GLYN MORRILL

DISCONTINUITY IN CATEGORIAL GRAMMAR[1]

ABSTRACT. Discontinuity refers to the character of many natural language constructions wherein signs differ markedly in their prosodic and semantic forms. As such it presents interesting demands on monostratal computational formalisms which aspire to descriptive adequacy. Pied piping, in particular, is argued by Pollard (1988) to motivate phrase structure-style feature percolation. In the context of categorial grammar, Bach (1981, 1984), Moortgat (1988, 1990, 1991) and others have sought to provide categorial operators suited to discontinuity. These attempts encounter certain difficulties with respect to model theory and/or proof theory, difficulties which the current proposals are intended to resolve.

Lambek calculus is complete for interpretation by *residuation* with respect to the adjunction operation of groupoid algebras (Buszkowski 1986). In Moortgat and Morrill (1991) it is shown how to give calculi for families of categorial operators, each defined by residuation with respect to an operation of prosodic adjunction (associative, non-associative, or with interactive axioms). The present paper treats discontinuity in this way, by residuation with respect to three adjunctions: + (associative), $(.,.)$ (split-point marking), and W (wrapping) related by the equation $s_1 + s_2 + s_3 = (s_1, s_3)Ws_2$. We show how the resulting methods apply to discontinuous functors, quantifier scope and quantifier scope ambiguity, pied piping, and subject and object antecedent reflexivisation.

1. Introduction

In order to specify a model of the relation between forms and meanings in natural language it is necessary to list those associations in which a meaning is not attributable to meanings of parts. In general we expect a lexical enumeration of properties of "words", but insofar as morphological semantics may be systematic, and phrasal idioms may be truly idiomatic, lexical items may be smaller or larger than an intuitive or lexicographic construal of "word". When an element with such an unanalysable meaning is not continuous in the speech stream the phenomenon of discontinuity is exhibited. Examples from English are provided by phrasal verbs (or: particle verbs) such as those in (1), discontinuous idioms such as those in (2), and coordination particle combinations such as those in (3).

(1)a. Mary *rang* John/the man *up*.
b. Mary *put* John/the man *down*.

(2)a. Mary *gave* John/the man *the cold shoulder*.
b. Mary *gets* John's/the man's *goat*.

[1] This paper in its current form has benefitted from extensive reviewing for *Linguistics and Philosophy*. I thank the reviewers and the editor for their attention.

(2)c. Mary *put* John/the man *down*.

(3)a. Mary *neither* walks *nor* talks.
b. Mary *either* walks *or* talks.
c. Mary *both* walks *and* talks.

Conceivably a categorematic treatment of (3) could tie the semantics to one particle and subcategorise it specifically for the other, semantically vacuous, particle. More simply however, we would just like to say that the particles deliver the coordination semantics jointly, but are realised discontinuously. Similarly the italicised elements in (2) have idiomatic meanings (respectively shun, annoy, and criticise/insult) not attributable to the meanings of their parts. And for the particle verbs a compositional derivation of the particle-verb meanings (respectively phone and lower to ground) appears unlikely (see however Jacobson 1987; for these examples there is in fact no discontinuity issue arising if a compositional analysis can be justified). The discontinuity of such constructions presents difficulties for any approach to grammar in which expressions are to be generated just by concatenation of adjacent elements (but see e.g. Wasow, Sag and Nunberg 1983).

These phenomena are in fact just the simplest kind of discontinuity that arises in a monostratal setting. Thus, if by discontinuity we understand mismatch between prosodic form and semantic form,[2] the question arises as to how a quantifier is to obtain sentential semantic scope while the corresponding quantifier phrase is embedded:

(4)a. John likes everything.
b. $\forall x((\mathbf{like}\ x)\ \mathbf{j})$

Furthermore, at least on the standard view, there must be admitted the quantifier scope ambiguity of (5a), and also that of (5b).

(5)a. Everyone loves something.
b. John believes someone walks.

[2] Prosodic form might alternatively be referred to as phonetic form or phonological form though in fact we shall consider no phonetics, phonology or prosodics beyond word order. We appeal to the term in its traditional connotation (cf. prose), but with an understanding that strictly prosodic properties such as intonation also require characterisation at the level at which we are dealing. We reserve the term syntax for the derivational structures that relate prosodic form and semantic form. Semantic form might also be referred to as logical form, but we wish to acknowledge that not just truth conditions (as in this paper), but also extra-logical properties such as information structure require representation at this level; and it is not just treatment of object language semantics, but all grammar, which is formally

Classical quantifier scope ambiguity, between subject wide scope and object wide scope, is exemplified by (5a). For (5b) there is a *de dicto* or non-specific reading with "someone" within the scope of "believes", and a *de re* or specific reading with "someone" outside the scope of "believes"; the latter necessitates quantifier "raising" to the level of a superordinate sentence.

The particular difficulty that quantifier raising represents can be evaded to some extent by appeal to non-monostratal architecture and/or relaxations of compositionality. But what is essentially the same puzzle is presented with an inescapable prosodic realisation in pied piping, that is relativisation in which additional material accompanies the "fronted" relative pronoun:

(6)a. (the man) for whom John works
b. (the contract) the loss of which after so much wrangling John would finally have to pay for (with his job)
c. (the thesis) the height of the lettering on the cover of which is prescribed by university regulations

In such a construction the relative pronoun needs to take semantic scope at the level of the entire fronted constituent in order to bind the gap in the body of the relative clause, i.e. some kind of raising is required, as for quantifiers. In addition, the category of the gap (e.g. prepositional phrase or noun phrase) needs to match that of the fronted constituent, creating, aside from semantics, a complex prosodic discontinuous dependency.

Finally, we consider also reflexivisation. Subject antecedent reflexivisation can be exercised from any object position (subject to certain locality conditions):

(7)a. John talked to Mary about Bill.
b. John talked to himself about Bill.
c. John talked to Mary about himself.

Perhaps more puzzling is object-antecedent reflexivisation:

(8)a. John showed Mary the book.
b. John showed Mary herself.
c. *John showed herself Mary.

Example (8b) is acceptable, whereas (8c) is not. In order for reflexivisation to occur in the semantics, it is necessary for a reflexive to combine with, and reflexivise, a predicate, before the predicate applies to the antecedent: the other way around, the antecedent semantics is not accessible for duplication. The facts in (8) are thus precisely the opposite of those expected for a compositional semantics based on a syntactic analysis concatenating the verb first with its adjacent complement, and then with the remote one. This motivates an analysis of such verbs (see e.g. Bach 1979, 1980; Dowty 1979; Pollard 1984) in which they combine with the prosodically remote complement first, and then "head-wrap" around the other complement. Note however that in (9) head-wrapping does not seem to be quite the right concept since 'Mary' must be inserted into 'talked to about herself' not after the head 'talked', but after 'talked to'.

(9) John talked to Mary about herself.

This example also illustrates both how the reflexive may be "pied piped" within its constituent, and need not actually be c-commanded by its antecedent. Our treatment of reflexives will support such scoping.

Discontinuity represents much that is problematic in natural language syntax, and it is not the pretence of the present paper to provide a comprehensive account of even the instances cited above. However, it does aim to show how each is rendered ammenable in its basic form in the context of categorial grammar in the logical tradition, that is the tradition of Lambek calculus and extended Lambek calculus (see e.g. Moortgat 1988, van Benthem 1991, Morrill 1992a). Moortgat (1988) attempted to place on a more logical footing the earlier proposals for discontinuous categorial operators of e.g. Bach (1981, 1984). But the logic was incomplete; indeed the sequent proof format used seemed in principle incapable of supporting a full logic for discontinuity. We shall see that for the proposals made here this turns out not to be the case after all. The treatments of quantifier raising and subject antecedent reflexivisation were anticipated in Moortgat (1990, 1991), though still without satisfactory technical formulation. We shall present an alternative algebraic interpretation, together with "user-friendly" labelled natural deduction in Prawitz-style and Fitch-style.

Section 2 sketches the basic view of type logical categorial grammar against which the present proposals are set. Section 3 discusses proof theoretic presentations and section 4 illustrates linguistically. Section 5 explains the multimodal generalisation providing the framework for the specific calculus of discontinuity of section 6. Section 7 elucidates linguistic applications: discontinuous functors, quantifier raising, pied piping and

2. Logical Categorial Grammar

The view of grammar to be presented here is distinguished by the fact that it retains no remnant of non-logical X-bar schema or their analogue inherited from the tradition of the Chomsky hierarchy or phrase structure grammar. The central tenet of categorial grammar in the logical tradition, as it is to be assumed here, is that the theory of formation, i.e. the set of operations projecting the language model from the lexicon, can be defined model theoretically. This stance occupies an extreme minimalist lexicalist position in which there is no syntactic component playing a definitional role in the specification of a language model. There is just a lexicon and a theory of formation under which the lexicon is to be closed to specify the language model, declaratively defined by the interpretation of categorial types. Rules of syntax serve to calculate, but not to define. Ultimately a computational and developmental gain is anticipated. When a lexicon defines a language model in interaction with a variable syntactic component, universal parsers and generators can only be optimised up to that structure which is common to all syntactic parameterisations, and grammar development must constantly readdress the possible tradeoff between lexical and syntactic variation. Such advantages cannot be proved *a priori* however, so we must be content now with the interest as regards structure of analyses of a model theoretic perspective on grammar.

2.1. *Division Operators and Groupoid Prosodic Interpretation*

Assume a set $\mathcal{F}$ of categorial syntactic types or ("category") formulas freely generated from a set $\mathcal{A}$ of atomic formulas thus:

(10) $\mathcal{F} ::= \mathcal{A} | \mathcal{F} \backslash \mathcal{F} | \mathcal{F}/\mathcal{F}$

(Such equations are a concise notation, referred to in computer science as Backus–Naur form, for recursive definitions.) We consider interpretation with respect to model structures starting with a groupoid algebra $\langle L, +\rangle$, which is simply a set L closed under a binary operation $+$. An interpretation is a mapping D of formulas into subsets of L such that (cf. e.g. Lambek 1988):[3]

[3] Note that we keep to the categorial notation as originally used by Lambek whereby argument types cancel under adjacency, rather than that of Steedman and others whereby argument types are uniformly right hand operands. The former, correlating with the left and right operands of adjunction, will allow a systematic reflection in type laws of the properties of residuation. To minimise technical volume we suppress treatment of products.

(11) $D(A \backslash B) = \{s | \forall s' \in D(A), s' + s \in D(B)\}$
$D(B/A) = \{s | \forall s' \in D(A), s + s' \in D(B)\}$

Such an interpretation for directional implications (or: divisions) \ ("under") and / ("over") is referred to as interpretation by residuation with respect to the groupoid operation +. Consider a semantic consequence relation $\models$ between formulas:

(12) $A \models B$ iff in all interpretations $D(A) \subseteq D(B)$

Then interpretation by residuation delivers the following (see Lambek 1958, 1961, 1988):[4]

(13) $A \models C/B$ iff $B \models A \backslash C$

Keeping the interpretation clauses and adjusting the algebra gives us alternative logics, in particular with groupoids we have the non-associative Lambek calculus **NL** (Lambek 1961). If we impose a condition of associativity on the algebra of interpretation, we are dealing with semigroup algebras $\langle L, +\rangle$:

(14) $s_1 + (s_2 + s_3) = (s_1 + s_2) + s_3$

This gives us associative Lambek calculus **L** (Lambek 1958), a fragment of a version of non-commutative linear logic (Girard 1989).

The interpretation in groupoids and semigroups corresponds to the prosodic dimension of signs (e.g. an algebra of binary trees is a groupoid, and an algebra of strings is a semigroup). For description of language we are interested also in a semantic dimension.

2.2. *Type Logical Semantic Interpretation*

The categorial division operators can be semantically (as opposed to prosodically) interpreted as spaces in simple type theory. Let a set $\mathscr{T}$ of semantic types be freely generated from a set $\mathscr{D}$ of basic semantic types thus:

(15) $\mathscr{T} ::= \mathscr{D} | \mathscr{T} \to \mathscr{T}$

A semantic algebra consists of a family $\{D_\tau\}_{\tau \in \mathscr{T}}$ of sets (semantic domains), called a frame, such that $D_{\tau_1 \to \tau_2}$ is the set of all (set theoretic) functions from D_{τ_1} to D_{τ_2} (a function space). The semantic types thus

[4] In fact the residuation laws are valid for an even more general interpretation scheme than that which we need here: they apply for ternary "accessibility" relations in general, not just ... (Dunn 1991; Moortgat 1992).

index the domains in a hierarchy of function spaces. A type map is a function T from category formulas to semantic types such that

(16) $T(A \backslash B) = T(B/A) = T(A) \to T(B)$

Working in two dimensions, each formula A has an interpretation $D(A)$ which is a set of ordered pairs of prosodic objects from L and semantic objects from $T(A)$ (cf. e.g. Morrill 1992a):

(17) $D(A \backslash B) = \{\langle s, m\rangle | \forall \langle s', m'\rangle \in D(A),\ \langle s' + s,\ m(m')\rangle \in D(B)\}$
$D(B/A) = \{\langle s, m\rangle | \forall \langle s', m'\rangle \in D(A),\ \langle s + s',\ m(m')\rangle \in D(B)\}$

3. Proof Theory

Lambek (1961) gives a Gentzen-style sequent proof theory for the non-associative calculus **NL**. A sequent is of the form $\Gamma \Rightarrow A$ where the succedent A is a type formula and the antecedent Γ is what we shall call a configuration, which in this case is a binary bracketed sequence of one or more type formulas. A sequent is read as stating that for any objects in the antecedent types the result of applying the (prosodic) operation implicit in the configuration is an object in the succedent type. The calculus is as follows. The parenthetical notation $\Gamma(\Delta)$ represents a configuration containing a distinguished subconfiguration Δ.

(18)a. $A \Rightarrow A$ id $\quad \dfrac{\Gamma \Rightarrow A \quad \Delta(A) \Rightarrow B}{\Delta(\Gamma) \Rightarrow B}\mathrm{Cut}$

b. $\dfrac{\Gamma \Rightarrow A \quad \Delta(B) \Rightarrow C}{\Delta([\Gamma, A \backslash B]) \Rightarrow C}\backslash\mathrm{L} \qquad \dfrac{[A, \Gamma] \Rightarrow B}{\Gamma \Rightarrow A \backslash B}\backslash\mathrm{R}$

c. $\dfrac{\Gamma \Rightarrow A \quad \Delta(B) \Rightarrow C}{\Delta([B/A, \Gamma]) \Rightarrow C}/\mathrm{L} \qquad \dfrac{[\Gamma, A] \Rightarrow B}{\Gamma \Rightarrow B/A}/\mathrm{R}$

A Gentzen sequent presentation has a left rule and a right rule for each connective. The former introduces an occurrence of the connective on the left hand side of the sequent, the latter on the right hand side. Or reading from conclusions to premisses, the former removes a connective occurrence on the left, and the latter removes a connective occurrence on the right. Apart from that, no new formula occurrences can be introduced in going from conclusion to premisses, except by the rule Cut. If a Cut-elimination result can be given, that is a demonstration that all theorems have a proof without Cut, a Gentzen sequent presentation may thus provide a decision procedure. Lambek (1961) proved Cut-elimination (a technical error is corrected by Kandulski 1988). By way of example, the following is a derivation of "lifting".

(19) $$\dfrac{\dfrac{A \Rightarrow A \quad B \Rightarrow B}{[A, A\backslash B] \Rightarrow B}\backslash L}{A \Rightarrow B/(A\backslash B)}/R$$

A Gentzen-style calculus for the associative Lambek calculus **L** can be obtained by adding a structural rule of associativity to the non-associative calculus **NL**:

(20) $$\dfrac{\Gamma([[\Delta_1, \Delta_2], \Delta_3]) \Rightarrow A}{\Gamma([\Delta_1, [\Delta_2, \Delta_3]]) \Rightarrow A}\mathrm{A}$$

(Double lines indicate the rule is valid reading both up and down.) Then, composition for example, which is not derivable in **NL**, can be obtained thus:

(21) $$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{C \Rightarrow C \quad A \Rightarrow A}{[A/C, C] \Rightarrow A}/L \quad B \Rightarrow B}{[B/A, [A/C, C]] \Rightarrow B}/L}{[[B/A, A/C], C] \Rightarrow B}\mathrm{A}}{[B/A, A/C] \Rightarrow B/C}/R}{}$$

Lambek (1958) gives a Gentzen-style sequent proof theory for the associative calculus in which associativity is made implicit. For this a configuration is an unbracketed sequence of one or more type formulas:

(22)a. $A \Rightarrow A$ id $\quad \dfrac{\Gamma \Rightarrow A \quad \Delta(A) \Rightarrow B}{\Delta(\Gamma) \Rightarrow B}\mathrm{Cut}$

b. $\dfrac{\Gamma \Rightarrow A \quad \Delta(B) \Rightarrow C}{\Delta(\Gamma, A\backslash B) \Rightarrow C}\backslash L \qquad \dfrac{A, \Gamma \Rightarrow B}{\Gamma \Rightarrow A\backslash B}\backslash R$

c. $\dfrac{\Gamma \Rightarrow A \quad \Delta(B) \Rightarrow C}{\Delta(B/A, \Gamma) \Rightarrow C}/L \qquad \dfrac{\Gamma, A \Rightarrow B}{\Gamma \Rightarrow B/A}/R$

Lambek (1958) proves Cut-elimination for this case. In this format the

(23) $$\dfrac{\dfrac{\dfrac{\dfrac{C \Rightarrow C \quad A \Rightarrow A}{A/C, C \Rightarrow A}/L \quad B \Rightarrow B}{B/A, A/C, C \Rightarrow B}/L}{B/A, A/C \Rightarrow B/C}/R}{}$$

Note that recursivity in presentation of rules is essential: the calculi cannot be characterised by replacing the left and right rules by some finite set of axiomatic rule schemata (non-finite axiomatisability; Zielonka 1981). There is thus an essential divergence from Generalised Phrase Structure Grammar (Gazdar et al. 1985), or Montagovian PTQ-style grammar (Montague 1973) which are usually understood to effectively limit themselves to a finite set of rule schemata (e.g. by blocking the recursive application of metarules in the former).

Rather than use these more standard Gentzen formulations of Lambek calculi, we shall for the most part use labelled deductive systems (LDSs) to present proof theory (Gabbay 1991; see Moortgat 1992 for categorial application). The philosophy of labelled deduction is "to bring semantics back into syntax". What that will mean for grammar is that prosodic terms and semantic terms, elements of term algebras of the algebras of interpretation, are explicitly managed, providing a formulation maximally comparable to a Montagovian PTQ perspective.

3.1. *Prosodic and Semantic Terms*

For groupoid models, a set $\mathcal{P}$ of prosodic terms (notated $\alpha, \beta, \gamma, \ldots$) is freely generated from a set $\mathcal{K}$ of prosodic constants and a denumerably infinite set $\mathcal{U}$ of prosodic variables thus:

(24) $$\mathcal{P} ::= \mathcal{U} | \mathcal{K} | (\mathcal{P} + \mathcal{P})$$

Each prosodic term has an interpretation as an object in a groupoid algebraic model structure, given in the obvious way.

To include the semantic side, typed lambda terms (notated $\phi, \psi, \chi, \ldots$) are defined and interpreted as usual. Starting from a set $\mathcal{C}_\tau$ of constants and a denumerably infinite set $\mathcal{V}_\tau$ of variables for each type τ, the set $\mathcal{S}_\tau$ of typed semantic terms for each type τ is freely generated by mutual recursion thus:

(25) $$\mathcal{S}_\tau ::= \mathcal{C}_\tau | \mathcal{V}_\tau | (\mathcal{S}_{\tau' \to \tau} \mathcal{S}_{\tau'})$$
$$\mathcal{S}_{\tau' \to \tau} ::= \lambda \mathcal{V}_{\tau'} . \mathcal{S}_\tau$$

We are now in a position to give labelled deductive systems for the

Lambek calculi. We do so in Prawitz-style natural deduction and Fitch-style natural deduction.[5]

3.2. *Prawitz-Style Natural Deduction*

Prawitz-style natural deduction (Prawitz 1965) represents derivation in a tree-like structure which eliminates irrelevant rule orderings by representing subderivations in parallel, in the same way that phrase structure trees represent equivalence classes of serial rewrite steps. Hypothetical reasoning is indicated by withdrawal of assumptions. First we say that any type assignment statement $\alpha - \phi\colon A$ is a derivation (of itself, from itself); then derivations are extended as follows:

(26)
$$\dfrac{\begin{matrix}\vdots & \vdots\\ \gamma - \chi\colon B/A & \alpha - \phi\colon A\end{matrix}}{(\gamma + \alpha) - (\chi\,\phi)\colon B}/\mathrm{E} \qquad \dfrac{\begin{matrix}\vdots & \vdots\\ \alpha - \phi\colon A & \gamma - \chi\colon A\backslash B\end{matrix}}{(\alpha + \gamma) - (\chi\,\phi)\colon B}\backslash\mathrm{E}$$

(27)
$$\dfrac{\begin{matrix}\overline{a - x\colon A}^{\,n}\\ \vdots\\ (\gamma + a) - \psi\colon B\end{matrix}}{\gamma - \lambda x\psi\colon B/A}/\mathrm{I}^n \qquad \dfrac{\begin{matrix}\overline{a - x\colon A}^{\,n}\\ \vdots\\ (a + \gamma) - \psi\colon B\end{matrix}}{\gamma - \lambda x\psi\colon A\backslash B}\backslash\mathrm{I}^n$$

The elimination rules combine two subderivations. The introduction rules bring about discharge of any single occurrence of the indicated assumption, provided its prosodic label is the left (\I) or right (/I) operand of + as indicated. Note that the ordering across the page (as opposed to the ordering in the labels) is not significant. Word order is represented in the labels; arrangement on the page as shown happens to correspond to word order for the basic calculi, but when we introduce discontinuity, there will be commutative interaction principles, so word order cannot always be read off as the yield of derivation structures.

We allow also relabelling under label equations:

(28)
$$\dfrac{\alpha - \phi\colon A}{\alpha' - \phi'\colon A}=, \text{ if } \alpha = \alpha' \;\&\; \phi = \phi'$$

Subject type lifting of a referring nominal is derivable thus in **NL**:

[5] The labelled systems can be correctly read as type assignment systems with basis the

(29)
$$\dfrac{\dfrac{a - x\colon N \quad \overline{b - y\colon \mathrm{N}\backslash \mathrm{S}}^{\,1}}{(a + b) - (y\,x)\colon S}\mathrm{E}\backslash}{a - \lambda y(y\,x)\colon \mathrm{S}/(\mathrm{N}\backslash\mathrm{S})}/\mathrm{I}^1$$

There are two ways of construing labelling for the associative as opposed to unassociative Lambek calculus, corresponding to the two sequent formulations. One is to add a label equation of associativity under which any prosodic (sub)term can be rewritten. The other is to represent associativity implicitly by replacing the binary prosodic term constructor + by its $n + 2$-ary associative generalisation: $\cdot + \ldots + \cdot$. The former leads to derivations which are made long-winded by explicit assocativity steps, so we shall adopt the latter. Then, for example, the following instance of composition is obtained:

(30)
$$\dfrac{\dfrac{d - w\colon \mathrm{VP/PP} \quad \dfrac{e - u\colon \mathrm{PP/N} \quad \overline{c - z\colon \mathrm{N}}^{\,1}}{e + c - (u\,z)\colon \mathrm{PP}}/\mathrm{E}}{d + e + c - (w\,(u\,z))\colon \mathrm{VP}}/\mathrm{E}}{d + e - \lambda z(w\,(u\,z))\colon \mathrm{VP/N}}/\mathrm{I}^1$$

3.2.1. *Fitch-style Natural Deduction*

Fitch-style natural deduction (Fitch 1952) represents reasoning serially, but with block structuring to indicate hypothetical reasoning. For labelled Fitch-style categorial derivation, lexical assignment, subderivation hypothesis, and term label equation rules are as follows:

(31) $n.\ \alpha - \phi\colon A$ for any lexical entry

$n.\ |\underline{a - x\colon A}$ H

$n.\ \dfrac{\alpha - \phi\colon A}{\alpha' - \phi'\colon A} = \mathrm{n}, \text{ if } \alpha = \alpha' \;\&\; \phi = \phi'$

As usual there are two rules for each operator: a rule of elimination (corresponding to the Gentzen left rule) showing how to use a formula with that operator as principal connective, and a rule of introduction (corresponding to the Gentzen right rule) showing how to prove a formula with that operator as principal connective.

(32)
$$\begin{array}{ll} n. & \alpha - \phi \colon A \\ m. & \gamma - \chi \colon A\backslash B \\ \hline & (\alpha + \gamma) - (\chi\, \phi)\colon B \quad \mathrm{E}\backslash\ n, m \end{array}$$

$$\begin{array}{ll|ll} n. & & a - x\colon A & \mathrm{H} \\ \cline{3-3} m. & & (a + \gamma) - \phi\colon B & \text{unique } a \text{ as indicated} \\ & \multicolumn{3}{l}{\gamma - \lambda x \phi\colon A\backslash B \quad \mathrm{I}\backslash\ n, m} \end{array}$$

(33)
$$\begin{array}{ll} n. & \alpha - \phi \colon A \\ m. & \gamma - \chi \colon B/A \\ \hline & (\gamma + \alpha) - (\chi\phi)\colon B \quad \mathrm{E}/\ n, m \end{array}$$

$$\begin{array}{ll|ll} n. & & a - x\colon A & \mathrm{H} \\ \cline{3-3} m. & & (\gamma + a) - \phi\colon\ B & \text{unique } a \text{ as indicated} \\ & \multicolumn{3}{l}{\gamma - \lambda x \phi\colon B/A \quad \mathrm{I}/\ n, m} \end{array}$$

The previous subject type lifting theorem is now derived:

(34)
$$\begin{array}{ll|ll} 1. & \multicolumn{2}{l}{a - x\colon \mathrm{N}} & \\ 2. & & b - y\colon \mathrm{N}\backslash\mathrm{S} & \mathrm{H} \\ \cline{3-3} 3. & & (a + b) - (y\, x)\colon \mathrm{S} & \mathrm{E}\backslash\ 1, 2 \\ 4. & \multicolumn{2}{l}{a - \lambda y(y\, x)\colon \mathrm{S}/(\mathrm{N}\backslash\mathrm{S})} & \mathrm{I}/\ 2, 3 \end{array}$$

Again, the associative Lambek calculus **L** may be obtained in two ways, either explicitly with a prosodic label equation of associativity, or implicitly with an $n+1$-ary associative constructor. The latter delivers composition as shown in (35):

(35)
$$\begin{array}{ll|ll} 1. & \multicolumn{2}{l}{d - w\colon \mathrm{VP/PP}} & \\ 2. & \multicolumn{2}{l}{e - u\colon \mathrm{PP/N}} & \\ 3. & & \mathrm{c - z\colon N} & \mathrm{H} \\ \cline{3-3} 4. & & e + c - (u\, z)\colon \mathrm{PP} & \mathrm{E}/\ 2, 3 \\ 5. & & d + e + c - (w\, (u\, z))\colon \mathrm{VP} & \mathrm{E}/\ 1, 4 \\ 6. & \multicolumn{2}{l}{d + e - \lambda z(w\, (u\, z))\colon \mathrm{VP/N}} & \mathrm{I}/\ 3, 5 \end{array}$$

We now have all we need in order to give linguistic illustration.

4. Linguistic Application: Left Extraction

The relativisation example (36a) exhibits the same kind of (long distance) dependency as other left extraction constructions such as interrogativisation (36b).

(36)a. which John talked about

$$\begin{array}{rl|ll}
1. & \multicolumn{2}{l}{\mathit{which} - \lambda x\lambda y\lambda z[(y\ z) \wedge (x\ z)]\colon (\mathrm{CN\backslash CN})/(\mathrm{S/N})} & \\
2. & \multicolumn{2}{l}{\mathit{John} - \mathbf{j}\colon \mathrm{N}} & \\
3. & \multicolumn{2}{l}{\mathit{talked} - \mathbf{talk}\colon (\mathrm{N\backslash S})/\mathrm{PP}} & \\
4. & \multicolumn{2}{l}{\mathit{about} - \mathbf{about}\colon \mathrm{PP/N}} & \\
5. & & a - x\colon \mathrm{N} & \mathrm{H} \\ \cline{3-3}
6. & & \mathit{about}{+}a - (\mathbf{about}\ x)\colon \mathrm{PP} & 4, 5\ \mathrm{E}/ \\
7. & & \mathit{talked}{+}\mathit{about}{+}a - (\mathbf{talk}\ (\mathbf{about}\ x))\colon \mathrm{N\backslash S} & 3, 6\ \mathrm{E}/ \\
8. & & \mathit{John}{+}\mathit{talked}{+}\mathit{about}{+}a - ((\mathbf{talk}\ (\mathbf{about}\ x))\ \mathbf{j})\colon \mathrm{S} & 2, 7\ \mathrm{E}\backslash \\
9. & \multicolumn{2}{l}{\mathit{John}{+}\mathit{talked}{+}\mathit{about} - \lambda x((\mathbf{talk}\ (\mathbf{about}\ x))\ \mathbf{j})\colon \mathrm{S/N}} & 5, 8\ \mathrm{I}/ \\
10. & \multicolumn{2}{l}{\mathit{which}{+}\mathit{John}{+}\mathit{talked}{+}\mathit{about}\ -} & \\
 & \multicolumn{2}{l}{(\lambda x\lambda y\lambda z[(y\ z) \wedge (x\ z)]\ \lambda x((\mathbf{talk}\ (\mathbf{about}\ x))\ \mathbf{j}))\colon \mathrm{CN\backslash CN}} & 1, 9\ \mathrm{E}/ \\
11. & \multicolumn{2}{l}{\mathit{which}{+}\mathit{John}{+}\mathit{talked}{+}\mathit{about}\ -} & \\
 & \multicolumn{2}{l}{\lambda y\lambda z[(y\ z) \wedge ((\mathbf{talk}\ (\mathbf{about}\ z))\ \mathbf{j})]\colon \mathrm{CN\backslash CN}} & =10
\end{array}$$

Fig. 1. Derivation of 'which John talked about'.

The relativisation can be derived as shown in Figure 1 in Fitch-style natural deduction **L** by assignment of the wh-element to a higher order type (CN\CN)/(S/N) (Steedman 1985). The derivation in labelled Prawitz-style is shown in (37).

(37)a.
$$\dfrac{\dfrac{\text{John}}{\mathit{John} - \mathbf{j}\colon \mathrm{N}} \qquad \dfrac{\dfrac{\text{talked}}{\mathit{talked} - \mathbf{talk}\colon (\mathrm{N\backslash S})/\mathrm{PP}} \qquad \dfrac{\dfrac{\text{about}}{\mathit{about} - \mathbf{about}\colon \mathrm{PP/N}} \qquad \dfrac{}{a - x\colon \mathrm{N}}{}^{1}}{\mathit{about}{+}a - (\mathbf{about}\ x)\colon \mathrm{PP}}/\mathrm{E}}{\mathit{talked}{+}\mathit{about}{+}a - (\mathbf{talk}\ (\mathbf{about}\ x))\colon \mathrm{N\backslash S}}/\mathrm{E}}{\dfrac{\mathit{John}{+}\mathit{talked}{+}\mathit{about}{+}a - ((\mathbf{talk}\ (\mathbf{about}\ x))\ \mathbf{j})\colon \mathrm{S}}{\mathit{John}{+}\mathit{talked}{+}\mathit{about} - \lambda x((\mathbf{talk}\ (\mathbf{about}\ x))\ \mathbf{j})\colon \mathrm{S/N}}/\mathrm{I}^{1}}\backslash\mathrm{E}$$

b.
$$\dfrac{\dfrac{\dfrac{\text{which}}{\mathit{which} - \lambda x\lambda y\lambda z[(y\ z) \wedge (x\ z)]\colon (\mathrm{CN\backslash CN})/(\mathrm{S/N})} \qquad \dfrac{\text{John talked about}}{\mathit{John}{+}\mathit{talked}{+}\mathit{about} - \lambda x((\mathbf{talk}\ (\mathbf{about}\ x))\ \mathbf{j})\colon \mathrm{S/N}}}{\mathit{which}{+}\mathit{John}{+}\mathit{talked}{+}\mathit{about} - (\lambda x\lambda y\lambda z[(y\ z) \wedge (x\ z)]\ \lambda x((\mathbf{talk}\ (\mathbf{about}\ x))\ \mathbf{j}))\colon \mathrm{CN\backslash CN}}}{\mathit{which}{+}\mathit{John}{+}\mathit{talked}{+}\mathit{about} - \lambda y\lambda z[(y\ z) \wedge ((\mathbf{talk}\ (\mathbf{about}\ z))\ \mathbf{j})]\colon \mathrm{CN\backslash CN}}=$$

Without going into details (but see e.g. Carpenter 1992), the wh-question can be obtained by assignment of the interrogative pronoun to I/(M/N) where M represents subject-auxiliary inverted sentences, and I interrogatives; i.e. in each case the fronting is triggered by a higher order assignment to the wh-element.

Elegant as such categorial grammar may be, it is more suggestive of an approach to computational linguistic grammar formalism than actually ple is limited to peripheral, as opposed to medial, extractions since S/N indicates sentences lacking nominals at the right frontier only. For resolution of such matters by means of structural operators, e.g. $\triangle$ for permutation, whence S/N in (CN\CN)/(S/N) may become S/$\triangle$N, see Morrill, Leslie, Hepple and Barry (1990) and Barry, Hepple, Leslie and Morrill (1991). Various other enrichments are proposed in e.g. Moortgat (1988, 1990, 1991), van Benthem (1990), Morrill (1990a, 1990b, 1992a, 1992b) and Moortgat and Morrill (1991). In particular, Moortgat (1988) advanced earlier discussion of discontinuity in e.g. Bach (1981, 1984) with a proposal for infixing and wrapping operators. The operators not only provide scope over these particular phenomena but also, as indicated in e.g. Moortgat (1991), seem to provide an underlying basis in terms of which operators for binding phenomena such as quantification and reflexivisation should be definable. The coverage of pied piping in Morrill (1992b) would also be definable in terms of these primitives, but all this depends on the resolution of certain technical issues which have been to date outstanding. Our treatment is based on development of categorial operators for more than one mode of prosodic adjunction. We first give an overall picture of such multimodality, and then formulate a particular discontinuity calculus within the space this provides.

5. Multimodal Categorial Logic

The essential idea of multimodal categorial logic is introduced in Moortgat and Morrill (1991). There it is shown how to give calculi for families of connectives each with their own structural properties, and perhaps with structural interactions. For example, we may juxtapose **L** and **NL** (this case was anticipated by Oehrle and Zhang 1989 and Morrill 1990b). More generally, we can define formulas $\mathcal{F}$ for n families of connectives thus:

(38) $$\mathcal{F} = \mathcal{A} | \mathcal{F}\backslash_1\mathcal{F} | \mathcal{F}/_1\mathcal{F} | \mathcal{F}\backslash_2\mathcal{F} | \mathcal{F}/_2\mathcal{F} | \ldots | \mathcal{F}\backslash_n\mathcal{F} | \mathcal{F}/_n\mathcal{F}$$

Prosodic interpretation is by residuation of each division with respect to its associated adjunction in an algebra $\langle L, \{+_i\}_{i\in\{1,\ldots,n\}}\rangle$, i.e. we use the residuation scheme exactly as before, as many times over as there are families of connectives:

(39) $$D(A\backslash_i B) = \{s | \forall s' \in D(A), s' +_i s \in D(B)\}$$
$$D(B/_i A) = \{s | \forall s' \in D(A), s +_i s' \in D(B)\}$$

As a consequence the residuation laws hold for each family:

For each mode the semantic interpretation takes place with respect to function formation in the same way as for the unimodal case.

Gentzen calculus is obtained by juxtaposing the usual rules for each family of connectives. The sequent punctuation needs to be conditioned in such a way as to indicate families. This is achieved by labelling the brackets in configurations with family indices:

a. $$\frac{\Gamma \Rightarrow A \quad \Delta(B) \Rightarrow C}{\Delta([_i\Gamma, A\backslash_i B]) \Rightarrow C}\backslash_i L \qquad \frac{[_iA, \Gamma] \Rightarrow B}{\Gamma \Rightarrow A\backslash_i B}\backslash_i R$$

b. $$\frac{\Gamma \Rightarrow A \quad \Delta(B) \Rightarrow C}{\Delta([_iB/_iA, \Gamma]) \Rightarrow C}/_i\mathrm{L} \qquad \frac{[_i\Gamma, A] \Rightarrow B}{\Gamma \Rightarrow B/_iA}/_i\mathrm{R}$$

Interactions between adjunctions will be represented by structural rules; we shall see examples when we come on to discontinuity.

The Prawitz-style and Fitch-style labelled deduction rules are as before, but with each connective correlated with its adjunction constructor in the labels:

(41) $$\frac{\begin{matrix}\vdots \\ \gamma - \chi\colon B/_iA\end{matrix} \quad \begin{matrix}\vdots \\ \alpha - \phi\colon A\end{matrix}}{(\gamma +_i \alpha) - (\chi\,\phi)\colon B}/_i\mathrm{E} \qquad \frac{\begin{matrix}\vdots \\ \alpha - \phi\colon A\end{matrix} \quad \begin{matrix}\vdots \\ \gamma - \chi\colon A\backslash_i B\end{matrix}}{(\alpha +_i \gamma) - (\chi\,\phi)\colon B}\backslash_i\mathrm{E}$$

(42) $$\frac{\begin{matrix}\overline{a - x\colon A}^{\,n} \\ \vdots \\ (\gamma +_i a) - \psi\colon B\end{matrix}}{\gamma - \lambda x\psi\colon B/_iA}/_i\mathrm{I}^n \qquad \frac{\begin{matrix}\overline{a - x\colon A}^{\,n} \\ \vdots \\ (a +_i \gamma) - \psi\colon B\end{matrix}}{\gamma - \lambda x\psi\colon A\backslash_i B}\backslash_i\mathrm{I}^n$$

(43)
$n.$	$\alpha - \phi\colon A$	
$m.$	$\gamma - \chi\colon A\backslash_i B$	
	$(\alpha +_i \gamma) - (\chi\phi)\colon B$	E$\backslash_i$ n, m
$n.$	$\vert\ a - x\colon A$	H
$m.$	$\vert\ (a +_i \gamma) - \phi\colon B$	unique a as indicated
	$\gamma - \lambda x\phi\colon A\backslash_i B$	I$\backslash_i$ n, m

(44)
$n.$	$\alpha - \phi\colon A$	
$m.$	$\gamma - \chi\colon B/_iA$	
	$(\gamma +_i \alpha) - (\chi\,\phi)\colon B$	E$/_i$ n, m
$n.$	$\vert\ a - x\colon A$	H
$m.$	$\vert\ (\gamma +_i a) - \phi\colon B$	unique a as indicated.
	$\gamma - \lambda x\phi\colon B/_iA$	I$/_i$ n, m

Structural properties and interactions between adjunctions are represented by rules rewriting labels. We set all this out explicitly in order to facilitate observation that the labelled proof theory for discontinuity will have precisely this pattern of implementation in Gentzen calculus and labelled natural deduction.

6. Discontinuity

Binary operators $\uparrow$ and $\downarrow$ are proposed in Moortgat (1988) such that $B \uparrow A$ signifies functors that wrap around their A arguments to form Bs, and (in our notation) $A \downarrow B$ signifies functors that infix themselves in their A arguments to form Bs. Assuming the semigroup algebra of associative Lambek calculus, there are two possibilities in each case, depending on whether we are free to insert anywhere (universal), or whether the relevant insertion points are fixed (existential). We leave the semantic dimension aside for the moment.

(45) Existential

$$D(B \uparrow_{\exists} A) = \{s \mid \exists s_1, s_2[s = s_1 + s_2 \wedge \forall s' \in D(A)\ s_1 + s' + s_2 \in D(B)]\}$$

Universal

$$D(B \uparrow_{\forall} A) = \{s \mid \forall s_1, s_2[s = s_1 + s_2 \rightarrow \forall s' \in D(A),\ s_1 + s' + s_2 \in D(B)]\}$$

(46) Existential

$$D(A \downarrow_{\exists} B) = \{s \mid \forall s' \in D(A), \exists s_1, s_2\ [s' = s_1 + s_2 \wedge s_1 + s + s_2 \in D(B)]\}$$

Universal

$$D(A \downarrow_{\forall} B) = \{s \mid \forall s' \in D(A), \forall s_1, s_2\ [s' = s_1 + s_2 \rightarrow s_1 + s + s_2 \in D(B)]\}$$

Inspecting the possibilities of sequent presentation, of the eight possible rules of inference (use and proof for each of four operators), only $\uparrow_{\exists}$R and $\downarrow_{\forall}$L are expressible in Gentzen sequent calculus:

(47) a.
$$\frac{\Gamma_1, A, \Gamma_2 \Rightarrow B}{\Gamma_1, \Gamma_2 \Rightarrow B \uparrow_{\exists} A}\ \uparrow_{\exists}\mathrm{R}$$

b.
$$\frac{\Gamma_1, \Gamma_2 \Rightarrow A \quad \Delta_1, B, \Delta_2 \Rightarrow C}{\Delta_1, \Gamma_1, A \downarrow_{\forall} B, \Gamma_2, \Delta_2 \Rightarrow C}\ \downarrow_{\forall}\mathrm{L}$$

This is the partial logic of Moortgat (1988). Note that the absence of a rule of use for existential wrapping means that we could not generate from discontinuous elements such as 'ring up' and 'give the cold shoulder'

rules could be formulated it is evident that $\uparrow_{\forall}$ would permit incorrect word order such as *'Mary gave the John cold shoulder'.) The problem with Gentzen sequent calculus appears to be that the implicit encoding of prosodic operations in ordered sequents is of limited expressivity. Accordingly, Moortgat (1992) seeks to improve the situation by means of explicit prosodic labelling. This does enable both rules for $\downarrow_{\forall}$ but still does not enable the useful $\uparrow_{\exists}$L: the remaining problem is, as noted by Versmissen (1991), that we need to have an insertion point somehow determinate from the prosodic label for an existential wrapper in order to perform a left reference.

In addition to these difficulties with proof theory for Moortgat's models there are also tantalising near misses as regards the suitability of the model theory for linguistic applications. In Moortgat (1991) it is observed how the term insertion of infix binders such as quantifier phrases seems *almost* definable as (S $\uparrow$ N) $\downarrow$ S: they infix themselves at N positions in Ss (and take semantic scope at the S level – that is why they must be quantified in). None of the interpretations above however enable expression of the requirement that the positions referred to by the two operator occurrences *are the same*. The universal and existential quantifications over positions for interpolation do not express that we want S $\uparrow$ N to signify the existence of exactly one such position, and (S $\uparrow$ N) $\downarrow$ S to indicate infixation at just this one position; the notion of unicity is lacking.

There are these problems then implicit in the lack of determinacy in Moortgat's model theory for the discontinuity operators; and over and above this there are problems with the proof theory. The model theoretic shortfall in linguistic suitability will be remedied by our revised formulation, which will furthermore admit of a full logic. The formulations in the next subsections will enable us in section 7 to deal with examples including all the following instances of discontinuity.

(48) a.	Mary rang John up.	Particle verb
b.	Mary gave John the cold shoulder.	Discontinuous idiom
c.	John likes everything. Everyone loves something. John believes someone walks.	Quantifier raising
d.	for whom John works; the loss of which after so much wrangling John would finally have to pay for	Pied piping

e. John votes for himself. Subject antecedent reflexivisation

f. John showed Mary herself. Object antecedent reflexivisation

6.1. *Model Theory for Discontinuity*

To formulate discontinuity we have a community comprising three families of divisions: the usual associative operators, 'split-point' non-associative operators, and discontinuity operators. The category formulas are:

(49) $\mathcal{F} ::= \mathcal{A} \mid \mathcal{F}\backslash\mathcal{F} \mid \mathcal{F}/\mathcal{F} \mid \mathcal{F} > \mathcal{F} \mid \mathcal{F} < \mathcal{F} \mid \mathcal{F}\downarrow\mathcal{F} \mid \mathcal{F}\uparrow\mathcal{F}$

The account of discontinuity as we present it now differs from Morrill and Solias (1993), and also the original interpretations proposed by Moortgat (1988), in that it treats wrapping adjunction as a *primitive*, rather than as a *defined*, operation in the prosodic algebra. Corresponding to the three families of multiplicatives there are three adjunctions, and there is an identity element for the associative adjunction +. Thus prosodic interpretation is in an algebra $\langle L^*, +, \epsilon, (.,.), W\rangle$ where $\langle L^*, +, \epsilon\rangle$ is a monoid, i.e. a semigroup $\langle L^*, +\rangle$ with an $\epsilon \in L^*$ such that $\epsilon + s = s + \epsilon = s$, and in addition to the associativity and identity conditions we have:

(50) $s_1 + s_2 + s_3 = (s_1, s_3)Ws_2$

Spelt out in full the interpretation is as follows by residuation with respect to each adjunction:

(51) $D(A\backslash B) = \{s \in L | \forall s' \in D(A), s' + s \in D(B)\}$
$D(B/A) = \{s \in L | \forall s' \in D(A), s + s' \in D(B)\}$

(52) $D(A\backslash B) = \{s \in L | \forall s' \in D(A), (s', s) \in D(B)\}$
$D(B/A) = \{s \in L | \forall s' \in D(A), (s, s') \in D(B)\}$

(53) $D(A\downarrow B) = \{s \in L | \forall s' \in D(A), s'Ws \in D(B)\}$
$D(B\uparrow A) = \{s \in L | \forall s' \in D(A), sWs' \in D(B)\}$

Here L is $L^* - \{\epsilon\}$. We shall need the identity element in order to be able to include instances of peripheral discontinuity under the general case. However the identity element itself must be excluded from the interpretation of types. If the identity element ϵ were allowed to belong to types it would inhabit all endocentric types A/A and $A\backslash A$ since concatenating ϵ with any element in A yields the same element, and hence an inhabitant of A. Consider two unfortunate effects this would have. First, [illegible] intensifier such as 'extremely' (as

in 'the extremely fat man') is (CN/CN)/(CN/CN); then the empty string could satisfy the intensifier's CN/CN valency in *'the extremely man'. Second, let us assume that expressions of like category can coordinate; then associativity allows (across-the-board exception, Ross 1967) coordination in N/N of e.g. 'the man who John met the brother of and the sister of' (cf. e.g. Steedman 1985); but then the empty string could constitute a second N/N conjunct in *'the man who John met the brother of and'.

The current proposal is a refinement of the proposal in Morrill and Solias (1993) which is to interpret discontinuous operators in a prosodic algebra $\langle L^*, +, \epsilon, \langle.,.\rangle, 1, 2\rangle$ where $\langle.,.\rangle$ is a pairing operator (introduced in Solias 1992) and so has associated projection functions 1 and 2. The latter defines an algebra of the present form by setting $(s, s') = \langle s, s'\rangle$ and $sWs' = 1s + s' + 2s$, but not vice-versa, so that the present form has more general models. There are two motives for the reformulation offered here. First, in order that defined wrap be a total operation we need 1 and 2 to be defined even for objects which are not the value of $\langle.,.\rangle$; but then we are not really dealing with pairing and projection. Second, when products are included there is incompleteness of non-associative Lambek calculus for interpretation as ordered trees (see e.g. Venema 1993). Since tree formation is isomorphic to pairing, the same incompleteness would arise in the treatment of discontinuity using tupling. The more general models do not impose the structure that validates the non-**NL**-theorems considered by Venema.

6.2. *Proof Theory for Discontinuity*

In order to deal with the three-family discontinuity system with identity in a Gentzen sequent presentation, we define sequents as of the form $\Gamma \Rightarrow A$ where A is a formula and Γ is a configuration, where configurations $\mathcal{O}$ are as follows.

(54) $\mathcal{O} ::= \epsilon \mid \mathcal{F} \mid \mathcal{O}, \mathcal{O} \mid [_n\mathcal{O}; \mathcal{O}] \mid [_w\mathcal{O}; \mathcal{O}]$

Thus a configuration is a sequence of one or more elements which are either formulas or null, with a partial labelled binary bracketing. Associative adjunction is represented by bracket-free $n + 2$-ary commas; then each bracket labelling is either n (the non-associative splitting mode) or w (the non-associative wrapping or interpolation mode); the non-associative separators (semicolons) need to be distinguished from the associative commas, otherwise $[_nA, B, C]$, for example, would be ambiguous. Corresponding to the split wrap interaction there is the following:

(55) $$\dfrac{\Gamma([_w[_n\Delta_1, \Delta_2], \Delta_3]) \Rightarrow A}{\Gamma(\Delta_1, \Delta_3, \Delta_2) \Rightarrow A}\mathrm{WN}$$

And for the identity properties there is (56).

(56) $$\dfrac{\Gamma(\Delta, \epsilon) \Rightarrow A}{\Gamma(\Delta) \Rightarrow A}\epsilon \qquad \dfrac{\Gamma(\epsilon, \Delta) \Rightarrow A}{\Gamma(\Delta) \Rightarrow A}\epsilon$$

Otherwise there is simply the generic scheme for multimodal calculi (with the usual identity and Cut rules):

(57) a. $$\dfrac{\Gamma \Rightarrow A \quad \Delta(B) \Rightarrow C}{\Delta(\Gamma, A\backslash B) \Rightarrow C}\backslash\mathrm{L} \qquad \dfrac{A, \Gamma \Rightarrow B}{\Gamma \Rightarrow A\backslash B}\backslash\mathrm{R}$$

b. $$\dfrac{\Gamma \Rightarrow A \quad \Delta(B) \Rightarrow C}{\Delta(B/A, \Gamma) \Rightarrow C}/\mathrm{L} \qquad \dfrac{\Gamma, A \Rightarrow B}{\Gamma \Rightarrow B/A}/\mathrm{R}$$

(58) a. $$\dfrac{\Gamma \Rightarrow A \quad \Delta(B) \Rightarrow C}{\Delta([_n\Gamma, A > B]) \Rightarrow C}>\mathrm{L} \qquad \dfrac{[_nA, \Gamma] \Rightarrow B}{\Gamma \Rightarrow A > B}>\mathrm{R}$$

b. $$\dfrac{\Gamma \Rightarrow A \quad \Delta(B) \Rightarrow C}{\Delta([_nB < A, \Gamma]) \Rightarrow C}<\mathrm{L} \qquad \dfrac{[_n\Gamma, A] \Rightarrow B}{\Gamma \Rightarrow B < A}<\mathrm{R}$$

(59) a. $$\dfrac{\Gamma \Rightarrow A \quad \Delta(B) \Rightarrow C}{\Delta([_w\Gamma, A \downarrow B]) \Rightarrow C}\downarrow\mathrm{L} \qquad \dfrac{[_wA, \Gamma] \Rightarrow B}{\Gamma \Rightarrow A \downarrow B}\downarrow\mathrm{R}$$

b. $$\dfrac{\Gamma \Rightarrow A \quad \Delta(B) \Rightarrow C}{\Delta([_wB \uparrow A, \Gamma]) \Rightarrow C}\uparrow\mathrm{L} \qquad \dfrac{[_w\Gamma, A] \Rightarrow B}{\Gamma \Rightarrow B \uparrow A}\uparrow\mathrm{R}$$

The following derivation derives an instance of lifting with respect to the discontinuity operators:

(60) $$\dfrac{\dfrac{\mathrm{A} \Rightarrow \mathrm{A} \quad \mathrm{B} \Rightarrow \mathrm{B}}{[_w\mathrm{A}, \mathrm{A} \downarrow \mathrm{B}] \Rightarrow \mathrm{B}}\downarrow\mathrm{L}}{\mathrm{A} \Rightarrow \mathrm{B} \uparrow (\mathrm{A} \downarrow \mathrm{B})}\uparrow\mathrm{R}$$

The derivation is exactly the same as (19), but for the interpolation adjunction. Thus the interactions giving rise to discontinuity do not destroy the exhibition within each family of type constructors of symmetric laws: $A \Rightarrow B/(A\backslash B)$ and $A \Rightarrow (B/A)\backslash B$ correspond precisely to $A \Rightarrow B \uparrow (A \downarrow B)$ and $A \Rightarrow (B \uparrow A) \downarrow B$. The notation is such that in each case the B context flanks the A. Observe that this pattern would be lost if we had maintained Moortgat's original notation, which has arguments to the right for both

same way that / and \ are defined with respect to + we use the same notational conventions for both.

By way of an example illustrating most aspects of the sequent calculus consider the following (partial) derivation. We will ultimately see its linguistic significance.

(61)
$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\mathrm{PP/N, N} \Rightarrow \mathrm{PP}}{\mathrm{PP/N, N}, \epsilon \Rightarrow \mathrm{PP}}\epsilon}{[_w[_n\mathrm{PP/N}; \epsilon]; \mathrm{N}] \Rightarrow \mathrm{PP}}\mathrm{WN}}{[_n\mathrm{PP/N}; \epsilon] \Rightarrow \mathrm{PP{\uparrow}N}}{\uparrow}\mathrm{R} \quad \dfrac{\dfrac{\mathrm{(VP/PP)/PP, PP/N} \Rightarrow \mathrm{(VP/PP)/N} \quad \dfrac{\mathrm{N} \Rightarrow \mathrm{N} \quad \mathrm{VP} \Rightarrow \mathrm{VP}}{[_w\mathrm{VP{\uparrow}N; N}] \Rightarrow \mathrm{VP}}\mathrm{E}{\uparrow}}{[_w[_n\mathrm{(VP/PP)/PP, PP/N}; \mathrm{((VP/PP)/N){>}(VP{\uparrow}N)}]; \mathrm{N}] \Rightarrow \mathrm{VP}}\mathrm{E}{>}}{\mathrm{(VP/PP)/PP, PP/N, N, ((VP/PP)/N){>}(VP{\uparrow}N)} \Rightarrow \mathrm{VP}}\mathrm{WN}}{\mathrm{(VP/PP)/PP, PP/N, N}, [_w[_n\mathrm{PP/N}; \epsilon]; \mathrm{(PP{\uparrow}N){\downarrow}(((VP/PP)/N){>}(VP{\uparrow}N))}] \Rightarrow \mathrm{VP}}{\downarrow}\mathrm{L}}{\mathrm{(VP/PP)/PP, PP/N, N, PP/N, (PP{\uparrow}N){\downarrow}(((VP/PP)/N){>}(VP{\uparrow}N))}, \epsilon \Rightarrow \mathrm{VP}}\mathrm{WN}}{\mathrm{(VP/PP)/PP, PP/N. N, PP/N, (PP{\uparrow}N){\downarrow}(((VP/PP)/N){>}(VP{\uparrow}N))} \Rightarrow \mathrm{VP}}\epsilon$$

For the labelled natural deduction presentations also, since + is the only associative constructor we will represent this by omitting its parentheses in labels. Consider first Prawitz-style. There are the following "structural" rules semantics and types, excluded, are the same in premise and conclusion:

(62) $$\dfrac{\begin{array}{c}\vdots\\ \delta(((\alpha, \beta)W\gamma))\end{array}}{\delta(\alpha + \gamma + \beta)}\mathrm{WN} \qquad \dfrac{\begin{array}{c}\vdots\\ \delta(\alpha + \gamma + \beta)\end{array}}{\delta(((\alpha, \beta)W\gamma))}\mathrm{WN}$$

(63) $$\dfrac{\begin{array}{c}\vdots\\ B(\alpha)\end{array}}{\beta(\alpha + \epsilon)}\epsilon \qquad \dfrac{\begin{array}{c}\vdots\\ \beta(\alpha)\end{array}}{\beta(\epsilon + \alpha)}\epsilon \qquad \dfrac{\begin{array}{c}\vdots\\ \beta(\alpha + \epsilon)\end{array}}{\beta(\alpha)}\epsilon \qquad \dfrac{\begin{array}{c}\vdots\\ \beta(\epsilon + \alpha)\end{array}}{\beta(\alpha)}\epsilon$$

Then there are these logical rules:

(64) $$\dfrac{\begin{array}{c}\vdots\\ \gamma - \chi\colon B/A\end{array} \quad \begin{array}{c}\vdots\\ \alpha - \phi\colon A\end{array}}{\gamma + \alpha - (\chi\,\phi)\colon B}/\mathrm{E} \qquad \dfrac{\begin{array}{c}\vdots\\ \alpha - \phi\colon A\end{array} \quad \begin{array}{c}\vdots\\ \gamma - \chi\colon A\backslash B\end{array}}{\alpha + \gamma - (\chi\,\phi)\colon B}\backslash\mathrm{E}$$

(65) $$\dfrac{\begin{array}{c}\overline{a - x\colon A}^{\,n}\\ \vdots\\ \gamma + a - \psi\colon B\end{array}}{\gamma - \lambda x\psi\colon B/A}/\mathrm{I}^n \qquad \dfrac{\begin{array}{c}\overline{a - x\colon A}^{\,n}\\ \vdots\\ a + \gamma - \psi\colon B\end{array}}{\gamma - \lambda x\psi\colon A\backslash B}\backslash\mathrm{I}^n$$

(66)
$$\frac{\begin{array}{cc}\vdots & \vdots \\ \gamma - \chi: B < A & \alpha - \phi: A\end{array}}{(\gamma, \alpha) - (\chi\,\phi): B}{<}\mathrm{E} \qquad \frac{\begin{array}{cc}\vdots & \vdots \\ \alpha - \phi: A & \gamma - \chi: A > B\end{array}}{(\alpha, \gamma) - (\chi\,\phi): B}{>}\mathrm{E}$$

(67)
$$\frac{\begin{array}{c}\overline{a - x: A}^{n} \\ \vdots \\ (\gamma, a) - \psi: B\end{array}}{\gamma - \lambda x \psi: B < A}{<}\mathrm{I}^{n} \qquad \frac{\begin{array}{c}\overline{a - x: A}^{n} \\ \vdots \\ (a, \gamma) - \psi: B\end{array}}{\gamma - \lambda x \psi: A > B}{>}\mathrm{I}^{n}$$

(68)
$$\frac{\begin{array}{cc}\vdots & \vdots \\ \gamma - \chi: B \uparrow A & \alpha - \phi: A\end{array}}{(\gamma W \alpha) - (\chi\,\phi): B}\uparrow\mathrm{E} \qquad \frac{\begin{array}{cc}\vdots & \vdots \\ \alpha - \phi: A & \gamma - \chi: A \downarrow B\end{array}}{(\alpha W \gamma) - (\chi\,\phi): B}\downarrow\mathrm{E}$$

(69)
$$\frac{\begin{array}{c}\overline{a - x: A}^{n} \\ \vdots \\ (\gamma W a) - \psi: B\end{array}}{\gamma - \lambda x \psi: B \uparrow A}\uparrow\mathrm{I}^{n} \qquad \frac{\begin{array}{c}\overline{a - x: A}^{n} \\ \vdots \\ (a W \gamma) - \psi: B\end{array}}{\gamma - \lambda x \psi: A \downarrow B}\downarrow\mathrm{I}^{n}$$

Consider next Fitch-style. There are the following term label equations:

(70)
$$\begin{array}{l}\alpha + \beta + \gamma = ((\alpha, \gamma) W \beta) \\ \alpha + \epsilon = \epsilon + \alpha = \alpha\end{array}$$

The lexical assignment, subderivation hypotheses, and term rewriting rules are as before. The logical rules are as follows.

(71)
$$\begin{array}{lll} n. & \alpha - \phi: A & \\ m. & \underline{\gamma - \chi: A \backslash B} & \\ & \alpha + \gamma - (\chi\,\phi): B & \mathrm{E}\backslash\ n, m \end{array}$$

$$\begin{array}{lll} n. & \vert\,\underline{a - x: A} & \mathrm{H} \\ m. & \vert\,a + \gamma - \phi: B & \text{unique } a \text{ as indicated} \\ & \gamma - \lambda x \phi: A \backslash B & \mathrm{I}\backslash\ n, m \end{array}$$

(72)
$$\begin{array}{lll} n. & \alpha - \phi: A & \\ m. & \underline{\gamma - \chi: B/A} & \\ & \gamma + \alpha - (\chi\,\phi): B & \mathrm{E}/\ n, m \end{array}$$

$$\begin{array}{lll} n. & \vert\,\underline{a - x: A} & \mathrm{H} \\ m. & \vert\,\gamma + a - \phi: B & \text{unique } a \text{ as indicated} \end{array}$$

(73)
$$\begin{array}{lll} n. & \alpha - \phi: A & \\ m. & \underline{\gamma - \chi: A > B} & \\ & (\alpha, \gamma) - (\chi\phi): B & \mathrm{E}{>}\ n, m \end{array}$$

$$\begin{array}{lll} n. & \vert\,\underline{a - x: A} & \mathrm{H} \\ m. & \vert\,(a, \gamma) - \phi: B & \text{unique } a \text{ as indicated} \\ & \gamma - \lambda x \phi: A > B & \mathrm{I}{>}\ n, m \end{array}$$

(74)
$$\begin{array}{lll} n. & \alpha - \phi: A & \\ m. & \underline{\gamma - \chi: B < A} & \\ & (\gamma, \alpha) - (\chi\phi): B & \mathrm{E}{<}\ n, m \end{array}$$

$$\begin{array}{lll} n. & \vert\,\underline{a - x: A} & \mathrm{H} \\ m. & \vert\,(\gamma, a) - \phi: B & \text{unique } a \text{ as indicated} \\ & \gamma - \lambda x \phi: B < A & \mathrm{I}{<}\ n, m \end{array}$$

(75)
$$\begin{array}{lll} n. & \alpha - \phi: A & \\ m. & \underline{\gamma - \chi: A \downarrow B} & \\ & (\alpha W \gamma) - (\chi\,\phi): B & \mathrm{E}\downarrow\ n, m \end{array}$$

$$\begin{array}{lll} n. & \vert\,\underline{a - x: A} & \mathrm{H} \\ m. & \vert\,(a W \gamma) - \phi: B & \text{unique } a \text{ as indicated} \\ & \gamma - \lambda x \phi: A \downarrow B & \mathrm{I}\downarrow\ n, m \end{array}$$

(76)
$$\begin{array}{lll} n. & \alpha - \phi: A & \\ m. & \underline{\gamma - \chi: B \uparrow A} & \\ & (\gamma W \alpha) - (\chi\,\phi): B & \mathrm{E}\uparrow\ n, m \end{array}$$

$$\begin{array}{lll} n. & \vert\,\underline{a - x: A} & \mathrm{H} \\ m. & \vert\,(\gamma W a) - \phi: B & \text{unique } a \text{ as indicated} \\ & \gamma - \lambda x \phi: B \uparrow A & \mathrm{I}\uparrow\ n, m \end{array}$$

Since the linguistic examples in the next section are derived using these formats we move straight on without giving abstract illustration of deductions.

7. Linguistic Application

7.1. *Discontinuous Functors*

In order to introduce the general method we begin with the three examples of discontinuity we cited at the beginning of the paper.

The particle verb example 'Mary rang John up' is derived as follows. The particle verb has a complex lexical form constructed out of the splitting adjunction, and its lexical type is that of a wrapping functor. After combination with the object at line 3, prosodic evaluation at line 4 gives the discontinuous word order.

(77)

1. $(\textit{rang}, \textit{up}) - \mathbf{phone}$: $(\mathrm{N}\backslash\mathrm{S})\uparrow\mathrm{N}$
2. $\textit{John} - \mathbf{j}$: N
3. $\textit{Mary} - \mathbf{m}$: N
4. $((\textit{rang}, \textit{up})W\,\textit{John}) - (\mathbf{phone}\ \mathbf{j})$: $\mathrm{N}\backslash\mathrm{S}$ 1, 2 E$\uparrow$
5. $\textit{rang} + \textit{John} + \textit{up} - (\mathbf{phone}\ \mathbf{j})$: $\mathrm{N}\backslash\mathrm{S}$ = 4
6. $\textit{Mary} + \textit{rang} + \textit{John} + \textit{up} - ((\mathbf{phone}\ \mathbf{j})\ \mathbf{m})$: S 3, 5 E$\backslash$

In Prawitz-style the derivation becomes the following:

(78)

$$\dfrac{\textit{Mary} - \mathbf{m}: \mathrm{N} \qquad \dfrac{\dfrac{(\textit{rang}, \textit{up}) - \mathbf{phone}: (\mathrm{N}\backslash\mathrm{S})\uparrow\mathrm{N} \qquad \textit{John} - \mathbf{j}: \mathrm{N}}{((\textit{rang}, \textit{up})W\,\textit{John}) - (\mathbf{phone}\ \mathbf{j}): \mathrm{N}\backslash\mathrm{S}}\mathrm{E}\uparrow}{\textit{rang}+\textit{John}+\textit{up} - (\mathbf{phone}\ \mathbf{j}): \mathrm{N}\backslash\mathrm{S}}=}{\textit{Mary}+\textit{rang}+\textit{John}+\textit{up} - ((\mathbf{phone}\ \mathbf{j})\ \mathbf{m}): \mathrm{S}}\mathrm{E}\backslash$$

A discontinuous idiom construction such as 'Mary gave John the cold shoulder' is treated in exactly the same way:

(79)

1. $(\textit{gave}, \textit{the} + \textit{cold} + \textit{shoulder}) - \mathbf{give\text{-}tcs}$: $(\mathrm{N}\backslash\mathrm{S})\uparrow\mathrm{N}$
2. $\textit{John} - \mathbf{j}$: N
3. $\textit{Mary} - \mathbf{m}$: N
4. $((\textit{gave}, \textit{the} + \textit{cold} + \textit{shoulder})W\,\textit{John}) -$ $(\mathbf{give\text{-}tcs}\ \mathbf{j})$: $\mathrm{N}\backslash\mathrm{S}$ 1, 2, E$\uparrow$
5. $\textit{gave} + \textit{John} + \textit{the} + \textit{cold} + \textit{shoulder} -$ $(\mathbf{give\text{-}tcs}\ \mathbf{j})$: $\mathrm{N}\backslash\mathrm{S}$ = 4
6. $\textit{Mary} + \textit{gave} + \textit{John} + \textit{the} + \textit{cold} + \textit{shoulder} -$ $((\mathbf{gave\text{-}tcs}\ \mathbf{j})\ \mathbf{m})$: S 3, 5 E$\backslash$

An example such as 'Mary neither walks nor talks' is obtained as indicated by the Prawitz-style derivation (80).

(80)

$$\dfrac{\dfrac{\dfrac{(\textit{neither}, \textit{nor}) - \lambda x\lambda y\lambda z\neg[(x\ z) \vee (y\ z)]: (\mathrm{VP}/\mathrm{VP})\uparrow\mathrm{VP} \qquad \textit{walks} - \mathbf{walk}: \mathrm{VP}}{(\textit{neither}, \textit{nor})W\,\textit{walks} - (\lambda x\lambda y\lambda z\neg[(x\ z) \vee (y\ z)]\ \mathbf{walk}): \mathrm{VP}/\mathrm{VP}}\mathrm{E}\uparrow}{\textit{neither}+\textit{walks}+\textit{nor} - \lambda y\lambda z\neg[(\mathbf{walk}\ z) \vee (y\ z)]: \mathrm{VP}/\mathrm{VP}}= \qquad \textit{talks} - \mathbf{talk}: \mathrm{VP}}{\textit{neither}+\textit{walks}+\textit{nor}+\textit{talks} - \lambda z\neg[(\mathbf{walk}\ z) \vee (\mathbf{talk}\ z)]: \mathrm{VP}}\mathrm{E}$$

7.2. *Quantifier Raising*

In Moortgat (1990) a binary operator which we write here as $\Uparrow$ is defined

ian treatment of quantifier-scoping is achieved by assignment of a quantifier phrase like 'something' to $\mathrm{N}\Uparrow\mathrm{S}$, and assignment of determiners like 'every' to $(\mathrm{N}\Uparrow\mathrm{S})/\mathrm{CN}$. As we already noted, in Moortgat (1991) it is suggested that a category such as $A\Uparrow B$ might be definable as $(B\uparrow A)\downarrow B$, but Moortgat observed that this definability does not hold for the given interpretation, for which, furthermore, the logic is problematic. On the present formulation however, these intuitions are realised. The category $(\mathrm{S}\uparrow\mathrm{N})\downarrow\mathrm{S}$ is a suitable type for a quantifier phrase such as 'everything' or 'some man', achieving sentential quantifier scope, and quantificational ambiguity. Consider first 'Every man walks':

(81)

1. $\textit{every} - \lambda x\lambda y\forall z[(x\ z) \to (y\ z)]$: $((\mathrm{S}\uparrow\mathrm{N})\downarrow\mathrm{S})/\mathrm{CN}$
2. $\textit{man} - \mathbf{man}$: CN
3. $\textit{walks} - \mathbf{walk}$: $\mathrm{N}\backslash\mathrm{S}$
4. $\textit{every} + \textit{man} - (\lambda x\lambda y\forall z[(x\ z) \to (y\ z)]\ \mathbf{man})$: $(\mathrm{S}\uparrow\mathrm{N})\downarrow\mathrm{S}$ E/ 1, 2
5. $\textit{every} + \textit{man} - \lambda y\forall z[(\mathbf{man}\ z) \to (y\ z)]$: $(\mathrm{S}\uparrow\mathrm{N})\downarrow\mathrm{S}$ = 4
6. $a - x$: N H
7. $a + \textit{walks} - (\mathbf{walk}\ x)$: S E$\backslash$ 3, 6
8. $\epsilon + a + \textit{walks} - (\mathbf{walk}\ x)$: S = 7
9. $((\epsilon, \textit{walks})Wa) - (\mathbf{walk}\ x)$: S = 8
10. $(\epsilon, \textit{walks}) - \lambda x(\mathbf{walk}\ x)$: $\mathrm{S}\uparrow\mathrm{N}$ I$\uparrow$ 6, 9
11. $((\epsilon, \textit{walks})W\,\textit{every} + \textit{man}) -$ $(\lambda y\forall z[(\mathbf{man}\ z) \to (y\ z)]\ \lambda x(\mathbf{walk}\ x))$: S E$\downarrow$ 5, 10
12. $\epsilon + \textit{every} + \textit{man} + \textit{walks} - \forall z[(\mathbf{man}\ z) \to (\mathbf{walk}\ z)]$: S = 11
13. $\textit{every} + \textit{man} + \textit{walks} - \forall z[(\mathbf{man}\ z) \to (\mathbf{walk}\ z)]$: S =12

The generation up to line 5 of 'every man' with the standard semantics and type $(\mathrm{S}\uparrow\mathrm{N})\downarrow\mathrm{S}$ is straightforward. In lines 7 to 9 a sentence is constructed on the basis of the nominal $a - x$ hypothesised at line 6. Prosodic equations are used to show that the prosodics can be expressed in a form in which W is the main constructor, and in which a is the right hand operand. The left hand operand is thus a split string term in which a is to be interpolated. Now because the wrap connective is the divisional residuation with respect to the right hand operand of W, this split string term is derivable at line 10 as of the wrap type $\mathrm{S}\uparrow\mathrm{N}$, by I$\uparrow$. Since 'every man' is an infix functor over $\mathrm{S}\uparrow\mathrm{N}$, it can combine by E$\downarrow$ (line 11), and on prosodic evaluation interpolates itself at the position in which the hypothesised nominal was used in the subderivational sentence. Thus the quantifier semantically binds a variable for the position in which it occurs prosodically.

There can be no deviance from this pattern, that is, a quantifier phrase cannot bind the wrong position, for there can be no way that the last line of the relevant subderivation can have the form required for I$\uparrow$, i.e. $(\alpha W a) - \phi$ where $a - x$ is the hypothesis, without α being a split string marking the interpolation position for the prosodics that corresponds to semantics ϕ in terms of x: the equations do not allow anything else. So when a quantifier phrase infixes itself, it will always semantically bind the position it occupies prosodically.

The following derivation shows the object position binding of 'John likes everything'.

(82)

1.	$John - \mathbf{j}: \mathrm{N}$	
2.	$likes - \mathbf{like}: (\mathrm{N}\backslash \mathrm{S})/\mathrm{N}$	
3.	$everything - \lambda x \forall y(x\, y): (\mathrm{S}\uparrow \mathrm{N})\downarrow \mathrm{S}$	
4.	$a - x: \mathrm{N}$	H
5.	$likes + a - (\mathbf{like}\, x): \mathrm{N}\backslash \mathrm{S}$	2, 4 E/
6.	$John + likes + a - ((\mathbf{like}\, x)\, \mathbf{j}): \mathrm{S}$	1, 5 E\
7.	$John + likes + a + \epsilon - ((\mathbf{like}\, x)\, \mathbf{j}): \mathrm{S}$	= 6
8.	$((John + likes, \epsilon)Wa) - ((\mathbf{like}\, x)\, \mathbf{j}): \mathrm{S}$	= 7
9.	$(John + likes, \epsilon) - \lambda x((\mathbf{like}\, x)\, \mathbf{j}): \mathrm{S}\uparrow \mathrm{N}$	4, 8 I$\uparrow$
10.	$((John + likes, \epsilon)W\, everything) - (\lambda x \forall y(x\, y)$ $\lambda x((\mathbf{like}\, x)\, \mathbf{j})): \mathrm{S}$	3, 9 E$\downarrow$
11.	$John + likes + everything - \forall y((\mathbf{like}\, y)\, \mathbf{j}): \mathrm{S}$	= 10

The next two derivations we consider will deliver the subject wide scope and object wide scope readings of 'Everyone loves something'. The first of these is thus:

(83)

1.	$everyone - \lambda x \forall z[(\mathbf{person}\, z) \rightarrow (x\, z)]:$ $(\mathrm{S}\uparrow \mathrm{N})\downarrow \mathrm{S}$	
2.	$loves - \mathbf{love}: (\mathrm{N}\backslash \mathrm{S})/\mathrm{N}$	
3.	$something - \lambda x \exists w[(\mathbf{thing}\, w) \wedge (x\, w)]:$ $(\mathrm{S}\uparrow \mathrm{N})\downarrow \mathrm{S}$	
4.	$b - y: \mathrm{N}$	H
5.	$a - x: \mathrm{N}$	H
6.	$loves + a - (\mathbf{love}\, x): \mathrm{N}\backslash \mathrm{S}$	E/ 2, 5
7.	$b + loves + a - ((\mathbf{love}\, x)\, y): \mathrm{S}$	E\ 4, 6
8.	$((b + loves, \epsilon)Wa) - ((\mathbf{love}\, x)\, y): \mathrm{S}$	= 7
9.	$(b + loves, \epsilon) - \lambda x((\mathbf{love}\, x)\, y): \mathrm{S}\uparrow \mathrm{N}$	I$\uparrow$ 5, 8

10.	$((b + loves, \epsilon)W\, something) -$ $(\lambda x \exists w[(\mathbf{thing}\, w) \wedge (x\, w)]\, \lambda x((\mathbf{love}\, x)\, y)): \mathrm{S}$	E$\downarrow$ 3, 9
11.	$b + loves + something - \exists w[(\mathbf{thing}\, w)$ $\wedge ((\mathbf{love}\, w)\, y)]: \mathrm{S}$	= 10
12.	$((\epsilon, loves + something)Wb) - \exists w[(\mathbf{thing}\, w)$ $\wedge ((\mathbf{love}\, w)\, y)]: \mathrm{S}$	= 11
13.	$(\epsilon, loves + something) - \lambda y \exists w[(\mathbf{thing}\, w)$ $\wedge ((\mathbf{love}\, w)\, y)]: \mathrm{S}\uparrow \mathrm{N}$	I$\uparrow$ 4, 12
14.	$everyone + loves + something -$ $\forall z[(\mathbf{person}\, z) \rightarrow \exists w[(\mathbf{thing}\, w)$ $\wedge ((\mathbf{love}\, w)\, z)]]: \mathrm{S}$	E$\downarrow$ 1, 13

In (83) a nominal hypothesis for the subject is made at line 4, and another subderivation hypothesis for the object at line 5. Since subderivations are first-in-last-out, the subject position is bound last, that is the subject wide scope reading is obtained. The sentence already with the object quantifier phrase is obtained at line 11 just like 'John likes everything' in the previous example, but the subject is a hypothesis variable not a lexical form, and we have worked nested one level down.

In (84) the hypothesis of the wider scope subderivation is used in object position, so that the object wide scope reading is obtained.

(84)

1.	$everyone - \lambda x \forall z[(\mathbf{person}\, z) \rightarrow (x\, z)]:$ $(\mathrm{S}\uparrow \mathrm{N})\downarrow \mathrm{S}$	
2.	$loves - \mathbf{love}: (\mathrm{N}\backslash \mathrm{S})/\mathrm{N}$	
3.	$something - \lambda x \exists w[(\mathbf{thing}\, w) \wedge (x\, w)]:$ $(\mathrm{S}\uparrow \mathrm{N})\downarrow \mathrm{S}$	
4.	$a - x: \mathrm{N}$	H
5.	$b - y: \mathrm{N}$	H
6.	$loves + a - (\mathbf{love}\, x): \mathrm{N}\backslash \mathrm{S}$	E/ 2, 4
7.	$b + loves + a - ((\mathbf{love}\, x)\, y): \mathrm{S}$	E\ 5, 6
8.	$((\epsilon, loves + a)Wb) - ((\mathbf{love}\, x)\, y): \mathrm{S}$	= 7
9.	$(\epsilon, loves + a) - \lambda y((\mathbf{love}\, x)\, y): \mathrm{S}\uparrow \mathrm{N}$	I$\uparrow$ 5, 8
10.	$((\epsilon, loves + a)W\, everyone) -$ $(\lambda x \forall z[(\mathbf{person}\, z)$ $\rightarrow (x\, z)]\, \lambda y((\mathbf{love}\, x)\, y)): \mathrm{S}$	E$\downarrow$ 1, 9
11.	$everyone + loves + a - \forall z[(\mathbf{person}\, z)$ $\rightarrow ((\mathbf{love}\, x)\, z)]: \mathrm{S}$	= 10
12.	$((everyone + loves, \epsilon)Wa) - \forall z[(\mathbf{person}\, z)$ $\rightarrow ((\mathbf{love}\, x)\, z)]: \mathrm{S}$	= 11
13.	$(everyone + loves, \epsilon) - \lambda x \forall z[(\mathbf{person}\, z)$ $\rightarrow ((\mathbf{love}\, x)\, z)]: \mathrm{S}\uparrow \mathrm{N}$	I$\uparrow$ 4, 12

14. *everyone* + *loves* + *something* − $\exists w[(\mathbf{thing}\ w) \wedge \forall z[(\mathbf{person}\ z) \rightarrow ((\mathbf{love}\ w)\ z)]]$: S — E↓ 3, 13

In the examples so far the quantifier is peripheral in the sentence and (in associative calculus) a category (S/N)\S could have been used for a quantifier phrase to appear in object position and S/(N\S) for the quantifier phrase to appear in subject position. But further assignments still would be required for a quantifier phrase to appear in sentence-medial positions. Some generality with respect to the latter can be achieved by assuming second-order polymorphic categories (see Emms 1990), but two assignments, one forward-looking and another backward looking would nevertheless be uniformly required by all quantifiers. The single assignment we have given allows appearance in all N positions without further ado, and allows all the relative quantifier scopings at S nodes. Thus for the example 'John believes someone walks', the first derivation to follow gives the narrow scope, non-specific, quantifier reading, but the second, the wide scope, specific reading, which involves the quantifier raising to the superordinate sentence, in which it is medial.

(85)

1. *John* − $\mathbf{j}$: N
2. *believes* − $\mathbf{believe}$: (N\S)/S
3. *someone* − $\lambda x \exists y(x\ y)$: $(S \uparrow N) \downarrow S$
4. *walks* − $\mathbf{walk}$: N\S
5. | *a* − x: N — H
6. | *a* + *walks* − $(\mathbf{walk}\ x)$: S — E\ 4, 5
7. | ((ε, *walks*)*Wa*) − $(\mathbf{walk}\ x)$: S — = 6
8. (ε, *walks*) − $\lambda x(\mathbf{walk}\ x)$: $S \uparrow N$ — I↑ 5, 7
9. *someone* + *walks* − $\exists y(\mathbf{walk}\ y)$: S — E↓ 3, 8
10. *believes* + *someone* + *walks* − $(\mathbf{believe}\ \exists y(\mathbf{walk}\ y))$: N\S — E/ 2, 9
11. *John* + *believes* + *someone* + *walks* − $((\mathbf{believe}\ \exists y(\mathbf{walk}\ y))\ \mathbf{j})$: S — E\ 1, 10

(86)

1. *John* − $\mathbf{j}$: N
2. *believes* − $\mathbf{believe}$: (N\S)/S
3. *someone* − $\lambda x \exists y(x\ y)$: $(S \uparrow N) \downarrow S$
4. *walks* − $\mathbf{walk}$: N\S
5. | *a* − x: N — H
6. | *a* + *walks* − $(\mathbf{walk}\ x)$: S — E\ 4, 5
7. | *believes* + *a* + *walks* − $(\mathbf{believe}\ (\mathbf{walk}\ x))$: N\S — E/ 2, 6

8. | *John* + *believes* + *a* + *walks* − $((\mathbf{believe}\ (\mathbf{walk}\ x))\ \mathbf{j})$: S — E\ 7, 1
9. | ((*John* + *believes*, *walks*)*Wa*) − $((\mathbf{believe}\ (\mathbf{walk}\ x))\ \mathbf{j})$: S — = 8
10. (*John* + *believes*, *walks*) − $\lambda x((\mathbf{believe}\ (\mathbf{walk}\ x))\ \mathbf{j})$: $S \uparrow N$ — I↑ 5, 9
11. *John* + *believes* + *someone* + *walks* − $\exists y((\mathbf{believe}\ (\mathbf{walk}\ y))\ \mathbf{j})$: S — E\ 3, 10

7.3. *Pied Piping*

Historically, pied piping has played a crucial role in the promotion of feature percolation and phrase structural approaches (Gazdar et al. 1985; Pollard and Sag 1987, 1994) over categorial grammar. Pollard (1988, p. 412) for example regards it as exposing a critical inadequacy:

(87) "Evidently, there is no principled analysis of pied piping in an extended categorial framework like Steedman's without the addition of a feature-passing mechanism for unbounded dependencies."

On the phrase structural view, a relative pronoun introduces information which may percolate up normal constituent structure to endow larger phrases with the relativisation property of occurring fronted and binding a gap of the same category as the entire fronted constituent. Cases in which there is no pied piping are, notably, obtained as the special case where the fronted constituent comprises only the relative pronoun. That is, a single categorisation covers both pied piping and non-pied piping cases such as (88).

(88) a. (the contract) the loss of which after so much wrangling John would finally have to pay for
b. (the contract) which John would finally have to pay for the loss of

In Moortgat (1991) a three-place operator is considered which is like $A \Uparrow B$, except that quantifying-in changes the category of the context expression. Morrill (1992b) shows that this enables capture of pied piping. It follows from the nature of the present proposals that $(B \uparrow C) \downarrow A$ presents the desired complicity between the operators. As a result, the treatment of Morrill (1992b) can be presented in these terms.

As a first example, note how the following pied piping assignment generates ‘about which John talked’ with the same semantics as ‘which John talked about’, considered earlier.

(89)

1.	$about - \mathbf{about}$: PP/N	
2.	$which - \lambda x \lambda y \lambda z \lambda w[(z\ w) \wedge (y\ (x\ w))]$: $(\mathrm{PP} \uparrow \mathrm{N}) \downarrow (\mathrm{R/(S/PP)})$	
3.	$John - \mathbf{j}$: N	
4.	$talked - \mathbf{talk}$: $(\mathrm{N \backslash S})/\mathrm{PP}$	
5.	$a - x$: N	H
6.	$about + a - (\mathbf{about}\ x)$: PP	1, 5 E/
7.	$about + a + \epsilon - (\mathbf{about}\ x)$: PP	= 6
8.	$((about, \epsilon)Wa) - (\mathbf{about}\ x)$: PP	= 7
9.	$(about, \epsilon) - \lambda x(\mathbf{about}\ x)$: $\mathrm{PP} \uparrow \mathrm{N}$	5, 8 I$\uparrow$
10.	$((about, \epsilon)W\ which) - (\lambda x \lambda y \lambda z \lambda w[(z\ w) \wedge (y\ (x\ w))]\ \lambda x(\mathbf{about}\ x))$: R/(S/PP)	2, 9 E$\downarrow$
11.	$about + which - \lambda y \lambda z \lambda w[(z\ w) \wedge (y\ (\mathbf{about}\ w))]$: R/(S/PP)	= 10
12.	$a - x$: PP	H
13.	$talked + a - (\mathbf{talk}\ x)$: $\mathrm{N \backslash S}$	4, 12 E/
14.	$John + talked + a - ((\mathbf{talk}\ x)\ \mathbf{j})$: S	3, 13 E$\backslash$
15.	$John + talked - \lambda x((\mathbf{talk}\ x)\ \mathbf{j})$: S/PP	12, 14 I/
16.	$about + which + John + talked - (\lambda y \lambda z \lambda w[(z\ w) \wedge (y\ (\mathbf{about}\ w))]\lambda x((\mathbf{talk}\ x)\ \mathbf{j}))$: R	11, 15 E/
17.	$about + which + John + talked - \lambda z \lambda w[(z\ w) \wedge ((\mathbf{talk}\ (\mathbf{about}\ w))\ \mathbf{j})]$: R	= 16

This example is potentially manageable in any categorial grammar with composition, by assignment of type $(\mathrm{PP/N}) \backslash ((\mathrm{CN \backslash CN})/(\mathrm{S/N}))$. Such assignments are an obvious possibility in the light of Szabolsci (1987, 1992) for example. Such an assignment must be additional to the regular one, a situation to be improved if possible. But furthermore an example like (88a) where the relative pronoun is not peripheral in the pied piped material would be problematic. It would need to be arranged by a further lexical assignment that ‘after so much wrangling’ modifies ‘loss’. Such treatment is not impossible; indeed any undergeneration can be approached on a case by case basis adding extra lexical assignments. But the issue, of course, is not just whether treatment is possible, but whether a formalism supports the generalisations that analysis of the data indicates. We suggest that medial pied piping would push augmentation of lexical assignments beyond credibility.

examples of the crucially problematic medial pied piping cases, but see e.g. (90).

(90) (a statue) for the transport of which by rail John would have to pay $10,000

In other medial cases the pied piped constituent occupies subject position:

(91)a. (a supermarket) the opening of which by the queen/in June was heralded a moving and historical occasion

b. (a woman) the painting of whom by Matisse fetched a fortune

c. (a boy) the yelling of whom outside could be heard throughout the sermon

If in reality there were no such cases, which would be to say that pied piping noun phrases always occur right-peripherally in the fronted constituent, a rudimentary treatment like that deriving from Szabolcsi would suffice for categorial grammar. Furthermore all existing phrase structure accounts would be erroneous in that none predict such right-peripherality. Thus for phrase structural approaches there would be “no principled analysis of pied piping” possible without the addition of directional constraints on feature inheritance. However since we judge the examples in the text to be acceptable, we do not regard this implication as going through.

The solution, in terms of infixing and wrapping, is much the same as that for quantification. It is also, at least with respect to semantics, much the same as that of Szabolcsi (which is in turn attributed partly to Steedman). Indeed, monostratal compositional semantics simply demands that a pied piped relativiser be a functor over its pied piping domain, and any categorial grammar will make this explicit. Where we are able to improve on the Szabolcsi/Steedman account is in finding a single assignment which generates peripheral, medial and also no-pied piping cases. This is done by shifting a relative pronoun R/(S/N) not to $(X/\mathrm{N}) \backslash (\mathrm{R}/(\mathrm{S}/X))$, i.e. with respect to the usual directional divisions, but to $(X \uparrow \mathrm{N}) \downarrow (\mathrm{R}/(\mathrm{S}/X))$, i.e. with respect to the discontinuity divisions.

There is the following derivation for ‘the loss of which after so much wrangling John would finally have to pay for’, given the relative pronoun assignment at line 4 (*asmw* abbreviates *after* + *so* + *much* + *wrangling* and *wfhtpf* abbreviates *would* + *finally* + *have* + *to* + *pay* + *for*).

(92)

1. $the - \lambda x \iota y(x\ y)$: N/CN
2. $loss - \mathbf{loss}$: CN
3. $of - \mathbf{of}$: $(\mathrm{CN \backslash CN})/\mathrm{N}$

4.	$\mathit{which} - \lambda x\lambda y\lambda z\lambda w[(z\,w) \wedge (y\,(x\,w))]$: $(\mathrm{N}\uparrow\mathrm{N})\downarrow((\mathrm{CN}\backslash\mathrm{CN})/(\mathrm{S}/\mathrm{N}))$	
5.	$\mathit{asmw} - \mathbf{asmw}$: $\mathrm{CN}\backslash\mathrm{CN}$	
6.	$\mathit{John} - \mathbf{j}$: N	
7.	$\mathit{wfhtpf} - \mathbf{wfhtpf}$: $(\mathrm{N}\backslash\mathrm{S})/\mathrm{N}$	
8.	$a - x$: N	H
9.	$\mathit{of} + a - (\mathbf{of}\,x)$: $\mathrm{CN}\backslash\mathrm{CN}$	E/ 3, 8
10.	$\mathit{loss} + \mathit{of} + a - ((\mathbf{of}\,x)\,\mathbf{loss})$: CN	E\ 2, 9
11.	$\mathit{loss} + \mathit{of} + a + \mathit{asmw} - (\mathbf{asmw}\,((\mathbf{of}\,x)\,\mathbf{loss}))$: CN	E\ 5, 10
12.	$\mathit{the} + \mathit{loss} + \mathit{of} + a + \mathit{asmw} - \iota y((\mathbf{asmw}\,((\mathbf{of}\,x)\,\mathbf{loss}))\,y)$: N	E\ 1, 11
13.	$(\mathit{the} + \mathit{loss} + \mathit{of}, \mathit{asmw})Wa - \iota y((\mathbf{asmw}\,((\mathbf{of}\,x)\,\mathbf{loss}))\,y)$: N	= 12
14.	$(\mathit{the} + \mathit{loss} + \mathit{of}, \mathit{asmw}) - \lambda x\iota y((\mathbf{asmw}\,((\mathbf{of}\,x)\,\mathbf{loss}))\,y)$: $\mathrm{N}\uparrow\mathrm{N}$	I↑ 8, 13
15.	$\mathit{the} + \mathit{loss} + \mathit{of} + \mathit{which} + \mathit{asmw} - \lambda y\lambda z\lambda w[(z\,w) \wedge (y\,\iota u((\mathbf{asmw}\,((\mathbf{of}\,w)\,\mathbf{loss}))\,u))]$: $(\mathrm{CN}\backslash\mathrm{CN})/(\mathrm{S}/\mathrm{N})$	E↓ 4, 14
16.	$a - x$: N	H
17.	$\mathit{wfhtpf} + a - (\mathbf{wfhtpf}\,x)$: $\mathrm{N}\backslash\mathrm{S}$	E/ 7, 16
18.	$\mathit{John} + \mathit{wfhtpf} + a - ((\mathbf{wfhtpf}\,x)\,\mathbf{j})$: S	E\ 6, 17
19.	$\mathit{John} + \mathit{wfhtpf} - \lambda x((\mathbf{wfhtpf}\,x)\,\mathbf{j})$: S/N	I/ 16, 18
20.	$\mathit{the} + \mathit{loss} + \mathit{of} + \mathit{which} + \mathit{asmw} + \mathit{John} + \mathit{wfhtpf} - \lambda z\lambda w[(z\,w) \wedge ((\mathbf{wfhtpf}\,\iota u((\mathbf{asmw}\,((\mathbf{of}\,w)\,\mathbf{loss}))\,u))\,\mathbf{j})]$: $\mathrm{CN}\backslash\mathrm{CN}$	E/ 15, 19

In addition, this same assignment generates non-pied piping cases, such as 'which John would finally have to pay for the loss of'. Lines 7 to 11 of the following show that the regular relative pronoun category is derivable from the nominal pied piping one because $(\epsilon, \epsilon) \in D(\mathrm{N}\uparrow\mathrm{N})$.

(93)

1.	$\mathit{which} - \lambda x\lambda y\lambda z\lambda w[(z\,w) \wedge (y\,(x\,w))]$: $(\mathrm{N}\uparrow\mathrm{N})\downarrow((\mathrm{CN}\backslash\mathrm{CN})/(\mathrm{S}/\mathrm{N}))$	
2.	$\mathit{John} - \mathbf{j}$: N	
3.	$\mathit{wfhtpf} - \mathbf{wfhtpf}$: $(\mathrm{N}\backslash\mathrm{S})/\mathrm{N}$	
4.	$\mathit{the} - \lambda x\iota y(x\,y)$: N/CN	
5.	$\mathit{loss} - \mathbf{loss}$: CN	
6.	$\mathit{of} - \mathbf{of}$: $(\mathrm{CN}\backslash\mathrm{CN})/\mathrm{N}$	
7.	$a - x$: N	H
8.	$(\epsilon, \epsilon)Wa - x$: N	= 7
		I↑ 7, 8

10.	$((\epsilon, \epsilon)W\,\mathit{which}) - (\lambda x\lambda y\lambda z\lambda w[(z\,w) \wedge (y\,(x\,w))]\,\lambda xx)$: $(\mathrm{CN}\backslash\mathrm{CN})/(\mathrm{S}/\mathrm{N})$	E↓ 1, 9
11.	$\mathit{which} - \lambda y\lambda z\lambda w[(z\,w) \wedge (y\,w)]$: $(\mathrm{CN}\backslash\mathrm{CN})/(\mathrm{S}/\mathrm{N})$	
12.	$a - x$: N	H
13.	$\mathit{of} + a - (\mathbf{of}\,x)$: $\mathrm{CN}\backslash\mathrm{CN}$	E/ 6, 12
14.	$\mathit{loss} + \mathit{of} + a - ((\mathbf{of}\,x)\,\mathbf{loss})$: CN	E\ 5, 13
15.	$\mathit{the} + \mathit{loss} + \mathit{of} + a - \iota y(((\mathbf{of}\,x)\,\mathbf{loss})\,y)$: N	E/ 4, 14
16.	$(\mathit{wfhtpf} + \mathit{the} + \mathit{loss} + \mathit{of} + a - (\mathbf{wfhtpf}\,\iota y(((\mathbf{of}\,x)\,\mathbf{loss})\,y))$: $\mathrm{N}\backslash\mathrm{S}$	E/ 3, 15
17.	$\mathit{John} + \mathit{wfhtpf} + \mathit{the} + \mathit{loss} + \mathit{of} + a - ((\mathbf{wfhtpf}\,\iota y(((\mathbf{of}\,x)\,\mathbf{loss})\,y))\,\mathbf{j})$: S	E\ 2, 16
18.	$\mathit{John} + \mathit{wfhtpf} + \mathit{the} + \mathit{loss} + \mathit{of} - \lambda x((\mathbf{wfhtpf}\,\iota y(((\mathbf{of}\,x)\,\mathbf{loss})\,y))\,\mathbf{j})$:S/N	I/ 12, 17
19.	$\mathit{which} + \mathit{John} + \mathit{wfhtpf} + \mathit{the} + \mathit{loss} + \mathit{of} - \lambda z\lambda w[(z\,w) \wedge ((\mathbf{wfhtpf}\,\iota u(((\mathbf{of}\,w)\,\mathbf{loss})\,u))\,\mathbf{j})]$: $\mathrm{CN}\backslash\mathrm{CN}$	E/ 11, 18

Thus prepositional pied piping, nominal pied piping, and no-pied piping examples are all obtained by assignment to just the following two types:

(94) $(\mathrm{N}\uparrow\mathrm{N})\downarrow((\mathrm{CN}\backslash\mathrm{CN})/(\mathrm{S}/\mathrm{N}))$
$(\mathrm{PP}\uparrow\mathrm{N})\downarrow((\mathrm{CN}\backslash\mathrm{CN})/(\mathrm{S}/\mathrm{PP}))$

The semantics is the same in each case, so all the examples considered are obtained by a single restricted second-order quantification assignment as in (95).

(95) $\mathit{which} - \lambda x\lambda y\lambda z\lambda w[(z\,w) \wedge (y\,(x\,w))]$:
$\forall^2 X{:}\,\{\mathrm{N}, \mathrm{PP}\}((X\uparrow\mathrm{N})\downarrow((\mathrm{CN}\backslash\mathrm{CN})/(\mathrm{S}/X)))$

The relative pronoun 'that' cannot pied pipe, and so should be assigned the regular type:

(96) $\mathit{that} - \lambda x\lambda y\lambda z[(y\,z) \wedge (x\,z)]$: $(\mathrm{CN}\backslash\mathrm{CN})/(\mathrm{S}/\mathrm{N})$

With interrogatives, there is prepositional, but not nominal, pied piping:

(97) a. Who did John buy the ticket for?
b. For whom did John buy the ticket?
c. *The ticket for whom did John buy?

Thus each interrogative pronoun should have assignment to some combination of (98), but none a nominal pied piping assignment

(98) $I/(M/N)$
$(PP \uparrow N) \downarrow (I/(M/PP))$

There are further issues; for instance interrogative pied piping is better in matrix than in embedded contexts. The usual kinds of featural distinctions may be brought to bear on such facts, but since this would be a diversion from the demands of discontinuity itself we shall not pursue such details here.

7.4. *Reflexivisation*

In this subsection we consider how the apparatus given can be applied to intrasentential bound anaphora as represented by reflexive pronouns. Consider the following subject-oriented examples:

(99) a. John/every man likes himself.
b. John/every man awards the prize to himself.
c. John/every man awards himself the prize.
d. John/every man talked about himself to Mary.

In each case the reflexive pronoun has as antecedent the subject noun phrase; as Reinhart (1983) observes, since the antecedent may be a non-referring quantified noun phrase the problem should be looked upon as one of semantic binding rather than just coreference. Clearly assigning a type such as $(VP/N)\backslash VP$ (with $VP = N\backslash S$) and semantics $\lambda x\lambda y((x\ y)\ y)$ to a reflexive pronoun yields (99a) and, given the relevant associativity, (99b); see Szabolsci (1987, 1992). However further types, e.g. $((VP/X)/N)\backslash(VP/X)$ with $X = N$ for (99c) and $X = PP$ for (99d), both with semantics $\lambda x\lambda y\lambda z(((x\ z)\ y)\ z)$, are needed for other cases. Thus, envisaging reflexives as Szabolsci argument reducers $(Y/N)\backslash Y$ requires a semantic polymorphism somehow schematised according to the polyadicity of Y.

The discontinuity apparatus enables us to realise this polymorphism by formulating the view that reflexives occur in verb phrases and can be bound by the subject through assignment to the single type $(VP \uparrow N) \downarrow VP$ with semantics $\lambda x\lambda y((x\ y)\ y)$; see Moortgat (1990, 1991). Argument reducer types $(VP/N)\backslash VP$, $((VP/X)/N)\backslash(VP/X)$, and so forth are derived from this basic assignment with their desired semantics. This yields all of the examples in (99). For example (99c) is obtained thus Prawitz-style (*tp*

(100)

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\mathit{awards} - \mathbf{award}: (VP/N)/N \quad \overline{a - x: N}^{1}}{\mathit{awards{+}a} - (\mathbf{award}\ x): VP/N}E/ \quad \mathit{tp} - \mathbf{tp}: N}{\mathit{awards{+}a{+}tp} - ((\mathbf{award}\ x)\ \mathbf{tp}): VP}E/}{((\mathit{awards, tp})W a) - ((\mathbf{award}\ x)\ \mathbf{tp}): VP}=}{(\mathit{awards, tp}) - \lambda x((\mathbf{award}\ x)\ \mathbf{tp}): VP\uparrow N}I\uparrow^{1} \quad \mathit{himself} - \lambda x\lambda y((x\ y)\ y): (VP\uparrow N)\downarrow VP}{\dfrac{((\mathit{awards, tp})W \mathit{himself}) - (\lambda x\lambda y((x\ y)\ y)\ \lambda x((\mathbf{award}\ x)\ \mathbf{tp})): VP}{\mathit{awards{+}himself{+}tp} - \lambda y(((\mathbf{award}\ y)\ \mathbf{tp})\ y): VP}=}E\downarrow
$$

The Moortgat characterisation, like the Szabolsci one, does not, unless associativity is somehow restricted, prevent long distance reflexivisation, which is ungrammatical (in English). It is therefore a further issue, not addressed here, how locality conditions can be imposed; see Morrill (1990a, 1992b, 1994b).

What is of relevance to discontinuity however is that reflexives can also take object antecedents:

(101) a. John showed Mary herself.
b. *John showed herself Mary.

(102) a. John showed Mary to herself.
b. *John showed herself to Mary.

Attempts have been made to associate constraints on object-oriented reflexivisation with reference to abstract ordering of complements (see e.g., the obliqueness hierarchy of Pollard and Sag 1994, cf. Dowty 1979, 1982): (101) and (102) switch the order in which surface positions are mapped into semantic arguments of a common ditransitive relation, but the generalisation with respect to reflexivisation is that in both cases the reflexive constituent must be more oblique than its antecedent constituent. Yet it is always interesting to see if a characterisation can be based on a generalisation which evades the supposition of primitive abstract structure. Thus in the present case we simply interpret the data as subject to the condition that the antecedent must precede the reflexive.

The fact that the antecedent separates the verb and the reflexive is paradoxical from the point of view of concatenative syntax and compositional semantics since the pronoun needs to reflexivise the predicate before the predicate applies to the antecedent. We can treat a case such as (101a) however by assignment of the reflexive to $((VP/N)/N) > (VP \uparrow N)$, using the splitting functor constructor. Thus in the following the reflexive forms a split string with the verb, and this complex wraps around the antecedent to give the required word order.

(103)
1. *John* – **j**: N
2. *showed* – **show**: ((N\S)/N)/N
3. *herself* – $\lambda x\lambda y((x\ y)\ y)$: $(((N\backslash S)/N)/N) > ((N\backslash S)\uparrow N)$
4. *Mary* – **m**: N
5. (*showed*, *herself*) – ($\lambda x\lambda y((x\ y)\ y)$ **show**): $(N\backslash S)\uparrow N$ — 2, 3 E>
6. (*showed*, *herself*) – λy((**show** y) y): $(N\backslash S)\uparrow N$ — = 5
7. ((*showed*, *herself*)*W Mary*) – (λy((**show** y) y) **m**): N\S — 4, 6 E↑
8. *showed* + *Mary* + *herself* – ((**show m**) **m**): N\S — = 7
9. *John* + *showed* + *Mary* + *herself* – (((**show m**) **m**) **j**): S — 1, 8 E\

Note that the unacceptable word order *'John showed herself Mary' cannot be derived.

The formulation we have given here requires a duplication of assignments for subject-oriented and object-oriented cases. This seems less than ideal for English, but it should be observed that in many languages (e.g. Dutch and Norwegian) the subject-oriented reflexives have a different form ('zichzelf' and 'seg selv', "himself") than the object-oriented ones ('hemself' and 'ham selv', "himself").

The object antecedent type given so far however is not adequate for instances such as (104) in which the reflexive does not comprise the entire constituent following its antecedent, but is embedded within this constituent (cf. also Szabolcsi 1987, 1992).

(104) John showed Mary to herself.

Generalising the earlier perspective, analysis here requires the reflexive to act as a functor over the preposition before undergoing splitting adjunction with the verb and subsequent wrapping around the antecedent. In fact the situation is just like that for pied piping in relativisation: the "pied piped" reflexive may be medial ('shows Mary a picture of herself by Picasso') and the constituent in which the reflexive is embedded may be of a variety of categories (e.g. noun phrase: 'shows Mary a picture of herself'). Thus we may propose assignment of the kind in (105), and exactly as for the relativisation before, with $X = \mathrm{N}$ this subsumes and therefore obviates the need for the original assignment.

(105) *herself* – $\lambda x\lambda y\lambda z((y\ z)\ (x\ z))$:
$\forall^2 X \in \{N, PP, \ldots\}((X\uparrow N)\downarrow(((VP/X)/N) > (VP\uparrow N)))$

By way of a final illustration we give the following derivation of (non-c-commanding) object-oriented reflexivisation 'talks to Mary about herself'.

(106)a.

$$\dfrac{\dfrac{\mathit{talks} - \mathbf{talk}: (VP/PP)/PP \qquad \dfrac{\mathit{to} - \mathbf{to}: PP/N \qquad \overline{a - x: N}^{\,1}}{\mathit{to{+}a} - (\mathbf{to}\ x): PP}\,E/}{\mathit{talks{+}to{+}a} - (\mathbf{talk}\ (\mathbf{to}\ x)): VP/PP}\,E/}{\mathit{talks{+}to} - \lambda x(\mathbf{talk}\ (\mathbf{to}\ x)): (VP/PP)/N}\,I/^{1}$$

b.

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathit{about} - \mathbf{about}: PP/N \qquad \overline{e - u: N}^{\,2}}{\mathit{about{+}e} - (\mathbf{about}\ u): PP}\,E/}{((\mathit{about}, \epsilon)We) - (\mathbf{about}\ u): PP}\,=}{(\mathit{about}, \epsilon) - \lambda u(\mathbf{about}\ u): PP\uparrow N}\,I\uparrow^{2} \qquad \begin{array}{c}\mathit{herself} - \lambda x\lambda y\lambda z((y\ z)\ (x\ z)):\\ (PP\uparrow N)\downarrow(((VP/PP)/N)>(VP\uparrow N))\end{array}}{\mathit{about{+}herself} - \lambda y\lambda z((y\ z)\ (\mathbf{about}\ z)): ((VP/PP)/N)>(VP\uparrow N)}\,E\downarrow$$

c.

$$\dfrac{\dfrac{\begin{array}{c}\mathit{talks{+}to} - \lambda x(\mathbf{talk}\ (\mathbf{to}\ x)):\\ (VP/PP)/N\end{array} \qquad \begin{array}{c}\mathit{about{+}herself} - \lambda y\lambda z((y\ z)\ (\mathbf{about}\ z)):\\ ((VP/PP)/N)>(VP\uparrow N)\end{array}}{(\mathit{talks{+}to}, \mathit{about{+}herself}) - \lambda z((\mathbf{talk}\ (\mathbf{to}\ z))\ (\mathbf{about}\ z)): VP\uparrow N}\,E> \qquad \mathit{Mary} - \mathbf{m}: N}{\mathit{talks{+}to{+}Mary{+}about{+}herself} - ((\mathbf{talk}\ (\mathbf{to}\ \mathbf{m}))\ (\mathbf{about}\ \mathbf{m})): VP}\,E\uparrow$$

This is the derivation that was presented in sequent calculus format before as (61).

8. Comparisons

The notion of wrapping, and/or head-wrapping has arisen a number of times in categorial grammar and other formalisms. It is therefore worthwhile to try to relate the formulation and applications here to other proposals. Consider the following sentiment expressed in Jacobson (1992, p. 142–143) and reiterated on p. 163:

(107) ". . . certain flexible CGs have the interesting property of structural completeness, and it is not at all clear that this would be preserved (or even meaningful) with the incorporation of wrap. Even in flexible CGs without structural completeness, wrap appears problematic, for it is not immediately obvious how to fold such an operation into a grammar with type-lifting, composition (and/or division) etc."

Structural completeness here refers to associativity. In a sense the present work can be seen as affirming that this property is in fact both meaningful and preserved in the context of wrap, but we need to be clear that we have addressed non-head-oriented discontinuity whereas categorial literature has generally considered head-oriented varieties.

Bach (1979) appeals to wrap in order that certain generalisations concerning control and agreement can be expressed; the generalisation sought is that a controlled expression, e.g. an infinitival verb phrase complement, is controlled by (and therefore agrees with) the noun phrase that its verb next combines with. Consider the following familiar paradigm:

(108) a. John promised Mary to wash himself/*herself.
b. John persuaded Mary to wash herself/*himself.

Under Bach's generalisation the order of application indicated by assignment of the control verb to the type $((N\backslash S)/VP)/N$ suffices for the subject control (108a), but for the object control (108b) Bach's generalisation motivates (in our notation) $((N\backslash S)\uparrow N) < VP$. In the present system however these two types are, as the reader may check, mutually derivable and indistinguishable as regards prosodic interpretation. So unless some special significance is added to the form of lexical types, as opposed to what they represent, the present system cannot be seen as supporting the Bach proposal.

A similar point can be made with respect to Dowty's (1982) definition of grammatical functions in terms of order of combination (subject last to combine with the verb, direct object penultimate, and so on). But as before, a prepositional ditransitive verb type $(VP/PP)/N$ is equivalent to $(VP\uparrow N) < PP$. The types do not define a unique order of combination in derivation.

In fact the flexibility of (associative) Lambek calculus already dissolves rigid ordering of combination since a transitive verb type $(N\backslash S)/N$ applying first to the object is equivalent to $N\backslash(S/N)$ applying first to the subject. The present tools then do not support implementation of the Dowty proposals either, unless the particular type chosen for a lexical assignment is taken to be significant, beyond its denotational equivalence with other types.

Given that associativity erases rigid ordering of combination, let us consider how data analysed in such terms might be reanalysed lexically. We have already seen how the kinds of matter that might be dealt with in relation to grammatical functions, such as reflexivisation, can be approached without such appeal.

The concerns of Bach (1979) with agreement are achievable by means familar from unification grammar; for example by lexical structure sharing

suade'. Related to the analysis however is Bach's (1980) invocation of wrap in order to define direct (i.e. passivisable) object as the noun phrase which completes the saturation of an intransitive verb phrase, i.e. $VP\uparrow N$ undergoes passivisation. Suppose that a verb triggering (agentless) passive, such as the English copula, were of category $VP/(VP\uparrow N)$. Then since 'given to Mary' and 'given the book' are derivable as $VP\uparrow N$ in our system we would generate the prosodic forms (109).

(109) a. *the + book + was + (given, to + Mary)*
b. *Mary + was + (given, the + book)*

Furthermore, in view of the identity element allowing inference such as (110) we would similary obtain (111).

(110) *liked*: $VP/N \Rightarrow$ (*liked*, ϵ): $VP\uparrow N$

(111) *John + was + (liked*, ϵ)

By the same token however we would also obtain (112).

(112) a. *Mary + was + (given + the + book + to*, ϵ)
b. *the + book + was + (given + Mary*, ϵ)

All these prosodic forms already differ from those for well-formed expressions before in that they include the splitting constructor; in the prosodically sorted refinement presented in the appendix they would not be obtainable. But it is clear that the style of analysis is not in any case tenable in the given context because $VP\uparrow N$ does not identify passivisable objects. On the other hand, assuming some kind of case marking, for example a feature 'do' for direct (= passivisable) object, a passive trigger type $VP/(VP\uparrow N{+}do)$ would suffice, at least up to the issue of using splitting constructors in generated prosodic forms. Then the following fundamental distinction would be realised by assignment of 'persuade' to $(VP/VP)/N{+}do$ and 'promise' to $(VP/VP)/N{-}do$.

(113) a. John was persuaded to go.
b. *John was promised to go.

But then use of structural modalities as for extraction generally (Morrill et al. 1990, Barry et al. 1991), with passive trigger type $VP/(VP/\triangle N{+}do)$, serves equally well, and avoids the matter of splitting.

Jacobson (1987) notes that those objects which can undergo heavy noun phrase shift are those which are passivisable:

(114) a. John persuaded to go all those who had shown inter

Prosodically the heavy shifting demands a particular phrasing, and semantically the shifted noun phrase appears to be in focus. However this is realised, the word order at least is obtained once the heavy noun phrase acquires a trigger type $(\mathrm{VP}\uparrow \mathrm{N{+}do})\backslash \mathrm{VP}$ or $(\mathrm{VP}/\triangle\mathrm{N{+}do})\backslash \mathrm{VP}$, i.e. a treatment is available just like that for passive.

In either case locality requirements apply: the object passivised cannot appear in a clause subordinate to the trigger, and heavy shift, like rightward movement in general, is bounded to its clause by a right roof constraint (Ross 1967). These requirements may be pursued by appeal to the apparatus for locality referenced earlier.

From this sketch it seems at least possible that passive and heavy shift can be treated through a lexical specification of direct object without appeal to the notion of head. But the latter option might be pursued also.

Pollard (1984) presents a technical proposal for head-wrapping based on the notion of headed strings. Such head-wrapping affords four cases: the infix may prefix or post-fix to the head of the host, and the head of the result may be inherited from either the host or the infix. Jacobson (1992) discusses categorial connectives for the notion of head-wrapping, and accordingly there are four varieties. For extraction of direct objects the relevant device is host inheriting postfixation. Where we notate this $\uparrow_h$ a passive trigger would be $\mathrm{VP}/(\mathrm{VP}\uparrow_h \mathrm{N{+}do})$ and a heavy shifted noun phrase $(\mathrm{VP}\uparrow_h \mathrm{N{+}do})\backslash \mathrm{VP}$. (The feature for direct object is still necessary for the persuade/promise distinction.) Such a view, furthermore, automatically respects the right roof constraint since the head of a verb phrase is always the matrix verb.

A specific formulation of such a view in the context of logical categorial grammar would presumably share the residuation characterisation of discontinuity operators given here, and connect with the headed categorial calculus of Moortgat and Morrill (1991). But such head-oriented apparatus seems unlikely to render redundant that for non head-oriented discontinuity such as coordination particles, quantifier raising, or reflexivisation.

At the time of writing then it is probably fair to say that categorial logic of head-wrapping as formulated in Pollard (1984) is still an open issue. However Pollard's Head Grammars are studied in Weir, Vijay-Shanker and Joshi (1986), and Joshi et al. define closely related Modified Head Grammars which mark splitting points rather than head elements. This latter is the closest relative of the current proposals, which in a sense show the logic of such splitting and interpolation in the same way that Lambek calculus shows the logic of concatenative subcategorisation. By appeal to an infixation partner to wrapping, and their interaction, we have shown

Appendix: Prosodic Sorting

Our aim here has been to balance logic and linguistics, letting neither get too far ahead of the other, to show how the apparatus presented provides the basic tools for a range of discontinuity phenomena. In relation to computation, little has been said, but Morrill (1994a) provides an implementation. We conclude here with an indication of how the calculus may be stream-lined with a view to improving computational properties.

The formulation we have given is one in which there is a single domain of prosodic objects on which all adjunctions are total functions and all type constructors recurse freely. This use of a total prosodic algebra shows that the various type constructors have a uniform logic, and are not just an ad hoc assembly.

Not all formulas however generate prosodic terms which are (normalisable to) W-free terms. For example, while $d\colon (A\uparrow B) < C$ generates $((d, c)Wb) = d + b + c$ by application to $c\colon C$ and $b\colon B$, $e\colon (A\uparrow B)/C$ generates $(e + cWb)$ which is not simplifiable. Linguistically formulas and terms such as the latter do not define a word order and are not used. We have a natural space within which to work, but it includes abstractions which are not required, and while it is conceptually harmless, computationally such unwanted abstraction creates unwarranted complexity.

Quite generally, when this situation arises it is normal to impose a discipline of typing or sorting on a formalism. In this appendix we indicate how that can be done in the present case. All of the previous analyses will be preserved unaltered, but the sorted formalism should be simpler to implement computationally.

As in Solias (1992) and Morrill and Solias (1993), the wrapping operation is *defined* in terms of a semigroup operation and a pairing operation. However, the formation of categorial types here is restricted and they fall into two sorts according to the data type of their inhabitants: string and split string. Given a semigroup operation $+$ of functionality $L, L \to L$ we define $(.\,,.)$ and W by putting $(s_1, s_2) = \langle s_1, s_2\rangle$ and $(s_1, s_3)Ws_2 = s_1 + s_2 + s_3$. The functionality of $(.\,,.)$ is $L, L \to L \times L$ and that of W, $L \times L, L \to L$. Interpretation of formulas is by residuation with respect to $+$, $(.\,,.)$ and W as before but they now come in two sorts: string ($\mathscr{F}$; prosodically interpreted as subsets of L) and split string ($\mathscr{G}$; prosodically interpreted as subsets of $L \times L$), and some previous formulas will be lost. For example $A\uparrow B$ will be of sort split string, $(A\uparrow B) < C$ will be of sort string, but $(A\uparrow B)/C$ will not be of either sort, i.e. it will not be a well-sorted formula. Let us assume that atomic formulas $\mathscr{A}$ are of sort string. Then the sorted formulas are defined by mutual recursion as follows:

(115) $\mathscr{F} ::= \mathscr{A} | \mathscr{F}/\mathscr{F} | \mathscr{F}\backslash\mathscr{F} | \mathscr{G} < \mathscr{F} | \mathscr{F} > \mathscr{G} | \mathscr{G} \downarrow \mathscr{F}$
$\mathscr{G} ::= \mathscr{F} \uparrow \mathscr{F}$

Labelled calculi are given much as before except that types of sort split string have their two components in the labels. As was the case for Moortgat (1988) however, an (unlabelled) Gentzen sequent formulation cannot be given because encoding prosodics by the position of a type cannot reflect the two components of a splitting type.

References

Bach, Emmon: 1979, 'Control in Montague Grammar', *Linguistic Inquiry* **10**, 515–531.

Bach, Emmon: 1980, 'In Defense of Passive', *Linguistics and Philosophy* **3**, 297–341.

Bach, Emmon: 1981, 'Discontinuous Constituents in Generalized Categorial Grammars', *Proceedings of the 11th Annual Meeting of the North Eastern Linguistics Society*, New York, pp. 1–12.

Bach, Emmon: 1984, 'Some Generalizations of Categorial Grammars', in Fred Landman and Frank Veltman (eds.), *Varieties of Formal Semantics*, Foris, Dordrecht, pp. 1–23.

Barry, Guy, Mark Hepple, Neil Leslie, and Glyn Morrill: 1991, 'Proof Figures and Structural Operators for Categorial Grammar', in *Proceedings of the Fifth Conference of the European Chapter of the Association for Computational Linguistics*, Berlin.

van Benthem, Johan: 1990 'Categorial Grammar Meets Unification', Report No. CSLI-90-142, Center for the Study of Language and Information, Stanford University.

van Benthem, Johan: 1991, *Language in Action: Categories, Lambdas and Dynamic Logic*, Studies in Logic and the Foundations of Mathematics Volume 130, North-Holland, Amsterdam.

Buszkowski, Wojciech: 1986, 'Completeness Results for Lambek Syntactic Calculus', *Zeitschrift für mathematische Logik und Grundlagen der Mathematik*, **32**, 13–28.

Carpenter, Bob: 1992, 'Categorial Grammars, Lexical Rules and the English Predicative', in Robert Levine (ed.) *Formal Grammar: Theory and Implementation*, Oxford University Press, Oxford, pp. 168–242.

Dowty, David R.: 1979, 'Dative 'Movement' and Thomason's Extensions of Montague Grammar', in S. Davis and M. Methun (eds.), *Linguistics, Philosophy and Montague Grammar*, University of Texas Press, pp. 153–222.

Dowty, David R.: 1982, 'Grammatical Relations and Montague Grammar', in P. Jacobson anf G. K. Pullum (eds.), *The Nature of Syntactic Representation*, D. Reidel, Dordrecht, pp. 79–130.

Dunn, Michael J.: 1991, 'Gaggle Theory: An Abstraction of Galois Connections and Residuation, with Applications to Negation, Implication, and Various Logical Operators', in J. van Eijck (ed.), *Logics in AI, Proceedings of JELIA '90*, Lecture Notes in Artificial Intelligence **478**, Springer-Verlag, pp. 31–51.

Emms, Martin: 1990 'Polymorphic Quantifiers', in Martin Stokhof and Leen Torenvliet (eds.), *Proceedings of the Seventh Amsterdam Colloquium*, Institute for Language, Logic and Information, Universiteit van Amsterdam, pp. 139–163, also in Guy Barry and Glyn Morrill (eds.), *Studies in Categorial Grammar*, Edinburgh Working Papers in Cognitive Science, Volume 5, Centre for Cognitive Science, University of Edinburgh, pp. 65–111.

Fitch, Frederic Brenton: 1952, *Symbolic Logic, An Introduction*, The Ronald Press Company, New York.

Gazdar, Gerald, Ewan Klein, Geoffrey Pullum, and Ivan Sag: 1985, *Generalised Phrase Structure Grammar*, Basil Blackwell, Oxford.

Girard, Jean-Yves: 1989, 'Towards a Geometry of Interaction', *Contemporary Mathematics* Volume 92, American Mathematical Society, pp. 69–108.

Hindley, J. Roger and Jonathan P. Seldin: 1986, *Introduction to Combinators and λ-Calculus*, London Mathematical Society Student Texts 1, Cambridge University Press, Cambridge.

Jacobson, Pauline: 1987, 'Phrase Structure, Grammatical Relations, and Discontinuous Constituents', in G. Huck and A. Ojeda (eds.), *Syntax and Semantics 20: Discontinuous Constituency*, Academic Press, New York, pp. 27–69.

Jacobson, Pauline: 1992, 'Flexible Categorial Grammars: Questions and Prospects', in Robert Levine (ed.), *Formal Grammar: Theory and Implementation*, Oxford University Press, Oxford, pp. 168–242.

Kandulski, Maciej: 1988, 'The Non-Associative Lambek Calculus', in W. Buszkowski, W. Marciszewski and J. van Benthem (eds.), *Categorial Grammar*, Linguistic and Literary Studies in Eastern Europe 25, John Benjamins, Amsterdam, pp. 141–151.

Lambek, J.: 1958, 'The Mathematics of Sentence Structure', *American Mathematical Monthly* **65**, 154–170.

Lambek, J.: 1961, 'On the Calculus of Syntactic Types', in R. Jakobson (ed.), *Structure of Language and its Mathematical Aspects*, Proceedings of the Symposia in Applied Mathematics **XII**, American Mathematical Society, pp. 166–178.

Lambek, J.: 1988, 'Categorial and Categorical Grammars', in Richard T. Oehrle, Emmon Bach and Deidre Wheeler (eds.), *Categorial Grammars and Natural Language Structures*, Studies in Linguistics and Philosophy Volume 32, D. Reidel, Dordrecht, pp. 297–317.

Montague, Richard: 1973, 'The Proper Treatment of Quantification in Ordinary English', in J. Hintikka, J. M. E. Moravcsik and P. Suppes (eds.), *Approaches to Natural Language*, D. Reidel, Dordrecht, reprinted in R. H. Thomason (ed.): 1974, *Formal Philosophy: Selected Papers of Richard Montague*, Yale University Press, New Haven, pp. 247–270.

Moortgat, Michael: 1988, *Categorial Investigations: Logical and Linguistic Aspects of the Lambek Calculus*, Foris, Dordrecht.

Moortgat, Michael: 1990, 'The Quantification Calculus: Questions of Axiomatisation', in Deliverable R1.2.A of DYANA Dynamic Interpretation of Natural Language, ESPRIT Basic Research Action BR3175.

Moortgat, Michael: 1991, 'Generalised Quantification and Discontinuous Type Constructors', to appear in Sijtsma and Van Horck (eds.), *Proceedings of the Tilburg Symposium on Discontinuous Constituency*, Walter de Gruyter, Berlin.

Moortgat, Michael: 1992, 'Labelled Deductive Systems for Categorial Theorem Proving', Working Paper OTS-WP-CL-92-002, Onderzoeksinstituut voor Taal en Spraak, Rijksuniversiteit Utrecht, also in *Proceedings of the Eighth Amsterdam Colloquium*, Institute for Language, Logic and Information, Universiteit van Amsterdam.

Moortgat, Michael and Glyn Morrill: 1991, 'Heads and Phrases: Type Calculus for Dependency and Constituency', to appear in *Journal of Language, Logic and Information*.

Morrill, Glyn: 1990a, 'Intensionality and Boundedness', *Linguistics and Philosophy* **13**, 699–726.

Morrill, Glyn: 1990b, 'Grammar and Logical Types', in Martin Stokhof and Leen Torenvliet (eds.), *Proceedings of the Seventh Amsterdam Colloquium*, Institute for Language, Logic and Information, Universiteit van Amsterdam, pp. 429–450, also in Guy Barry and Glyn Morrill (eds.), *Studies in Categorial Grammar*, Edinburgh Working Papers in Cognitive Science Volume 5, Centre for Cognitive Science, University of Edinburgh, pp. 127–148.

Morrill, Glyn: 1990c, 'Rules and Derivations: Binding Phenomena and Coordination in Categorial Logic', Deliverable R1.2.D of DYANA Dynamic Interpretation of Natural Language, ESPRIT Basic Research Action BR3175.

Morrill, Glyn: 1992a, *Type Logical Grammar*, Report de Recerca LSI-92-5-R, Departament de Llenguatges i Sistemes Informàtics, Universitat Politècnica de Catal

Paper OTS-WP-CL-92-002, Onderzoeksinstituut voor Taal en Spraak, Rijksuniversiteit Utrecht, monograph to appear, Kluwer, Dordrecht.
Morrill, Glyn: 1992b, 'Categorial Formalisation of Relativisation: Pied Piping, Islands and Extraction Sites', Report de recerca LSI-92-23-R, Departament de Llenguatges i Sistemes Informàtics, Universitat Politècnica de Catalunya.
Morrill, Glyn: 1994a, 'Clausal Proofs and Discontinuity', to appear in the *Bulletin of the Interest Group in Propositional and Predicate Logic*.
Morrill, Glyn: 1994b, 'Structural Facilitation and Structural Inhibition', Report de recerca LSI-94-26-R, Departament de Llenguatges i Sistemes Informatics, Universitat Politècnica de Catalunya.
Morrill, Glyn, Neil Leslie, Mark Hepple, and Guy Barry: 1990, 'Categorial Deductions and Structural Operations', in Guy Barry and Glyn Morrill (eds.), *Studies in Categorial Grammar*, Edinburgh Working Papers in Cognitive Science, Volume 5, Centre for Cognitive Science, University of Edinburgh, 1–21.
Morrill, Glyn and Teresa Solias: 1993, 'Tuples, Discontinuity and Gapping', in *Proceedings of European Chapter of the Association for Computational Linguistics*, Utrecht, pp. 287–297.
Oehrle, Richard T. and Shi Zhang: 1989, 'Lambek Calculus and Preposing of Embedded Subjects', *Chicago Linguistic Society* **25**, Chicago.
Pollard, Carl J.: 1984, *Generalized Phrase Structure Grammar, Head Grammars and Natural Languages*, Stanford University, Stanford.
Pollard, Carl J.: 1988, 'Categorial Grammar and Phrase Structure Grammar: An Excursion on the Syntax-Semantics Frontier', in Richard T. Oehrle, Emmon Bach and Deidre Wheeler (eds.), *Categorial Grammars and Natural Language Structures*, Studies in Linguistics and Philosophy, Volume 32, D. Reidel, Dordrecht, pp. 391–415.
Pollard, Carl and Ivan A. Sag: 1987, *Information-Based Approach to Syntax and Semantics*, CSLI Lecture Notes 13, Center for the Study of Language and Information, Stanford.
Pollard, Carl and Ivan A. Sag: 1994, *Head-Driven Phrase Structure Grammar*, Studies in Contemporary Linguistics, University of Chicago Press, Chicago.
Prawitz, D.: 1965, *Natural Deduction: A Proof-Theoretical Study*, Almqvist and Wiksell, Uppsala.
Reinhart, Tanya: 1983, 'Coreference and Bound Anaphora: A Restatement of the Anaphora Questions', *Linguistics and Philosophy* **6**, 47–88.
Ross, J. R.: 1967, *Constraints on Variables in Syntax*, Ph.D. dissertation, Massachusetts Institute of Technology, Indiana University Linguistics Club.
Solias, Teresa: 1992, *Gramáticas Categoriales, Coordinación Generalizada y Elisión*, doctorate thesis, Departamento de Lingüística, Lógica, Lenguas Modernas y Fílosofia de la Ciencia, Universidad Autónoma de Madrid.
Steedman, Mark: 1985, 'Dependency and Coordination in the Grammar of Dutch and English', *Language* **61**(3), 523–568.
Szabolcsi, Anna: 1987, 'Bound Variables in Syntax (Are There Any?)', in Jeroen Groenendijk, Dick de Jongh and Martin J.B. Stokhof (eds.), *Proceedings of the Sixth Amsterdam Colloquium*, Institute for Language, Logic and Information, University of Amsterdam, pp. 331–351, revised version in Renate Bartsch et al. (eds): 1989, Foris, Dordrecht.
Szabolcsi, Anna: 1992, 'Combinatory Grammar and Projection from the Lexicon', in Ivan A. Sag and Anna Szabolcsi (eds.), *Lexical Matters*, CSLI Lecture Notes No. 24, Center for the Study of Language and Information, Stanford, pp. 241–268.
Venema, Yde: 1993, 'Tree Models and (Labelled) Categorial Grammar', *Proceedings of the Ninth Amsterdam Colloquium*.
Versmissen, Koen: 1991, 'Discontinuous Type Constructors in Categorial Grammar', ms. Onderzoeksinstituut voor Taal en Spraak, Rijksuniversiteit Utrecht.
Wasow, Thomas, Ivan A. Sag and Geoffrey Nunberg: 1983, 'Idioms: An Interim Report', in Shiro Hattori and Kazuko Inoue (eds.), *Proceedings of the XIIIth International Congress*

Weir, D. J., K. Vijay-Shanker and A. K. Joshi: 1986, 'The Relationship between Tree Adjoining Grammars and Head Grammars', in *Proceedings of the 24th Meeting of the Association for Computational Linguistics*.
Zielonka, Wojciech: 1981, 'Axiomatizability of Ajdukiewicz–Lambek Calculus by Means of Cancellation Schemes', *Zeitschrift für mathematische Logik und Grundlagen der Mathematik* **27**, 215–224.

Universitat Politècnica de Catalunya
Dept. de Llenguatges i Sistemes Informàtics
Pau Gargallo 5
08028 Barcelona
Spain