Sg \mid \blacksquare Nt(s(f))) / (\langle \rangle Nt(s(f)) \setminus Sg)) : \lambda AA$

slowly : $\Box \forall a \forall f (\Box (\langle \rangle N a \setminus S f) \setminus (\langle \rangle \Box N a \setminus S f)) : \hat{\lambda} A \lambda B (\sim \text{slowly } \hat{(\sim A \sim B)})$
such+that : $\blacksquare \forall n ((C N n \setminus C N n) / (S f | \blacksquare N t(n))) : \lambda A \lambda B \lambda C [(B C) \wedge (A C)]$
talks : $\Box (\langle \rangle \exists g N t(s(g)) \setminus S f) : \hat{\lambda} A (\sim \text{talk } A)$
that : $\blacksquare (C P \text{that} / \Box S f) : \lambda A A$
the : $\blacksquare \forall n (N t(n) / C N n) : \iota$
to : $\blacksquare ((P P \text{to} / \exists a N a) \Box \forall n ((\langle \rangle N n \setminus S i) / (\langle \rangle N n \setminus S b))) : \lambda A A$
tries : $\Box ((\langle \rangle \exists g N t(s(g)) \setminus S f) / \Box (\langle \rangle \exists g N t(s(g)) \setminus S i)) : \hat{\lambda} A \lambda B ((\sim \text{try } \hat{(\sim A B)}) B)$
unicorn : $\Box C N s(n) : \text{unicorn}$
walk : $\Box (\langle \rangle \exists a N a \setminus S b) : \hat{\lambda} A (\sim \text{walk } A)$
walks : $\Box (\langle \rangle \exists g N t(s(g)) \setminus S f) : \hat{\lambda} A (\sim \text{walk } A)$
woman : $\Box C N s(f) : \text{woman}$

3 Performing the Montague Test

CatLog2 is a type logical parser/theorem prover with a web interface at <http://www.cs.upc.edu/~morrill/CatLog/CatLog2/index.php>. It:

- comprises 6000 lines of prolog
- has 20 primitive categorial connectives, 29 defined connectives, and 1 metalogical connective: a total of 50 connectives
- has typically 2 rules for each connective: a rule of use and a rule of proof: roughly $50 \times 2 = 100$ rules
- uses backward chaining sequent proof search and uses *focusing* (Andreoli 1992); for the focused rules—about half of them—for a binary connective there are 4 cases of “polarity”: +/+, +/−, −/+, −/−: $50 + 50 \times 4 =$ a total of about 250 rules

At CSSP in Paris on 9 October 2015, the Montague Test was performed by CatLog2 version “gmontague” with input in the following format; note that currently it is necessary to give syntactic domains in the input to CatLog2 (though these play no role in Montague’s grammar):

str(dwp('7-7')), [b([john]), walks], s(f)).
 str(dwp('7-16')), [b([every, man]), talks], s(f)).
 str(dwp('7-19')), [b([the, fish]), walks], s(f)).
 str(dwp('7-32')), [b([every, man]), b([b([walks, or, talks]])]), s(f)).

str(dwp('7-34')), [b([b([b([every, man]), walks, or, b([every, man]), talks])])], s(f)).
 str(dwp('7-39')), [b([b([b([a, woman]), walks, and, b([she]), talks])])], s(f)).
 str(dwp('7-43, 45')), [b([john]), believes, that, b([a, fish]), walks], s(f)).
 str(dwp('7-48, 49, 52')), [b([every, man]), believes, that, b([a, fish]), walks], s(f)).
 str(dwp('7-57')), [b([every, fish, such, that, b([it]), walks]), talks], s(f)).
 str(dwp('7-60, 62')), [b([john]), seeks, a, unicorn], s(f)).
 str(dwp('7-73')), [b([john]), is, bill], s(f)).
 str(dwp('7-76')), [b([john]), is, a, man], s(f)).
 str(dwp('7-83')), [necessarily, b([john]), walks], s(f)).
 str(dwp('7-86')), [b([john]), walks, slowly], s(f)).
 str(dwp('7-91')), [b([john]), tries, to, walk], s(f)).
 str(dwp('7-94')), [b([john]), tries, to, b([b([catch, a, fish, and, eat, it])])], s(f)).
 str(dwp('7-98')), [b([john]), finds, a, unicorn], s(f)).
 str(dwp('7-105')), [b([every, man, such, that, b([he]), loves, a, woman]), loses, her], s(f)).
 str(dwp('7-110')), [b([john]), walks, in, a, park], s(f)).
 str(dwp('7-116, 118')), [b([every, man]), doesnt, walk], s(f)).

The \LaTeX output generated was as follows. Each item comes in the form of its identifier and the prosodic form of its input, followed by each semantically labelled sequent that results from lexical lookup. Where there is a derivation or derivations for a sequent, these appear in figures with the semantic forms delivered by the analysis in the main text. CatLog2 observes the proof search discipline of *focusing* (Andreoli 1992, Morrill & Valentín 2015b): in the derivations the focused types are boxed, which means that when a complex type in a conclusion is boxed, it is the active type of the inference. For reasons of space, some derivations are omitted.

(dwp((7-7))) [john]+walks : Sf

[$\blacksquare Nt(s(m)) : j$], $\square(\langle \rangle \exists g Nt(s(g)) \setminus Sf) : \hat{\lambda} A (\check{walk} A) \Rightarrow Sf$

For the derivation, see figure 1.

$$\begin{array}{c}
\frac{\boxed{Nt(s(m))} \Rightarrow Nt(s(m))}{\blacksquare L} \\
\frac{\boxed{\blacksquare Nt(s(m))} \Rightarrow Nt(s(m))}{\exists R} \\
\frac{\blacksquare Nt(s(m)) \Rightarrow \boxed{\exists g Nt(s(g))}}{\langle \rangle R} \\
\frac{\boxed{[\blacksquare Nt(s(m))]} \Rightarrow \boxed{\langle \rangle \exists g Nt(s(g))} \quad \boxed{Sf} \Rightarrow Sf}{\setminus L} \\
\frac{[\blacksquare Nt(s(m))], \boxed{\langle \rangle \exists g Nt(s(g)) \setminus Sf} \Rightarrow Sf}{\square L} \\
\boxed{[\blacksquare Nt(s(m))], \boxed{\square(\langle \rangle \exists g Nt(s(g)) \setminus Sf)}} \Rightarrow Sf
\end{array}$$

Figure 1 Derivation of (dwp((7-7)))

(\checkmark walk j)

(dwp((7-16))) [**every+man**]+**talks** : Sf

$[\blacksquare \forall g(\forall f((Sf \uparrow Nt(s(g))) \downarrow Sf)/CNs(g)) : \lambda A \lambda B \forall C[(A C) \rightarrow (B C)],$
 $\square CNs(m) : man], \square(\langle \rangle \exists g Nt(s(g)) \setminus Sf) : \hat{\lambda} D(\checkmark talk D) \Rightarrow Sf$

For the derivation, see figure 2.

$\forall C[(\checkmark man C) \rightarrow (\checkmark talk C)]$

(dwp((7-19))) [**the+fish**]+**walks** : Sf

$[\blacksquare \forall n(Nt(n)/CNn) : \iota, \square CNs(n) : fish], \square(\langle \rangle \exists g Nt(s(g)) \setminus Sf) :$
 $\hat{\lambda} A(\checkmark walk A) \Rightarrow Sf$

(Derivation omitted)

(\checkmark walk (ι \checkmark fish))

(dwp((7-32))) [**every+man**]+[[**walks+or+talks**]] : Sf

$[\blacksquare \forall g(\forall f((Sf \uparrow Nt(s(g))) \downarrow Sf)/CNs(g)) : \lambda A \lambda B \forall C[(A C) \rightarrow (B C)],$
 $\square CNs(m) : man], [[\square(\langle \rangle \exists g Nt(s(g)) \setminus Sf) : \hat{\lambda} D(\checkmark walk D),$
 $\blacksquare \forall f((\blacksquare ?Sf \setminus [\]^{-1} [\]^{-1} Sf)/\blacksquare Sf) : (\Phi^{n+} o or), \square(\langle \rangle \exists g Nt(s(g)) \setminus Sf) :$
 $\hat{\lambda} E(\checkmark talk E)] \Rightarrow Sf$

$[\blacksquare \forall g(\forall f((Sf \uparrow Nt(s(g))) \downarrow Sf)/CNs(g)) : \lambda A \lambda B \forall C[(A C) \rightarrow (B C)],$

$$\begin{array}{c}
\frac{\overline{Nt(s(m)) \Rightarrow Nt(s(m))}}{\overline{Nt(s(m)) \Rightarrow \exists g Nt(s(g))}} \exists R \\
\frac{\overline{Nt(s(m)) \Rightarrow \exists g Nt(s(g))}}{[Nt(s(m))] \Rightarrow \langle \exists g Nt(s(g)) \rangle} \langle \exists R \\
\frac{[Nt(s(m))] \Rightarrow \langle \exists g Nt(s(g)) \rangle \quad \overline{Sf} \Rightarrow Sf}{[Nt(s(m))], \langle \exists g Nt(s(g)) \rangle \backslash Sf \Rightarrow Sf} \backslash L \\
\frac{[Nt(s(m))], \langle \exists g Nt(s(g)) \rangle \backslash Sf \Rightarrow Sf}{[Nt(s(m))], \square \langle \exists g Nt(s(g)) \rangle \backslash Sf \Rightarrow Sf} \square L \\
\frac{[Nt(s(m))], \square \langle \exists g Nt(s(g)) \rangle \backslash Sf \Rightarrow Sf}{[1], \square \langle \exists g Nt(s(g)) \rangle \backslash Sf \Rightarrow Sf \uparrow Nt(s(m))} \uparrow R \\
\frac{[1], \square \langle \exists g Nt(s(g)) \rangle \backslash Sf \Rightarrow Sf \uparrow Nt(s(m)) \quad \overline{Sf} \Rightarrow Sf}{\square CNs(m) \Rightarrow CNs(m)} \downarrow L \\
\frac{\overline{CNs(m)} \Rightarrow CNs(m)}{\square CNs(m) \Rightarrow CNs(m)} \square L \\
\frac{[\square (Sf \uparrow Nt(s(m))) \downarrow Sf], \square \langle \exists g Nt(s(g)) \rangle \backslash Sf \Rightarrow Sf}{[\forall f ((Sf \uparrow Nt(s(m))) \downarrow Sf)], \square \langle \exists g Nt(s(g)) \rangle \backslash Sf \Rightarrow Sf} \forall L \\
\frac{[\forall f ((Sf \uparrow Nt(s(m))) \downarrow Sf) / CNs(m)], \square CNs(m), \square \langle \exists g Nt(s(g)) \rangle \backslash Sf \Rightarrow Sf}{[\forall g (\forall f ((Sf \uparrow Nt(s(g))) \downarrow Sf) / CNs(g)), \square CNs(m), \square \langle \exists g Nt(s(g)) \rangle \backslash Sf \Rightarrow Sf} \forall L \\
\frac{[\forall g (\forall f ((Sf \uparrow Nt(s(g))) \downarrow Sf) / CNs(g)), \square CNs(m), \square \langle \exists g Nt(s(g)) \rangle \backslash Sf \Rightarrow Sf}{[\blacksquare \forall g (\forall f ((Sf \uparrow Nt(s(g))) \downarrow Sf) / CNs(g)), \square CNs(m), \square \langle \exists g Nt(s(g)) \rangle \backslash Sf \Rightarrow Sf} \blacksquare L
\end{array}$$

Figure 2 Derivation of (dwp((7-16)))

$$\begin{array}{l}
\square CNs(m) : man, [[\square \langle \exists g Nt(s(g)) \rangle \backslash Sf] : \wedge \lambda D (\checkmark walk D), \\
\blacksquare \forall a \forall f ((\blacksquare ? \langle \exists g Nt(s(g)) \rangle \backslash Sf) \backslash [\]^{-1} [\]^{-1} \langle \exists g Nt(s(g)) \rangle \backslash Sf) / \blacksquare \langle \exists g Nt(s(g)) \rangle \backslash Sf) : (\Phi^{n+} (s o) or), \\
\square \langle \exists g Nt(s(g)) \rangle \backslash Sf : \wedge \lambda E (\checkmark talk E)] \Rightarrow Sf
\end{array}$$

(Derivation omitted)

$$\forall C [(\checkmark man C) \rightarrow [(\checkmark walk C) \vee (\checkmark talk C)]]$$

$$\begin{array}{l}
[\blacksquare \forall g (\forall f ((Sf \uparrow Nt(s(g))) \downarrow Sf) / CNs(g)) : \lambda A \lambda B \forall C [(A C) \rightarrow (B C)], \\
\square CNs(m) : man, [[\square \langle \exists g Nt(s(g)) \rangle \backslash Sf] : \wedge \lambda D (\checkmark walk D), \\
\blacksquare \forall f ((\blacksquare ? (Sf / \langle \exists g Nt(s(g)) \rangle \backslash Sf) \backslash [\]^{-1} [\]^{-1} (Sf / \langle \exists g Nt(s(g)) \rangle \backslash Sf))) / \\
\blacksquare (Sf / \langle \exists g Nt(s(g)) \rangle \backslash Sf) : (\Phi^{n+} (s o) or), \square \langle \exists g Nt(s(g)) \rangle \backslash Sf : \\
\wedge \lambda E (\checkmark talk E)] \Rightarrow Sf
\end{array}$$

$$(dwp((7-34))) [[[\mathbf{every+man}]+\mathbf{walks+or}+[\mathbf{every+man}]+\mathbf{talks}] : Sf$$

$$\begin{array}{l}
[[[\blacksquare \forall g (\forall f ((Sf \uparrow Nt(s(g))) \downarrow Sf) / CNs(g)) : \lambda A \lambda B \forall C [(A C) \rightarrow (B C)], \\
\square CNs(m) : man, \square \langle \exists g Nt(s(g)) \rangle \backslash Sf : \wedge \lambda D (\checkmark walk D), \\
\blacksquare \forall f ((\blacksquare ? Sf \backslash [\]^{-1} [\]^{-1} Sf) / \blacksquare Sf) : (\Phi^{n+} o or), \\
[\blacksquare \forall g (\forall f ((Sf \uparrow Nt(s(g))) \downarrow Sf) / CNs(g)) : \lambda E \lambda F \forall G [(E G) \rightarrow (F G)], \\
\square CNs(m) : man, \square \langle \exists g Nt(s(g)) \rangle \backslash Sf : \wedge \lambda H (\checkmark talk H)] \Rightarrow Sf
\end{array}$$

(Derivation omitted)

$$[\forall H[(\checkmark man H) \rightarrow (\checkmark walk H)] \vee \forall C[(\checkmark man C) \rightarrow (\checkmark talk C)]]$$

$$\begin{aligned} & [[[\blacksquare \forall g(\forall f((Sf \uparrow Nt(s(g))) \downarrow Sf) / CNs(g)) : \lambda A \lambda B \forall C[(A C) \rightarrow (B C)], \\ & \square CNs(m) : man], \square (\langle \rangle \exists g Nt(s(g)) \setminus Sf) : \hat{\lambda} D(\checkmark walk D), \\ & \blacksquare \forall a \forall f((\blacksquare ?(\langle \rangle Na \setminus Sf) \setminus [\]^{-1} [\]^{-1}(\langle \rangle Na \setminus Sf)) / \blacksquare (\langle \rangle Na \setminus Sf)) : (\Phi^{n+} (s o) or), \\ & [\blacksquare \forall g(\forall f((Sf \uparrow Nt(s(g))) \downarrow Sf) / CNs(g)) : \lambda E \lambda F \forall G[(E G) \rightarrow (F G)], \\ & \square CNs(m) : man], \square (\langle \rangle \exists g Nt(s(g)) \setminus Sf) : \hat{\lambda} H(\checkmark talk H)]] \Rightarrow Sf \end{aligned}$$

$$\begin{aligned} & [[[\blacksquare \forall g(\forall f((Sf \uparrow Nt(s(g))) \downarrow Sf) / CNs(g)) : \lambda A \lambda B \forall C[(A C) \rightarrow (B C)], \\ & \square CNs(m) : man], \square (\langle \rangle \exists g Nt(s(g)) \setminus Sf) : \hat{\lambda} D(\checkmark walk D), \\ & \blacksquare \forall f((\blacksquare ?(Sf / (\langle \rangle \exists g Nt(s(g)) \setminus Sf)) \setminus [\]^{-1} [\]^{-1}(Sf / (\langle \rangle \exists g Nt(s(g)) \setminus Sf))) / \\ & \blacksquare (Sf / (\langle \rangle \exists g Nt(s(g)) \setminus Sf)) : (\Phi^{n+} (s o) or), \\ & [\blacksquare \forall g(\forall f((Sf \uparrow Nt(s(g))) \downarrow Sf) / CNs(g)) : \lambda E \lambda F \forall G[(E G) \rightarrow (F G)], \\ & \square CNs(m) : man], \square (\langle \rangle \exists g Nt(s(g)) \setminus Sf) : \hat{\lambda} H(\checkmark talk H)]] \Rightarrow Sf \end{aligned}$$

$$(dwp((7-39))) [[[\mathbf{a+woman}]+\mathbf{walks+and+}[\mathbf{she}]+\mathbf{talks}]] : Sf$$

$$\begin{aligned} & [[[\blacksquare \forall g(\forall f((Sf \uparrow \blacksquare Nt(s(g))) \downarrow Sf) / CNs(g)) : \lambda A \lambda B \exists C[(A C) \wedge (B C)], \\ & \square CNs(f) : woman], \square (\langle \rangle \exists g Nt(s(g)) \setminus Sf) : \hat{\lambda} D(\checkmark walk D), \\ & \blacksquare \forall f((\blacksquare ?Sf \setminus [\]^{-1} [\]^{-1} Sf) / \blacksquare Sf) : (\Phi^{n+} o and), \\ & [\blacksquare [\]^{-1} \forall g((\blacksquare Sg \blacksquare Nt(s(f))) / (\langle \rangle Nt(s(f)) \setminus Sg)) : \lambda EE], \\ & \square (\langle \rangle \exists g Nt(s(g)) \setminus Sf) : \hat{\lambda} F(\checkmark talk F)]] \Rightarrow Sf \end{aligned}$$

(Derivation omitted)

$$\exists C[(\checkmark woman C) \wedge [(\checkmark walk C) \wedge (\checkmark talk C)]]$$

$$\begin{aligned} & [[[\blacksquare \forall g(\forall f((Sf \uparrow \blacksquare Nt(s(g))) \downarrow Sf) / CNs(g)) : \lambda A \lambda B \exists C[(A C) \wedge (B C)], \\ & \square CNs(f) : woman], \square (\langle \rangle \exists g Nt(s(g)) \setminus Sf) : \hat{\lambda} D(\checkmark walk D), \\ & \blacksquare \forall a \forall f((\blacksquare ?(\langle \rangle Na \setminus Sf) \setminus [\]^{-1} [\]^{-1}(\langle \rangle Na \setminus Sf)) / \blacksquare (\langle \rangle Na \setminus Sf)) : \\ & (\Phi^{n+} (s o) and), \\ & [\blacksquare [\]^{-1} \forall g((\blacksquare Sg \blacksquare Nt(s(f))) / (\langle \rangle Nt(s(f)) \setminus Sg)) : \lambda EE], \\ & \square (\langle \rangle \exists g Nt(s(g)) \setminus Sf) : \hat{\lambda} F(\checkmark talk F)]] \Rightarrow Sf \end{aligned}$$

$$(dwp((7-43, 45))) [\mathbf{john}]+\mathbf{believes+that+}[\mathbf{a+fish}]+\mathbf{walks} : Sf$$

$$\begin{aligned} & [\blacksquare Nt(s(m)) : j], \square ((\langle \rangle \exists g Nt(s(g)) \setminus Sf) / (CPthat \square \square Sf)) : \\ & \hat{\lambda} A \lambda B ((\checkmark believe A) B), \blacksquare (CPthat / \square Sf) : \lambda CC, \\ & [\blacksquare \forall g(\forall f((Sf \uparrow \blacksquare Nt(s(g))) \downarrow Sf) / CNs(g)) : \lambda D \lambda E \exists F[(D F) \wedge (E F)], \end{aligned}$$

$\square CNs(n) : fish], \square(\langle \rangle \exists g Nt(s(g)) \setminus Sf) : \hat{\lambda} G (\checkmark walk G) \Rightarrow Sf$

For the derivation, see figure 3.

$\exists C[(\checkmark fish C) \wedge ((\checkmark believe \hat{\ } (\checkmark walk C)) j)]$

For the derivation, see figure 4.

$((\checkmark believe \hat{\ } \exists F[(\checkmark fish F) \wedge (\checkmark walk F)])) j)$

$(dwp((7-48, 49, 52)))$ [**every+man+believes+that+[a+fish]+walks** : Sf

$[\blacksquare \forall g (\forall f ((Sf \uparrow Nt(s(g))) \downarrow Sf) / CNs(g)) : \lambda A \lambda B \forall C [(A C) \rightarrow (B C)],$

$\square CNs(m) : man], \square(\langle \rangle \exists g Nt(s(g)) \setminus Sf) / (CPthat \sqcup \square Sf) :$

$\hat{\lambda} D \lambda E ((\checkmark believe D) E), \blacksquare (CPthat / \square Sf) : \lambda FF,$

$[\blacksquare \forall g (\forall f ((Sf \uparrow \blacksquare Nt(s(g))) \downarrow Sf) / CNs(g)) : \lambda G \lambda H \exists I [(G I) \wedge (H I)],$

$\square CNs(n) : fish], \square(\langle \rangle \exists g Nt(s(g)) \setminus Sf) : \hat{\lambda} \lambda J (\checkmark walk J) \Rightarrow Sf$

(Derivation omitted)

$\exists C[(\checkmark fish C) \wedge \forall G[(\checkmark man G) \rightarrow ((\checkmark believe \hat{\ } (\checkmark walk C)) G)]]$

(Derivation omitted)

$\forall C[(\checkmark man C) \rightarrow \exists G[(\checkmark fish G) \wedge ((\checkmark believe \hat{\ } (\checkmark walk G)) C)]]$

(Derivation omitted)

$\forall C[(\checkmark man C) \rightarrow ((\checkmark believe \hat{\ } \exists J[(\checkmark fish J) \wedge (\checkmark walk J)]) C)]$

$(dwp((7-57)))$ [**every+fish+such+that+[it]+walks]+talks** : Sf

$[\blacksquare \forall g (\forall f ((Sf \uparrow Nt(s(g))) \downarrow Sf) / CNs(g)) : \lambda A \lambda B \forall C [(A C) \rightarrow (B C)],$

$\square CNs(n) : fish, \blacksquare \forall n ((CNn \setminus CNn) / (Sf | \blacksquare Nt(n))) : \lambda D \lambda E \lambda F [(E F) \wedge (D F)],$

$[\blacksquare \forall f \forall a (((\langle \rangle Na \setminus Sf) \uparrow \blacksquare Nt(s(n))) \downarrow (\blacksquare (\langle \rangle Na \setminus Sf) | \blacksquare Nt(s(n)))) : \lambda GG],$

$\square(\langle \rangle \exists g Nt(s(g)) \setminus Sf) : \hat{\lambda} \lambda H (\checkmark walk H)], \square(\langle \rangle \exists g Nt(s(g)) \setminus Sf) :$

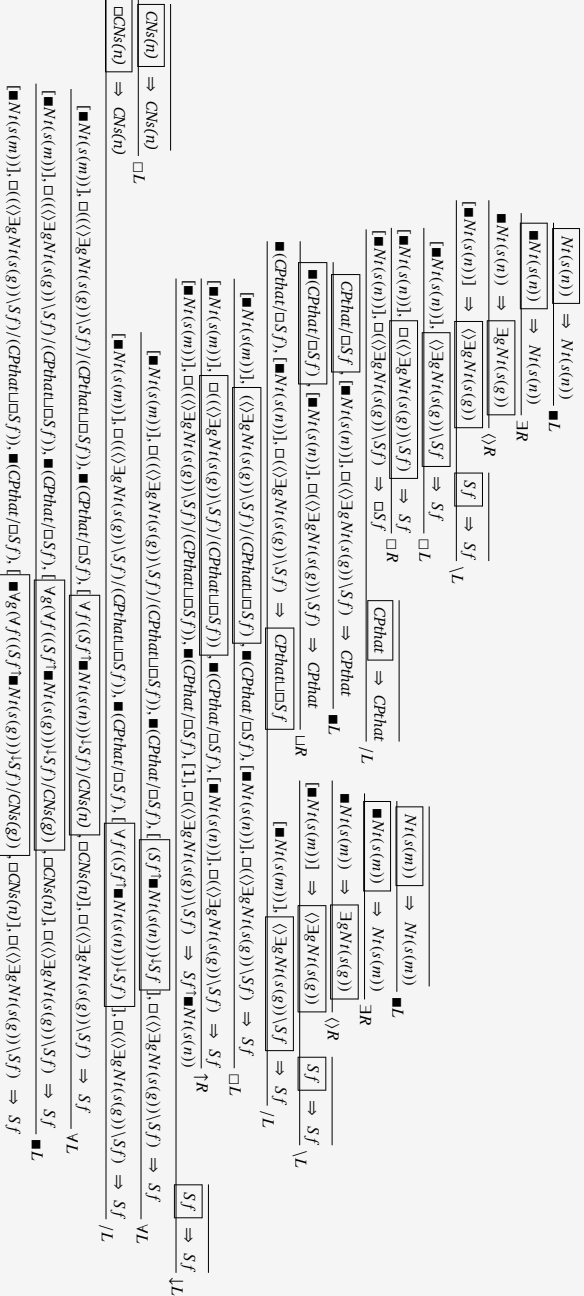
$\hat{\lambda} \lambda I (\checkmark talk I) \Rightarrow Sf$

$[\blacksquare \forall g (\forall f ((Sf \uparrow Nt(s(g))) \downarrow Sf) / CNs(g)) : \lambda A \lambda B \forall C [(A C) \rightarrow (B C)],$

$\square CNs(n) : fish, \blacksquare \forall n ((CNn \setminus CNn) / (Sf | \blacksquare Nt(n))) : \lambda D \lambda E \lambda F [(E F) \wedge (D F)],$

$[\blacksquare \square]^{-1} \forall f ((\blacksquare Sf | \blacksquare Nt(s(n))) / (\langle \rangle Nt(s(n)) \setminus Sf)) : \lambda GG],$

$\square(\langle \rangle \exists g Nt(s(g)) \setminus Sf) : \hat{\lambda} \lambda H (\checkmark walk H)], \square(\langle \rangle \exists g Nt(s(g)) \setminus Sf) :$


Figure 3 First derivation of $(dwp((7-43, 45)))$

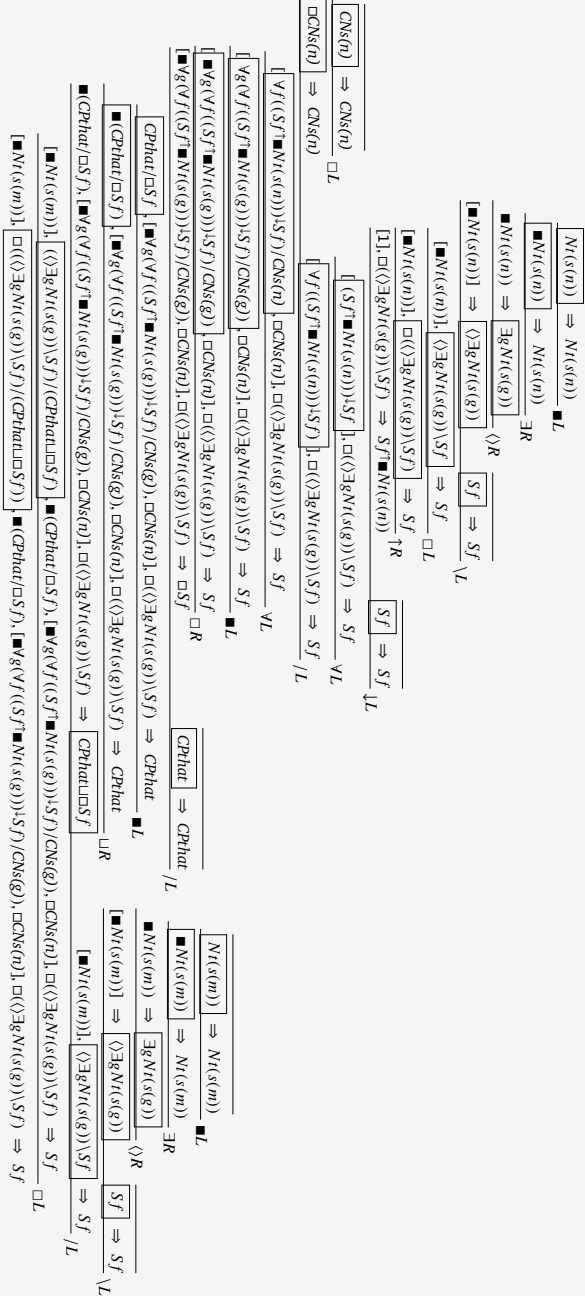


Figure 4 Second derivation of $(\text{dwp}((7-43, 45)))$

$$\wedge \lambda I (\wedge \text{talk } I) \Rightarrow Sf$$

(Derivation omitted)

$$\forall C [(\wedge \text{fish } C) \wedge (\wedge \text{walk } C)] \rightarrow (\wedge \text{talk } C)]$$

(dwp((7-60, 62))) **[john]+seeks+a+unicorn : Sf**

$$[\blacksquare Nt(s(m)) : j], \square((\langle \rangle \exists g Nt(s(g)) \setminus Sf) /$$

$$\square \forall a \forall f (((Na \setminus Sf) / \exists b Nb) \setminus (Na \setminus Sf))) :$$

$$\wedge \lambda A \lambda B ((\wedge \text{try } \wedge ((\wedge A \wedge \text{find } B)) B), \blacksquare \forall g (\forall f ((Sf \uparrow \blacksquare Nt(s(g))) \downarrow Sf) / CNs(g)) :$$

$$\lambda C \lambda D \exists E [(C E) \wedge (D E)], \square CNs(n) : \text{unicorn} \Rightarrow Sf$$

For the derivation, see figure 5.

$$\exists C [(\wedge \text{unicorn } C) \wedge ((\wedge \text{try } \wedge ((\wedge \text{find } C) j)) j)]$$

For the derivation, see figure 6.

$$((\wedge \text{try } \wedge \exists G [(\wedge \text{unicorn } G) \wedge ((\wedge \text{find } G) j)]) j)$$

(dwp((7-73))) **[john]+is+bill : Sf**

$$[\blacksquare Nt(s(m)) : j],$$

$$\blacksquare((\langle \rangle \exists g Nt(s(g)) \setminus Sf) / (\exists a Na \oplus (\exists g ((CNg / CNg) \sqcup (CNg \setminus CNg)) - I))) :$$

$$\lambda A \lambda B (A \rightarrow C.[B = C]; D.((D \lambda E [E = B]) B)), \blacksquare Nt(s(m)) : b \Rightarrow Sf$$

For the derivation, see figure 7.

$$[j = b]$$

(dwp((7-76))) **[john]+is+a+man : Sf**

$$[\blacksquare Nt(s(m)) : j],$$

$$\blacksquare((\langle \rangle \exists g Nt(s(g)) \setminus Sf) / (\exists a Na \oplus (\exists g ((CNg / CNg) \sqcup (CNg \setminus CNg)) - I))) :$$

$$\lambda A \lambda B (A \rightarrow C.[B = C]; D.((D \lambda E [E = B]) B)),$$

$$\blacksquare \forall g (\forall f ((Sf \uparrow \blacksquare Nt(s(g))) \downarrow Sf) / CNs(g)) : \lambda F \lambda G \exists H [(F H) \wedge (G H)],$$

$$\square CNs(m) : \text{man} \Rightarrow Sf$$

For the derivation, see figure 8.

$$\exists C [(\wedge \text{man } C) \wedge [j = C]]$$

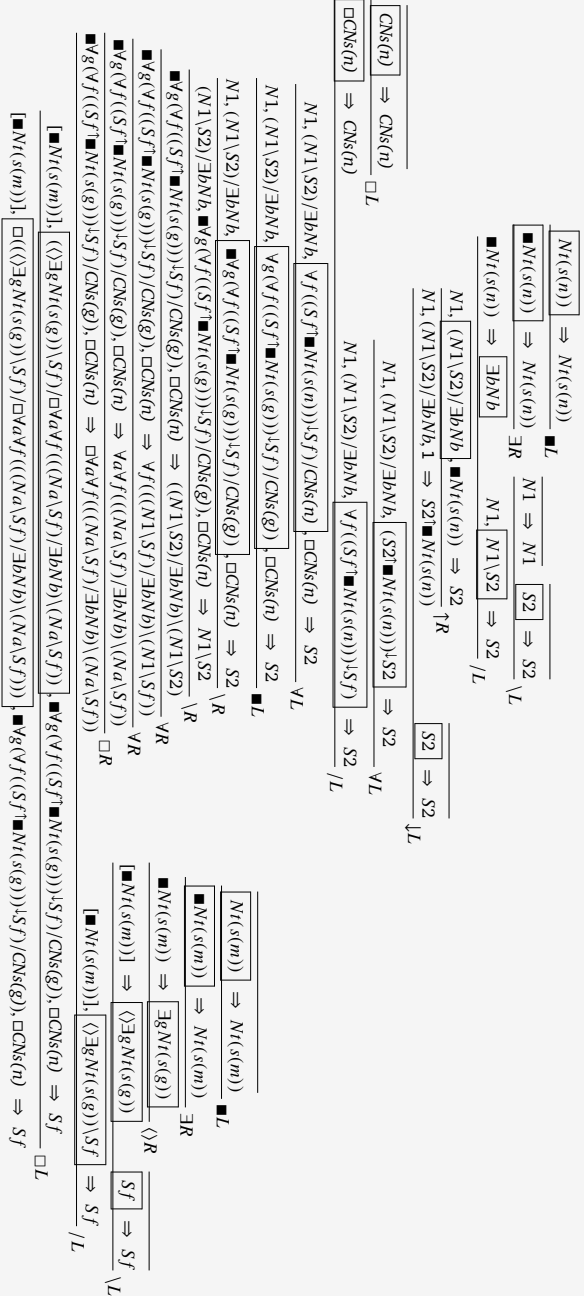


Figure 6 Second derivation of (dwp((7-60, 62)))

$$\begin{array}{c}
\frac{\frac{\frac{Nt(s(m)) \Rightarrow Nt(s(m))}{\blacksquare L}}{\blacksquare Nt(s(m)) \Rightarrow Nt(s(m))} \exists R}{\blacksquare Nt(s(m)) \Rightarrow \exists aNa} \exists R \\
\frac{\frac{\frac{Nt(s(m)) \Rightarrow Nt(s(m))}{\blacksquare L}}{\blacksquare Nt(s(m)) \Rightarrow Nt(s(m))} \exists R}{\blacksquare Nt(s(m)) \Rightarrow \exists g Nt(s(g))} \exists R \\
\frac{\frac{\frac{\frac{Nt(s(m)) \Rightarrow Nt(s(m))}{\blacksquare L}}{\blacksquare Nt(s(m)) \Rightarrow Nt(s(m))} \exists R}{\blacksquare Nt(s(m)) \Rightarrow \exists g Nt(s(g))} \exists R}{\blacksquare Nt(s(m)) \Rightarrow \langle \exists g Nt(s(g)) \rangle} \langle R \\
\frac{\frac{\frac{\frac{\frac{Nt(s(m)) \Rightarrow Nt(s(m))}{\blacksquare L}}{\blacksquare Nt(s(m)) \Rightarrow Nt(s(m))} \exists R}{\blacksquare Nt(s(m)) \Rightarrow \exists g Nt(s(g))} \exists R}{\blacksquare Nt(s(m)) \Rightarrow \langle \exists g Nt(s(g)) \rangle} \langle R}{\blacksquare Nt(s(m)) \Rightarrow \langle \exists g Nt(s(g)) \rangle Sf} \backslash L \\
\frac{\frac{\frac{\frac{\frac{\frac{Nt(s(m)) \Rightarrow Nt(s(m))}{\blacksquare L}}{\blacksquare Nt(s(m)) \Rightarrow Nt(s(m))} \exists R}{\blacksquare Nt(s(m)) \Rightarrow \exists g Nt(s(g))} \exists R}{\blacksquare Nt(s(m)) \Rightarrow \langle \exists g Nt(s(g)) \rangle} \langle R}{\blacksquare Nt(s(m)) \Rightarrow \langle \exists g Nt(s(g)) \rangle Sf} \backslash L}{\blacksquare Nt(s(m)) \Rightarrow \exists aNa \oplus (\exists g (CNg/CNg) \sqcup (CNg \setminus CNg)) - I} \oplus R \\
\frac{\frac{\frac{\frac{\frac{\frac{Nt(s(m)) \Rightarrow Nt(s(m))}{\blacksquare L}}{\blacksquare Nt(s(m)) \Rightarrow Nt(s(m))} \exists R}{\blacksquare Nt(s(m)) \Rightarrow \exists g Nt(s(g))} \exists R}{\blacksquare Nt(s(m)) \Rightarrow \langle \exists g Nt(s(g)) \rangle} \langle R}{\blacksquare Nt(s(m)) \Rightarrow \langle \exists g Nt(s(g)) \rangle Sf} \backslash L}{\blacksquare Nt(s(m)) \Rightarrow \exists aNa \oplus (\exists g (CNg/CNg) \sqcup (CNg \setminus CNg)) - I} \oplus R}{\blacksquare Nt(s(m)) \Rightarrow \langle \exists g Nt(s(g)) \rangle Sf} \backslash L \\
\frac{\frac{\frac{\frac{\frac{\frac{Nt(s(m)) \Rightarrow Nt(s(m))}{\blacksquare L}}{\blacksquare Nt(s(m)) \Rightarrow Nt(s(m))} \exists R}{\blacksquare Nt(s(m)) \Rightarrow \exists g Nt(s(g))} \exists R}{\blacksquare Nt(s(m)) \Rightarrow \langle \exists g Nt(s(g)) \rangle} \langle R}{\blacksquare Nt(s(m)) \Rightarrow \langle \exists g Nt(s(g)) \rangle Sf} \backslash L}{\blacksquare Nt(s(m)) \Rightarrow \exists aNa \oplus (\exists g (CNg/CNg) \sqcup (CNg \setminus CNg)) - I} \oplus R}{\blacksquare Nt(s(m)) \Rightarrow \langle \exists g Nt(s(g)) \rangle Sf} \backslash L} \blacksquare L \\
\frac{\frac{\frac{\frac{\frac{\frac{Nt(s(m)) \Rightarrow Nt(s(m))}{\blacksquare L}}{\blacksquare Nt(s(m)) \Rightarrow Nt(s(m))} \exists R}{\blacksquare Nt(s(m)) \Rightarrow \exists g Nt(s(g))} \exists R}{\blacksquare Nt(s(m)) \Rightarrow \langle \exists g Nt(s(g)) \rangle} \langle R}{\blacksquare Nt(s(m)) \Rightarrow \langle \exists g Nt(s(g)) \rangle Sf} \backslash L}{\blacksquare Nt(s(m)) \Rightarrow \exists aNa \oplus (\exists g (CNg/CNg) \sqcup (CNg \setminus CNg)) - I} \oplus R}{\blacksquare Nt(s(m)) \Rightarrow \langle \exists g Nt(s(g)) \rangle Sf} \backslash L} \blacksquare L
\end{array}$$

Figure 7 Derivation of (dwp((7-73)))

(dwp((7-83))) necessarily+[john]+walks : Sf

$\blacksquare(SA/\square SA) : Nec$, $\blacksquare Nt(s(m)) : j$, $\square(\langle \exists g Nt(s(g)) \rangle \setminus Sf) :$
 $\wedge \lambda B(\checkmark walk B) \Rightarrow Sf$

(Derivation omitted)

$(Nec \wedge ((\checkmark walk j)))$

(dwp((7-86))) [john]+walks+slowly : Sf

$\blacksquare Nt(s(m)) : j$, $\square(\langle \exists g Nt(s(g)) \rangle \setminus Sf) : \wedge \lambda A(\checkmark walk A)$,
 $\square \forall a \forall f(\square(\langle \exists a Na \setminus Sf \rangle) \setminus (\langle \square Na \setminus Sf \rangle)) : \wedge \lambda B \lambda C(\checkmark slowly \wedge (\checkmark B \checkmark C)) \Rightarrow Sf$

(Derivation omitted)

$(\checkmark slowly \wedge (\checkmark walk j))$

(dwp((7-91))) [john]+tries+to+walk : Sf

$\blacksquare Nt(s(m)) : j$, $\square(\langle \langle \exists g Nt(s(g)) \rangle \setminus Sf \rangle / \square(\langle \exists g Nt(s(g)) \rangle \setminus Si)) :$
 $\wedge \lambda A \lambda B((\checkmark try \wedge (\checkmark A B)) B)$, $\blacksquare((PPto/\exists aNa) \cap \forall n(\langle \langle Nn \setminus Si \rangle \rangle / (\langle \langle Nn \setminus Sb \rangle \rangle))) :$
 λCC , $\square(\langle \exists aNa \setminus Sb \rangle) : \wedge \lambda D(\checkmark walk D) \Rightarrow Sf$

(Derivation omitted)

$((\checkmark try \wedge (\checkmark walk j)) j)$

(dwp((7-94))) [john]+tries+to+[[catch+a+fish+and+eat+it]] : Sf

$\blacksquare Nt(s(m)) : j$, $\square(\langle \langle \exists g Nt(s(g)) \rangle \setminus Sf \rangle / \square(\langle \exists g Nt(s(g)) \rangle \setminus Si)) :$
 $\wedge \lambda A \lambda B((\checkmark try \wedge (\checkmark A B)) B)$, $\blacksquare((PPto/\exists aNa) \cap \forall n(\langle \langle Nn \setminus Si \rangle \rangle / (\langle \langle Nn \setminus Sb \rangle \rangle))) :$

$$\begin{aligned}
& \lambda CC, [[\square((\langle \rangle \exists aNa \setminus Sb) / \exists aNa) : \hat{\lambda} D \lambda E((\sim \text{catch } D) E), \\
& \blacksquare \forall g(\forall f((Sf \uparrow \blacksquare Nt(s(g))) \downarrow Sf) / CNs(g)) : \lambda F \lambda G \exists H[(F H) \wedge (G H)], \\
& \square CNs(n) : \text{fish}, \blacksquare \forall f((\blacksquare ?Sf \setminus []^{-1} []^{-1} Sf) / \blacksquare Sf) : (\Phi^{n+} \text{ o and}), \\
& \square((\langle \rangle \exists aNa \setminus Sb) / \exists aNa) : \hat{\lambda} I \lambda J((\sim \text{eat } I) J), \\
& \blacksquare \forall f \forall a((\langle \rangle Na \setminus Sf) \uparrow \blacksquare Nt(s(n))) \downarrow (\blacksquare (\langle \rangle Na \setminus Sf) | \blacksquare Nt(s(n))) : \lambda KK]] \Rightarrow \\
& Sf
\end{aligned}$$

$$\begin{aligned}
& [\blacksquare Nt(s(m)) : j], \square((\langle \rangle \exists g Nt(s(g)) \setminus Sf) / \square(\langle \rangle \exists g Nt(s(g)) \setminus Si)) : \\
& \hat{\lambda} A \lambda B((\sim \text{try } \hat{\sim} A B) B), \blacksquare((P Pto / \exists aNa) \square \forall n((\langle \rangle Nn \setminus Si) / (\langle \rangle Nn \setminus Sb))) : \\
& \lambda CC, [[\square((\langle \rangle \exists aNa \setminus Sb) / \exists aNa) : \hat{\lambda} D \lambda E((\sim \text{catch } D) E), \\
& \blacksquare \forall g(\forall f((Sf \uparrow \blacksquare Nt(s(g))) \downarrow Sf) / CNs(g)) : \lambda F \lambda G \exists H[(F H) \wedge (G H)], \\
& \square CNs(n) : \text{fish}, \blacksquare \forall f((\blacksquare ?Sf \setminus []^{-1} []^{-1} Sf) / \blacksquare Sf) : (\Phi^{n+} \text{ o and}), \\
& \square((\langle \rangle \exists aNa \setminus Sb) / \exists aNa) : \hat{\lambda} I \lambda J((\sim \text{eat } I) J), \\
& \blacksquare []^{-1} \forall f((\blacksquare Sf | \blacksquare Nt(s(n))) / (\langle \rangle Nt(s(n)) \setminus Sf)) : \lambda KK]] \Rightarrow Sf
\end{aligned}$$

$$\begin{aligned}
& [\blacksquare Nt(s(m)) : j], \square((\langle \rangle \exists g Nt(s(g)) \setminus Sf) / \square(\langle \rangle \exists g Nt(s(g)) \setminus Si)) : \\
& \hat{\lambda} A \lambda B((\sim \text{try } \hat{\sim} A B) B), \blacksquare((P Pto / \exists aNa) \square \forall n((\langle \rangle Nn \setminus Si) / (\langle \rangle Nn \setminus Sb))) : \\
& \lambda CC, [[\square((\langle \rangle \exists aNa \setminus Sb) / \exists aNa) : \hat{\lambda} D \lambda E((\sim \text{catch } D) E), \\
& \blacksquare \forall g(\forall f((Sf \uparrow \blacksquare Nt(s(g))) \downarrow Sf) / CNs(g)) : \lambda F \lambda G \exists H[(F H) \wedge (G H)], \\
& \square CNs(n) : \text{fish}, \blacksquare \forall a \forall f((\blacksquare ?(\langle \rangle Na \setminus Sf) \setminus []^{-1} []^{-1} (\langle \rangle Na \setminus Sf)) / \blacksquare (\langle \rangle Na \setminus Sf)) : \\
& (\Phi^{n+} (s \text{ o and}), \square((\langle \rangle \exists aNa \setminus Sb) / \exists aNa) : \hat{\lambda} I \lambda J((\sim \text{eat } I) J), \\
& \blacksquare \forall f \forall a((\langle \rangle Na \setminus Sf) \uparrow \blacksquare Nt(s(n))) \downarrow (\blacksquare (\langle \rangle Na \setminus Sf) | \blacksquare Nt(s(n))) : \lambda KK]] \Rightarrow \\
& Sf
\end{aligned}$$

(Derivation omitted)

$$\exists C[(\sim \text{fish } C) \wedge ((\sim \text{try } \hat{[(\sim \text{catch } C) j] \wedge ((\sim \text{eat } C) j)]}) j]$$

(Derivation omitted)

$$((\sim \text{try } \hat{\exists F}[(\sim \text{fish } F) \wedge ((\sim \text{catch } F) j) \wedge ((\sim \text{eat } F) j)]]) j)$$

$$((\sim \text{try } \hat{\exists H}[(\sim \text{fish } H) \wedge ((\sim \text{catch } H) j) \wedge ((\sim \text{eat } H) j)]]) j)$$

$$\begin{aligned}
& [\blacksquare Nt(s(m)) : j], \square((\langle \rangle \exists g Nt(s(g)) \setminus Sf) / \square(\langle \rangle \exists g Nt(s(g)) \setminus Si)) : \\
& \hat{\lambda} A \lambda B((\sim \text{try } \hat{\sim} A B) B), \blacksquare((P Pto / \exists aNa) \square \forall n((\langle \rangle Nn \setminus Si) / (\langle \rangle Nn \setminus Sb))) : \\
& \lambda CC, [[\square((\langle \rangle \exists aNa \setminus Sb) / \exists aNa) : \hat{\lambda} D \lambda E((\sim \text{catch } D) E), \\
& \blacksquare \forall g(\forall f((Sf \uparrow \blacksquare Nt(s(g))) \downarrow Sf) / CNs(g)) : \lambda F \lambda G \exists H[(F H) \wedge (G H)], \\
& \square CNs(n) : \text{fish}, \blacksquare \forall a \forall f((\blacksquare ?(\langle \rangle Na \setminus Sf) \setminus []^{-1} []^{-1} (\langle \rangle Na \setminus Sf)) / \blacksquare (\langle \rangle Na \setminus Sf)) :
\end{aligned}$$

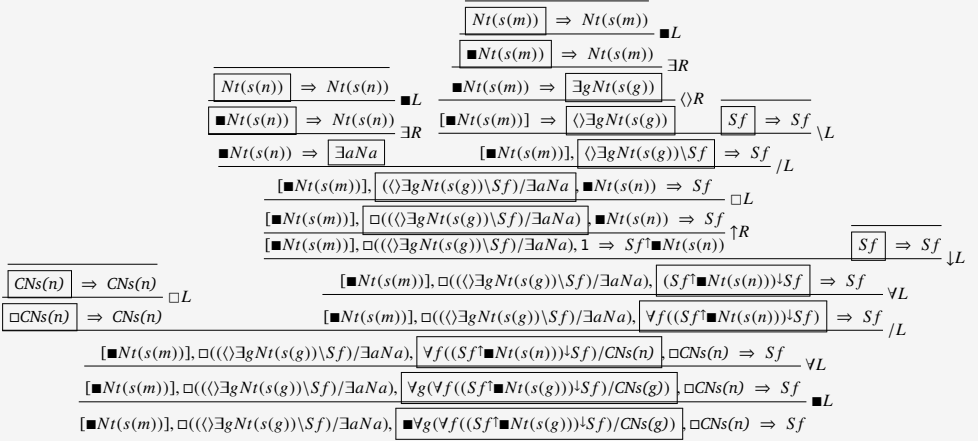


Figure 9 Derivation of (dwp((7-98)))

$(\Phi^{n+}(s\ o\ and), \square((\langle\langle\rangle\exists aNa\ Sb\rangle\rangle/\exists aNa) : \hat{\lambda}l\lambda I\lambda J((\check{v}\ eat\ I)\ J),$
 $\blacksquare[\]^{-1}\forall f((\blacksquare Sf|\blacksquare Nt(s(n)))/(\langle\langle\rangle Nt(s(n))\ Sf\rangle\rangle) : \lambda KK] \Rightarrow Sf$

(dwp((7-98))) [john]+finds+a+unicorn : Sf

$[\blacksquare Nt(s(m)) : j], \square((\langle\langle\rangle\exists gNt(s(g))\ Sf\rangle\rangle/\exists aNa) : \hat{\lambda}\lambda A\lambda B((\check{v}\ find\ A)\ B),$
 $\blacksquare\forall g(\forall f((Sf\uparrow\blacksquare Nt(s(g)))\downarrow Sf)/CNs(g)) : \lambda C\lambda D\exists E[(C\ E) \wedge (D\ E)],$
 $\square CNs(n) : unicorn \Rightarrow Sf$

For the derivation, see figure 9.

$\exists C[(\check{v}\ unicorn\ C) \wedge ((\check{v}\ find\ C)\ j)]$

(dwp((7-105))) [every+man+such+that+[he]+loves+a+woman]
 +loses+her : Sf

$[\blacksquare\forall g(\forall f((Sf\uparrow\blacksquare Nt(s(g)))\downarrow Sf)/CNs(g)) : \lambda A\lambda B\forall C[(A\ C) \rightarrow (B\ C)],$
 $\square CNs(m) : man, \blacksquare\forall n((CNn\ CNn)/(Sf|\blacksquare Nt(n)) : \lambda D\lambda E\lambda F[(E\ F) \wedge (D\ F)],$
 $[\blacksquare[\]^{-1}\forall g((\blacksquare Sg|\blacksquare Nt(s(m)))/(\langle\langle\rangle Nt(s(m))\ Sg\rangle\rangle) : \lambda GG],$
 $\square((\langle\langle\rangle\exists gNt(s(g))\ Sf\rangle\rangle/\exists aNa) : \hat{\lambda}H\lambda I((\check{v}\ love\ H)\ I),$
 $\blacksquare\forall g(\forall f((Sf\uparrow\blacksquare Nt(s(g)))\downarrow Sf)/CNs(g)) : \lambda J\lambda K\exists L[(J\ L) \wedge (K\ L)],$
 $\square CNs(f) : woman],$
 $\square((\langle\langle\rangle\exists gNt(s(g))\ Sf\rangle\rangle/\exists aNa) : \hat{\lambda}M\lambda N((\check{v}\ lose\ M)\ N),$
 $\blacksquare\forall g\forall a(((\langle\langle\rangle Na\ Sg\rangle\rangle\uparrow\blacksquare Nt(s(f)))\downarrow(\blacksquare(\langle\langle\rangle Na\ Sg)|\blacksquare Nt(s(f)))) : \lambda OO \Rightarrow Sf$

(Derivation omitted)

$$\exists C[(\checkmark \text{woman } C) \wedge \forall G[(\checkmark \text{man } G) \wedge ((\checkmark \text{love } C) G)] \rightarrow ((\checkmark \text{lose } C) G)]$$

(dwp((7-II0))) [**john**]+**walks+in+a+park** : Sf

$$\begin{aligned} & [\blacksquare Nt(s(m)) : j], \square(\langle \rangle \exists g Nt(s(g)) \setminus Sf) : \hat{\lambda} A (\checkmark \text{walk } A), \\ & \square(\forall a \forall f((\langle \rangle Na \setminus Sf) \setminus (\langle \rangle Na \setminus Sf)) / \exists a Na) : \hat{\lambda} B \lambda C \lambda D ((\checkmark \text{in } B) (C D)), \\ & \blacksquare \forall g (\forall f((Sf \uparrow \blacksquare Nt(s(g))) \downarrow Sf) / CNs(g)) : \lambda E \lambda F \exists G [(E G) \wedge (F G)], \\ & \square CNs(n) : \text{park} \Rightarrow Sf \end{aligned}$$

(Derivation omitted)

$$\exists C[(\checkmark \text{park } C) \wedge ((\checkmark \text{in } C) (\checkmark \text{walk } j))]$$

(dwp((7-II6, II8))) [**every+man**]+**doesnt+walk** : Sf

$$\begin{aligned} & [\blacksquare \forall g (\forall f((Sf \uparrow Nt(s(g))) \downarrow Sf) / CNs(g)) : \lambda A \lambda B \forall C [(A C) \rightarrow (B C)], \\ & \square CNs(m) : \text{man}], \blacksquare \forall g \forall a ((Sg \uparrow (\langle \rangle Na \setminus Sf) / (\langle \rangle Na \setminus Sb)) \downarrow Sg) : \\ & \lambda D \neg (D \lambda E \lambda F (E F)), \square(\langle \rangle \exists a Na \setminus Sb) : \hat{\lambda} G (\checkmark \text{walk } G) \Rightarrow Sf \end{aligned}$$

(Derivation omitted)

$$\forall C[(\checkmark \text{man } C) \rightarrow \neg(\checkmark \text{walk } C)]$$

(Derivation omitted)

$$\neg \forall G[(\checkmark \text{man } G) \rightarrow (\checkmark \text{walk } G)]$$

Appendix: Rules

The syntactic types of displacement logic are sorted $\mathcal{F}_0, \mathcal{F}_1, \mathcal{F}_2, \dots$ according to the number of points of discontinuity 0, 1, 2, ... their expressions contain. Each type predicate letter has a sort and an arity which are naturals, and a corresponding semantic type. Assuming ordinary terms to be already given, where P is a type predicate letter of sort i and arity n and t_1, \dots, t_n are terms, $Pt_1 \dots t_n$ is an (atomic) type of sort i of the corresponding semantic type. Compound types are formed by connectives as indicated in table 2,² and the structure preserving semantic type map T

²We list only connectives drawn from the first two rows of table 1, omitting some which are not central here.

1.	$\mathcal{F}_i ::= \mathcal{F}_{i+j}/\mathcal{F}_j$	$T(C/B) = T(B) \rightarrow T(C)$	over
2.	$\mathcal{F}_j ::= \mathcal{F}_i \setminus \mathcal{F}_{i+j}$	$T(A/C) = T(A) \rightarrow T(C)$	under
3.	$\mathcal{F}_{i+j} ::= \mathcal{F}_i \bullet \mathcal{F}_j$	$T(A \bullet B) = T(A) \& T(B)$	continuous product
4.	$\mathcal{F}_0 ::= I$	$T(I) = \top$	continuous unit
5.	$\mathcal{F}_{i+1} ::= \mathcal{F}_{i+j} \uparrow_k \mathcal{F}_j, 1 \leq k \leq i+j$	$T(C \uparrow_k B) = T(B) \rightarrow T(C)$	extract
6.	$\mathcal{F}_j ::= \mathcal{F}_{i+1} \downarrow_k \mathcal{F}_{i+j}, 1 \leq k \leq i+1$	$T(A \downarrow_k C) = T(A) \rightarrow T(C)$	infix
7.	$\mathcal{F}_{i+j} ::= \mathcal{F}_{i+1} \circ_k \mathcal{F}_j, 1 \leq k \leq i+1$	$T(A \circ_k B) = T(A) \& T(B)$	discontinuous product
8.	$\mathcal{F}_1 ::= J$	$T(J) = \top$	discontinuous unit
9.	$\mathcal{F}_i ::= \mathcal{F}_i \& \mathcal{F}_i$	$T(A \& B) = T(A) \& T(B)$	additive conjunction
10.	$\mathcal{F}_i ::= \mathcal{F}_i \oplus \mathcal{F}_i$	$T(A \oplus B) = T(A) + T(B)$	additive disjunction
11.	$\mathcal{F}_i ::= \bigwedge V \mathcal{F}_i$	$T(\bigwedge v A) = F \rightarrow T(A)$	1st order univ. qu.
12.	$\mathcal{F}_i ::= \bigvee V \mathcal{F}_i$	$T(\bigvee v A) = F \& T(A)$	1st order exist. qu.
13.	$\mathcal{F}_i ::= \square \mathcal{F}_i$	$T(\square A) = LT(A)$	universal modality
14.	$\mathcal{F}_i ::= \diamond \mathcal{F}_i$	$T(\diamond A) = MT(A)$	existential modality
15.	$\mathcal{F}_i ::= [\]^{-1} \mathcal{F}_i$	$T([\]^{-1} A) = T(A)$	univ. bracket modality
16.	$\mathcal{F}_i ::= \langle \rangle \mathcal{F}_i$	$T(\langle \rangle A) = T(A)$	exist. bracket modality
17.	$\mathcal{F}_0 ::= !\mathcal{F}_0$	$T(!A) = T(A)$	universal exponential
18.	$\mathcal{F}_0 ::= ?\mathcal{F}_0$	$T(?A) = T(A)^+$	existential exponential
19.	$\mathcal{F}_{i+j} ::= \mathcal{F}_{i+j} \mathcal{F}_j$	$T(B A) = T(A) \rightarrow T(B)$	contr. for anaph.
35.	$\mathcal{F}_i ::= \forall V \mathcal{F}_i$	$T(\forall v A) = T(A)$	sem. inactive 1st order univ. qu.
36.	$\mathcal{F}_i ::= \exists V \mathcal{F}_i$	$T(\exists v A) = T(A)$	sem. inactive 1st order exist. qu.
37.	$\mathcal{F}_i ::= \blacksquare \mathcal{F}_i$	$T(\blacksquare A) = T(A)$	sem. inactive universal modality
38.	$\mathcal{F}_i ::= \blacklozenge \mathcal{F}_i$	$T(\blacklozenge A) = T(A)$	sem. inactive existential modality

Table 2 Syntactic types

associates these with semantic types.

In Gentzen sequent configurations (Γ, Δ) for displacement calculus a discontinuous type is a mother, rather than a leaf, and dominates its discontinuous components marked off by curly brackets and colons.

In Gentzen sequent antecedents for displacement logic with bracket modalities (structural inhibition) and exponentials (structural facilitation) there is also a bracket constructor for the former and ‘stoups’ for the latter.

Stoups (cf. the linear logic of Girard 2011 (ζ)) are stores read as multisets for re-usable (nonlinear) resources which appear at the left of a configuration marked off by a semicolon (when the stoup is empty the semicolon may be omitted, as in the derivations of the previous section). The stoup of linear logic is for resources which can be contracted (copied) or weakened (deleted). By contrast, our stoup is for a linguistically motivated variant of contraction, and does not allow weakening. Furthermore, whereas linear logic is commutative, our logic is in general noncommutative and the stoup is used for resources which are also commutative.

A configuration together with a stoup is a *zone* (Ξ). The bracket constructor applies not to a configuration alone but to a configuration with a

stoup, i.e a zone: reusable resources are specific to their domain.

Stoups \mathcal{S} and configurations \mathcal{O} are defined by (\emptyset is the empty stoup; Λ is the empty configuration; the *separator* 1 marks points of discontinuity.³

$$\begin{aligned} \text{(1)} \quad \mathcal{S} &::= \emptyset \mid \mathcal{F}_0, \mathcal{S} \\ \mathcal{O} &::= \Lambda \mid \mathcal{T}, \mathcal{O} \\ \mathcal{T} &::= 1 \mid \mathcal{F}_0 \mid \mathcal{F}_{i>0} \underbrace{\{\mathcal{O} : \dots : \mathcal{O}\}}_{i \text{ } \mathcal{O}'\text{s}} \mid [\mathcal{S}; \mathcal{O}] \end{aligned}$$

For a type A , its sort $s(A)$ is the i such that $A \in \mathcal{F}_i$. For a configuration Γ , its sort $s(\Gamma)$ is $|\Gamma|_1$, that is, the number of points of discontinuity 1 which it contains. Sequents are of the form:

$$\text{(2)} \quad \mathcal{S}; \mathcal{O} \Rightarrow \mathcal{F} \text{ such that } s(\mathcal{O}) = s(\mathcal{F})$$

The figure \vec{A} of a type A is defined by:

$$\text{(3)} \quad \vec{A} = \begin{cases} A & \text{if } s(A) = 0 \\ A \underbrace{\{1 : \dots : 1\}}_{s(A) \text{ } 1'\text{s}} & \text{if } s(A) > 0 \end{cases}$$

Where Γ is a configuration of sort i and $\Delta_1, \dots, \Delta_i$ are configurations, the *fold* $\Gamma \otimes \langle \Delta_1 : \dots : \Delta_i \rangle$ is the result of replacing the successive 1's in Γ by $\Delta_1, \dots, \Delta_i$ respectively. Where Γ is of sort i , the hyperoccurrence notation $\Delta \langle \Gamma \rangle$ abbreviates $\Delta_0(\Gamma \otimes \langle \Delta_1 : \dots : \Delta_i \rangle)$, that is, a context configuration Δ (which is externally Δ_0 and internally $\Delta_1, \dots, \Delta_i$) with a potentially discontinuous distinguished subconfiguration Γ . Where Δ is a configuration of sort $i > 0$ and Γ is a configuration, the k th *metalinguistic intercalation* $\Delta \mid_k \Gamma$, $1 \leq k \leq i$, is given by:

$$\text{(4)} \quad \Delta \mid_k \Gamma =_{df} \Delta \otimes \underbrace{\langle 1 : \dots : 1 \rangle}_{k-1 \text{ } 1'\text{s}} : \Gamma : \underbrace{\langle 1 : \dots : 1 \rangle}_{i-k \text{ } 1'\text{s}}$$

that is, $\Delta \mid_k \Gamma$ is the configuration resulting from replacing by Γ the k th separator in Δ .

A semantically labelled sequent is a sequent in which the antecedent type occurrences A_1, \dots, A_n are labelled by distinct variables x_1, \dots, x_n

³Note that only types of sort 0 can go into the stoup; reusable types of other sorts would not preserve the sequent antecedent-succedent sort equality under contraction.

1.
$$\frac{\zeta_1; \Gamma \Rightarrow B: \psi \quad \zeta_2; \Delta \langle \vec{C}: z \rangle \Rightarrow D: \omega}{\zeta_1 \uplus \zeta_2; \Delta \langle \vec{C}/\vec{B}: x, \Gamma \rangle \Rightarrow D: \omega \{(x \psi)/z\}} /L \quad \frac{\zeta; \Gamma, \vec{B}: y \Rightarrow C: \chi}{\zeta; \Gamma \Rightarrow C/B: \lambda y \chi} /R$$
2.
$$\frac{\zeta_1; \Gamma \Rightarrow A: \phi \quad \zeta_2; \Delta \langle \vec{C}: z \rangle \Rightarrow D: \omega}{\zeta_1 \uplus \zeta_2; \Delta \langle \Gamma, \vec{A} \setminus \vec{C}: y \rangle \Rightarrow D: \omega \{(y \phi)/z\}} \setminus L \quad \frac{\zeta; \vec{A}: x, \Gamma \Rightarrow C: \chi}{\zeta; \Gamma \Rightarrow A \setminus C: \lambda x \chi} \setminus R$$
3.
$$\frac{\zeta; \Delta \langle \vec{A}: x, \vec{B}: y \rangle \Rightarrow D: \omega}{\zeta; \Delta \langle \vec{A} \bullet \vec{B}: z \rangle \Rightarrow D: \omega \{\pi_1 z/x, \pi_2 z/y\}} \bullet L \quad \frac{\zeta_1; \Gamma_1 \Rightarrow A: \phi \quad \zeta_2; \Gamma_2 \Rightarrow B: \psi}{\zeta_1 \uplus \zeta_2; \Gamma_1, \Gamma_2 \Rightarrow A \bullet B: (\phi, \psi)} \bullet R$$
4.
$$\frac{\zeta; \Delta \langle \Lambda \rangle \Rightarrow A: \phi}{\zeta; \Delta \langle \vec{T}: x \rangle \Rightarrow A: \phi} IL \quad \frac{}{\emptyset; \Lambda \Rightarrow I: 0} IR$$

Figure 10 Continuous multiplicatives

of types $T(A_1), \dots, T(A_n)$ respectively, and the succedent type A is labelled by a term of type $T(A)$ with free variables drawn from x_1, \dots, x_n . In this appendix we give the semantically labelled Gentzen sequent rules for some primary connectives, and indicate some linguistic applications.

The continuous multiplicatives of figure 10, the Lambek connectives (Lambek 1958, 1988), defined in relation to appending, are the basic means of categorial categorization and subcategorization. Note that here and throughout the active types in antecedents are figures (vectorial) whereas those in succedents are not; intuitively this is because antecedents are structured but succedents are not. The directional divisions over, /, and under, \, are exemplified by assignments such as **the**: N/CN for **the man**: N , **sings**: $N \setminus S$ for **John sings**: S , and **loves**: $(N \setminus S)/N$ for **John loves Mary**: S . The continuous product \bullet is exemplified by a ‘small clause’ assignment such as **considers**: $(N \setminus S)/(N \bullet (CN/CN))$.

The discontinuous multiplicatives of figure 11, the displacement connectives (Morrill & Valentín 2010, Morrill et al. 2011), are defined in relation to plugging. When the value of the k subindex indicates the first (leftmost) point of discontinuity, it may be omitted. Extraction, \uparrow , is exemplified by a discontinuous idiom assignment **gives+1+the+cold+shoulder**: $(N \setminus S) \uparrow N$ for **Mary gives John the cold shoulder**: S , and infixation, \downarrow , and extraction together are exemplified by a quantifier phrase assignment **everyone**: $(S \uparrow N) \downarrow S$, simulating Montague’s S_{14} treatment of quantifying in. Extrac-

5.
$$\frac{\zeta_1; \Gamma \Rightarrow B: \psi \quad \zeta_2; \Delta \langle \vec{C}: z \rangle \Rightarrow D: \omega}{\zeta_1 \uplus \zeta_2; \Delta \langle \overrightarrow{C \uparrow_k B}: x \mid_k \Gamma \rangle \Rightarrow D: \omega \{ (x \ \psi) / z \}} \uparrow_k L \quad \frac{\zeta; \Gamma \mid_k \vec{B}: y \Rightarrow C: \chi}{\zeta; \Gamma \Rightarrow C \uparrow_k B: \lambda y \chi} \uparrow_k R$$
6.
$$\frac{\zeta_1; \Gamma \Rightarrow A: \phi \quad \zeta_2; \Delta \langle \vec{C}: z \rangle \Rightarrow D: \omega}{\zeta_1 \uplus \zeta_2; \Delta \langle \Gamma \mid_k \overrightarrow{A \downarrow_k C}: y \rangle \Rightarrow D: \omega \{ (y \ \phi) / z \}} \downarrow_k L \quad \frac{\zeta; \vec{A}: x \mid_k \Gamma \Rightarrow C: \chi}{\zeta; \Gamma \Rightarrow A \downarrow_k C: \lambda x \chi} \downarrow_k R$$
7.
$$\frac{\zeta; \Delta \langle \vec{A}: x \mid_k \vec{B}: y \rangle \Rightarrow D: \omega}{\zeta; \Delta \langle A \odot_k B \rangle: z \rangle \Rightarrow D: \omega \{ \pi_1 z / x, \pi_2 z / y \}} \odot_k L \quad \frac{\zeta_1; \Gamma_1 \Rightarrow A: \phi \quad \zeta_2; \Gamma_2 \Rightarrow B: \psi}{\zeta_1 \uplus \zeta_2; \Gamma_1 \mid_k \Gamma_2 \Rightarrow A \odot_k B: (\phi, \psi)} \odot_k R$$
8.
$$\frac{\zeta; \Delta \langle 1 \rangle \Rightarrow A: \phi}{\zeta; \Delta \langle \vec{J}: x \rangle \Rightarrow A: \phi} JL \quad \frac{}{\emptyset; 1 \Rightarrow J: 0} JR$$

Figure 11 Discontinuous multiplicatives

9.
$$\frac{\Xi \langle \vec{A}: x \rangle \Rightarrow C: \chi}{\Xi \langle A \& B \rangle: z \rangle \Rightarrow C: \chi \{ \pi_1 z / x \}} \&L_1 \quad \frac{\Xi \langle \vec{B}: y \rangle \Rightarrow C: \chi}{\Xi \langle A \& B \rangle: z \rangle \Rightarrow C: \chi \{ \pi_2 z / y \}} \&L_2 \quad \frac{\Xi \Rightarrow A: \phi \quad \Xi \Rightarrow B: \psi}{\Xi \Rightarrow A \& B: (\phi, \psi)} \&R$$
10.
$$\frac{\Xi \langle \vec{A}: x \rangle \Rightarrow C: \chi_1 \quad \Xi \langle \vec{B}: y \rangle \Rightarrow C: \chi_2}{\Xi \langle A \oplus B \rangle: z \rangle \Rightarrow C: z \rightarrow x. \chi_1; y. \chi_2} \oplus L \quad \frac{\Xi \Rightarrow A: \phi}{\Xi \Rightarrow A \oplus B: \iota_1 \phi} \oplus R_1 \quad \frac{\Xi \Rightarrow B: \psi}{\Xi \Rightarrow A \oplus B: \iota_2 \psi} \oplus R_2$$

Figure 12 Additives

tion and discontinuous product, \odot , are shown together with the continuous unit in an assignment to a relative pronoun **that**: $(CN \setminus CN) / ((S \uparrow N) \odot I)$, allowing both peripheral and medial extraction, as in **that John likes**: $CN \setminus CN$ and **that John saw today**: $CN \setminus CN$.

In relation to the multiplicative rules, notice how the stoup is distributed reading bottom-up from conclusions to premise: it is partitioned between the two premises in the case of binary rules, copied to the premise in the case of unary rules, and empty in the case of nullary rules (axioms).

The remaining figures give rules for additives, quantifiers, normal modalities, bracket modalities, exponentials, and limited contraction for anaphora.

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$$\begin{array}{l}
11. \quad \frac{\Xi \langle \overrightarrow{A[t/v]} : x \rangle \Rightarrow B : \psi}{\Xi \langle \bigwedge v A : z \rangle \Rightarrow B : \psi \{(z t)/x\}} \wedge L \quad \frac{\Xi \Rightarrow A[a/v] : \phi}{\Xi \Rightarrow \bigwedge v A : \lambda v \phi} \wedge R^\dagger \\
12. \quad \frac{\Xi \langle \overrightarrow{A[a/v]} : x \rangle \Rightarrow B : \psi}{\Xi \langle \bigvee v A : z \rangle \Rightarrow B : \psi \{\pi_2 z/x\}} \vee L^\dagger \quad \frac{\Xi \Rightarrow A[t/v] : \phi}{\Xi \Rightarrow \bigvee v A : (t, \phi)} \vee R
\end{array}$$

Figure 13 Quantifiers, where † indicates that there is no a in the conclusion

$$\begin{array}{l}
13. \quad \frac{\Xi \langle \overrightarrow{A} : x \rangle \Rightarrow B : \psi}{\Xi \langle \overrightarrow{\square A} : z \rangle \Rightarrow B : \psi \{\vee z/x\}} \square L \quad \frac{\boxtimes \Xi \Rightarrow A : \phi}{\boxtimes \Xi \Rightarrow \square A : \wedge \phi} \square R \\
14. \quad \frac{\boxtimes \Xi \langle \overrightarrow{A} : x \rangle \Rightarrow \diamond B : \psi}{\boxtimes \Xi \langle \overrightarrow{\diamond A} : z \rangle \Rightarrow \diamond B : \psi \{\cup z/x\}} \diamond L \quad \frac{\Xi \Rightarrow A : \phi}{\Xi \Rightarrow \diamond A : \cap \phi} \diamond R
\end{array}$$

Figure 14 Normal modalities, where \boxtimes/\diamond marks a structure all the types of which have main connective a box/diamond

$$\begin{array}{l}
15. \quad \frac{\Xi \langle \overrightarrow{A} : x \rangle \Rightarrow B : \psi}{\Xi \langle \overrightarrow{[\]^{-1} A} : x \rangle \Rightarrow B : \psi} [\]^{-1} L \quad \frac{[\Xi] \Rightarrow A : \phi}{\Xi \Rightarrow [\]^{-1} A : \phi} [\]^{-1} R \\
16. \quad \frac{\Xi \langle \overrightarrow{[\] A} : x \rangle \Rightarrow B : \psi}{\Xi \langle \overrightarrow{\langle \rangle A} : x \rangle \Rightarrow B : \psi} \langle \rangle L \quad \frac{\Xi \Rightarrow A : \phi}{[\Xi] \Rightarrow \langle \rangle A : \phi} \langle \rangle R
\end{array}$$

Figure 15 Bracket modalities

$$\begin{array}{l}
17. \quad \frac{\Xi(\zeta \wp \{A : x\}; \Gamma_1, \Gamma_2) \Rightarrow B : \psi}{\Xi(\zeta; \Gamma_1, !A : x, \Gamma_2) \Rightarrow B : \psi} !L \quad \frac{\zeta; \Lambda \Rightarrow A : \phi}{\zeta; \Lambda \Rightarrow !A : \phi} !R \quad \frac{\Xi(\zeta; \Gamma_1, A : x, \Gamma_2) \Rightarrow B : \psi}{\Xi(\zeta \wp \{A : x\}; \Gamma_1, \Gamma_2) \Rightarrow B : \psi} !P \quad \frac{\Xi(\zeta \wp \{A : x\}; \Gamma_1, [\{A : y\}; \Gamma_2], \Gamma_3) \Rightarrow B : \psi}{\Xi(\zeta \wp \{A : x\}; \Gamma_1, \Gamma_2, \Gamma_3) \Rightarrow B : \psi \{x/y\}} !C \\
18. \quad \frac{\Delta(A : x) \Rightarrow D : \omega([x]) \quad \Delta(A : x, A : y) \Rightarrow D : \omega([x, y]) \quad \dots}{\Delta(?A : w) \Rightarrow D : \omega(w)} ?L \quad \frac{\Xi \Rightarrow A : \phi}{\Xi \Rightarrow ?A : [\phi]} ?R \quad \frac{\zeta; \Gamma \Rightarrow A : \phi \quad \zeta'; \Delta \Rightarrow ?A : \psi}{\zeta \wp \zeta'; \Gamma, \Delta \Rightarrow ?A : [\phi/\psi]} ?M
\end{array}$$

Figure 16 Exponentials

$$19. \frac{\zeta; \Gamma \Rightarrow A: \phi \quad \zeta'; \Delta(\vec{A}: x; \vec{B}: y) \Rightarrow D: \omega}{\zeta \wp \zeta'; \Delta(\Gamma; \vec{B}|\vec{A}: z) \Rightarrow D: \omega(\phi/x, (z \phi)/y)} |L \quad \frac{\zeta; \Gamma(\vec{B}_0: y_0; \dots; \vec{B}_n: y_n) \Rightarrow D: \omega}{\zeta; \Gamma(\vec{B}_0|\vec{A}: z_0; \dots; \vec{B}_n|\vec{A}: z_n) \Rightarrow D|\lambda x \omega\{(z_0 x)/y_0, \dots, (z_n x)/y_n\}} |R$$

Figure 17 Limited contraction for anaphora

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