GLYN MORRILL

INTENSIONALITY AND BOUNDEDNESS*

0. Introduction

The study of the syntax of discontinuity phenomena has led to the hypothesis of boundaries and bounded domains; such concepts can be motivated in relation to syntactic descriptive adequacy. For example, in English *wh*-clauses are islands to relativisation, and non-picture noun reflexives cannot take an antecedent from outside the tensed clause in which they occur:

(1)a. *This is the man who John knows whether Mary likes.
  b. *John thinks Mary likes himself.

When a perspective on grammar is taken which includes semantics, it may be observed that this field of itself contains the notion of intensional domains. Thus it might be assumed that each clause forms an intensional domain. My concern here is with the technical issue of the formalisation of intensionality, and the empirical issue of the possible relevance of intensional domains to syntactic description. The paper presents intensional categorial grammar and shows how intensionality provides an appropriate term of reference for boundaries to relativisation and boundedness of reflexivisation.

In categorial grammar, linguistic expressions are classified by recursively defined types. The relations between (extensional) types are such that a functional type-forming connective is a kind logical implication. Derivations of expressions are associated with functional terms representing meaning. Introduction and elimination of functional types corresponds to the two complementary computational operations of abstraction and application; this is the so-called Curry–Howard isomorphism, and the core of type-theory. Section 1 introduces categorial grammar and the calculus of Lambek (1958). A perspective is offered in which the isomorphism between deductions and functional terms holds essentially as for impli-

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cational intuitionistic logic and the simply typed lambda calculus. Accounts of relativisation and reflexivisation are presented, but it is observed that these fail to model islandhood and boundedness.

Intensionality is usually approached by adding to the functional types a basic type representing indices. Here however, it is claimed that intensionality can be formalised according to the Curry–Howard isomorphism with introduction and elimination of the logical connective $\Box$ of universal modality corresponding to the complementary operations of intension and extension. Thus if $X \rightarrow Y$ is an extensional type of functions mapping from X into Y, $\Box(X \rightarrow Y)$ is the corresponding intensional type. Section 2 presents intensional categorial grammar and characterises *wh*-islandhood in relativisation and clause-boundedness in (non-picture noun) reflexivisation.

1. Extensional categorial grammar

In extensional bidirectional categorial grammar, the set of **TYPES** is defined in terms of a set of primitive types as shown in (2).

(2)a. If X is a primitive type
then X is a type.

b. If X and Y are types
then X/Y and $Y\backslash X$ are types.

The categorial calculus AB, essentially that of Ajdukiewicz (1935) and Bar-Hillel (1953), contains just the following two rules which state that a derivation of (FUNCTOR) type X/Y followed by a derivation of (ARGUMENT) type Y can be combined to form a derivation of type X, and that a derivation of (ARGUMENT) type Y followed by a derivation of (FUNCTOR) type $Y\backslash X$ can be combined to form a derivation of type X.

(3)a.
$$\frac{\overset{\vdots}{X/Y} \qquad \overset{\vdots}{Y}}{X}\,/\mathrm{E}$$

b.
$$\frac{\overset{\vdots}{Y} \qquad \overset{\vdots}{Y\backslash X}}{X}\,\backslash\mathrm{E}$$

A type under vertical ellipses signifies a derivation of that type, and a sole type constitutes a derivation of itself. Ignoring directionality and writing types as implicational formulas emphasises their logical character; the slash elimination rules above are order-sensitive versions of the natural deduction rule of impliction elimination:

(4) $\vdots \qquad \vdots$

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Assume for example primitive types PP (prepositional phrase), NP (noun phrase), S (sentence), and N (common noun). A grammar may be developed by assigning types to words in the manner illustrated below:

(5)

for	:=	PP/NP
John, Mary	:=	NP
likes	:=	(NP\S)/NP
man	:=	N
the	:=	NP/N
thinks	:=	(NP\S)/S
votes	:=	(NP\S)/PP

Then *Mary likes John* and *Mary thinks John votes for the man* are derived as sentences as follows:

(6)
$$\dfrac{\dfrac{\textbf{Mary}}{NP} \qquad \dfrac{\dfrac{\textbf{likes}}{(NP\backslash S)/NP} \quad \dfrac{\textbf{John}}{NP}}{NP\backslash S}\,/E}{S}\,\backslash E$$

(7)
$$\dfrac{\dfrac{\textbf{Mary}}{NP} \qquad \dfrac{\dfrac{\textbf{thinks}}{(NP\backslash S)/S} \quad \dfrac{\dfrac{\textbf{John}}{NP} \quad \dfrac{\dfrac{\textbf{votes}}{(NP\backslash S)/PP} \quad \dfrac{\dfrac{\textbf{for}}{PP/NP} \quad \dfrac{\dfrac{\textbf{the}}{NP/N} \quad \dfrac{\textbf{man}}{N}}{NP}\,/E}{PP}\,/E}{NP\backslash S}\,/E}{S}\,\backslash E}{NP\backslash S}\,/E}{S}\,\backslash E$$

The structures generated by such an AB categorial grammar, which may be called CANONICAL DERIVATIONS, resemble traditional constituent structure. But there are discontinuity phenomena like coordination reduc-

(8)a. John likes, and Bill adores, Mary.
b. the man who John likes.

These cases exhibit 'incomplete' expressions such as *John likes* which are not canonical constituents. In addition to this apparent linguistic inadequacy, from a logical point of view the AB calculus is unbalanced in that while it contains elimination rules, it does not contain introduction rules. The symmetry is restored in the associative Lambek calculus L (Lambek 1958; the non-associative calculus is presented in Lambek 1961). Formulated as a kind of natural deduction system, L is obtained by adding the following slash introduction rules to the slash elimination rules of AB. These state that from a derivation of *Y* from assumptions including an (appropriate) occurrence of *X*, a further derivation is obtained by withdrawing that assumption occurrence.

(9)a. $$\dfrac{\begin{matrix}[Y]^n \\ \vdots \\ X\end{matrix}}{X/Y}/I^n$$ b. $$\dfrac{\begin{matrix}[Y]^n \\ \vdots \\ X\end{matrix}}{Y\backslash X}\backslash I^n$$

There is the condition that *Y* is respectively the rightmost and leftmost undischarged assumption above *X* in /I and \I. By way of illustration of such conditionalisation, the non-canonical constituent *John likes* can be derived as of type S/NP, and assignment of type (N\N)/(S/NP) to an object relative pronoun produces relativisation:

(10)
$$\dfrac{\dfrac{\text{the}}{NP/N} \quad \dfrac{\dfrac{\text{man}}{N} \quad \dfrac{\dfrac{\text{who}}{(N\backslash N)/(S/NP)} \quad \dfrac{\dfrac{\dfrac{\text{Mary}}{NP} \quad \dfrac{\dfrac{\text{likes}}{(NP\backslash S)/NP} \quad [NP]^1}{NP\backslash S}/E}{S}\backslash E}{S/NP}/I^1}{N\backslash N}/E}{N}\backslash E}{NP}/E$$

In addition to resulting in the generation of new strings, conditionalisation

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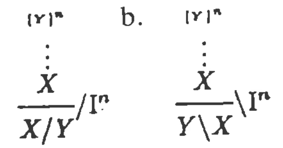

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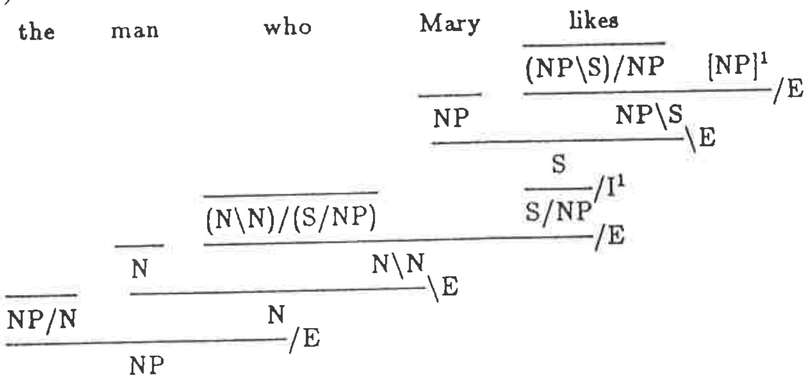
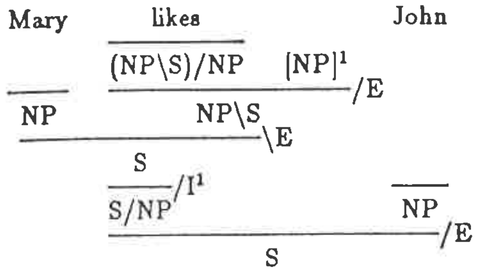


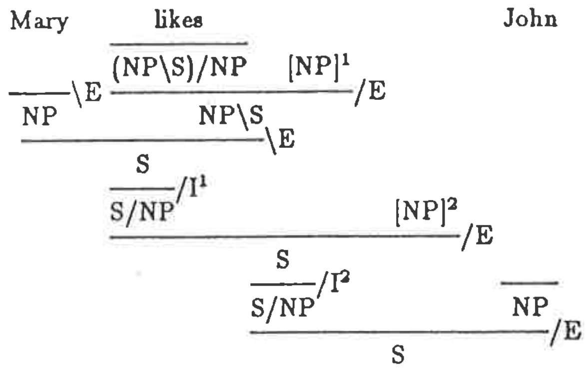
In addition to resulting in the generation of new strings, conditionalisation allows the same strings to be assigned different structures. Thus *Mary*

(11)



In fact an infinite number of structures are assigned, e.g. the previous string also has this derivation:

(12)



Prawitz (1965) observes that in intuitionistic natural deduction, feeding an introduction rule into an elimination rule produces a reducible proof, resulting in the notions of proof-reduction and normal-form proof. Curry and Howard (Curry and Feys 1958; Howard 1969) point out that there is a correspondence between intuitionistic natural deductions and typed functional terms of the lambda calculus. The inductive definition of the natural deduction proofs is the same as that of the typed lambda terms; furthermore proof reduction and the normal form of a proof correspond to lambda reduction and the normal form of a term (lambda program). This Curry-Howard isomorphism presents typed functional terms as corresponding to natural deductions with ‘proofs as programs’ and ‘formulas as types’.

The present case is similar. For the / connective of the Lambek calculus, the following proof-reduction is available when an introduction feeds into

(13)
$$\dfrac{\dfrac{\begin{matrix}[x]^n\\ \vdots\\ Y\end{matrix}}{Y/X}/\mathrm{I}^n \qquad \begin{matrix}\vdots\\ X\end{matrix}}{Y}/\mathrm{E} \qquad \triangleright \qquad \begin{matrix}\vdots\\ x\\ \vdots\\ Y\end{matrix}$$

This states that any proof can be reduced by replacing an occurrence of the form on the left (the REDEX) by the form on the right (the CONTRACTUM). Thus (12) ▷ (11) ▷ (6). The correspondence that holds in the Lambek calculus is a variation on the intuitionistic version. A Lambek natural deduction will be encoded as a single-bind, bidirectional lambda term by notating /E as ' (an infix operator interpreted as application of its left operand to its right), and \E as ‘ (an infix operator interpreted as application of its right operand to its left); a discharged assumption indexed n and of type X is notated as variable x_n of type X; $/\mathrm{I}^n$ is notated as prefix λx_n and $\backslash\mathrm{I}^n$ postfix $x_n\lambda$; such a lambda calculus for bidirectional categorial grammar has been proposed by Wojciech Buszkowski. The unary lambda operators are assumed to bind more tightly than the binary applications. Corresponding to the proof reduction in (13) there is a lambda reduction ($x\lambda$ is symmetrical):

(14) $\lambda x\alpha\,'\,\beta \triangleright \alpha[\beta/x]$

By way of illustration, the reduction (12) ▷ (11) ▷ (6) is encoded as follows (types omitted):

(15) $\lambda y[\lambda x[\text{Mary}\,‘\,(\text{likes}\,'\,x)]\,'\,y]\,'\,\text{John} \triangleright$
$\lambda x[\text{Mary}\,‘\,(\text{likes}\,'\,x)]\,'\,\text{John} \triangleright$
$\text{Mary}\,‘\,(\text{likes}\,'\,\text{John})$

From the point of view of psychology, the left-branching derivations are interesting because they follow a pattern of left-to-right incremental analysis. From the point of view of computational implementation, the multitude of derivations presents problems. This phenomenon of 'spurious ambiguity' or DERIVATIONAL EQUIVALENCE is treated in various works by computing just normal form derivations (Hepple and Morrill 1989; König 1989).

In addition to the above Prawitz style natural deduction formulation of categorial grammar, it is possible to take a Gentzen style sequent perspective (Lambek 1958; Moortgat 1988). A sequent $\Gamma \Rightarrow X$ states that there is a deduction of conclusion X from the antecedent sequence of assumptions

(13)
$$\dfrac{\dfrac{\begin{array}{c}[x]^n\\ \vdots\\ Y\end{array}}{Y/X}/\mathrm{I}^n \quad \begin{array}{c}\vdots\\ X\end{array}}{Y}/\mathrm{E} \quad \rhd \quad \begin{array}{c}\vdots\\ x\\ \vdots\\ Y\end{array}$$

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λx[Mary ' (likes ' x)] ' John ▷
Mary ' (likes ' John)

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(16) $X \Rightarrow X$ [Ax]

$\Delta(\Gamma) \Rightarrow Y$ [CUT]
  $\Gamma \Rightarrow X$
  $\Delta(X) \Rightarrow Y$

$\Gamma(X/Y\ \Delta) \Rightarrow Z$ [/L]
  $\Delta \Rightarrow Y$
  $\Gamma(X) \Rightarrow Z$

$\Gamma(\Delta\ Y\backslash X) \Rightarrow Z$ [\L]
  $\Delta \Rightarrow Y$
  $\Gamma(X) \Rightarrow Z$

$\Gamma \Rightarrow X/Y$ [/R]
  $\Gamma\ Y \Rightarrow X$

$\Gamma \Rightarrow Y\backslash X$ [\R]
  $Y\ \Gamma \Rightarrow X$

Parenthesised parameters to sequences indicate occurences, e.g. $\Gamma(\Delta)$ stands for a sequence with a distinguished occurence of a subsequence Δ. A sequent rule states that the top line sequent follows from the indented sequents. The Ax and CUT rules represent the reflexivity and transitivity of derivability. Lambek (1958) shows that the sequent formulation of L has the property of cut-elimination, i.e. every sequent which is a theorem has a cut-free proof. The cut-free formulation provides a decision procedure since each sequent rule has premisses containing fewer connectives than the conclusion. In intuitionistic logic, cut elimination in the sequent formulation corresponds to removal of the formula which disappears in the replacement of a natural deduction redex by its contractum. The situation seems to be the same for L (cf. Moortgat 1990).

It is possible to locate the Lambek calculus at the bottom of a hierarchy of implicational logics (Wansing 1989, Ono to appear, van Benthem 1987). As a sequent system intuitionistic logic includes the structural rules of weakening (W), contraction (C), and exchange or permutation (P):

(17) $\Gamma\ Y \Rightarrow X$ [W]
  $\Gamma \Rightarrow X$

(18) $\Gamma\ Y \Rightarrow X$ [C]

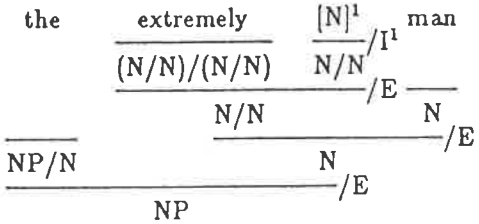
(19) $\Gamma(Y\ X) \Rightarrow Z$ [P]

$$\Gamma(X\ Y) \Rightarrow Z$$

In terms of natural deduction, weakening corresponds to the possibility of conditionalising without withdrawing an assumption (vacuous abstraction in lambda calculus). Dropping weakening takes intuitionistic logic into relevance logic (Anderson and Belnap 1975). Contraction corresponds to withdrawing more than one assumption (binding of more than one variable occurrence in lambda calculus). Dropping also contraction turns a relevance logic into an occurrence logic such as linear logic (Girard 1987; Girard and Lafont 1987; Avron 1988); conditionalisation must withdraw exactly one assumption, and lambdas always bind a single variable. This implicational logic is much discussed by van Benthem (e.g. 1983, 1988a) and has been called the Lambek-van Benthem calculus. The antecedents of sequents are treated as multisets and the theorems are closed under permutation of their assumptions. In non-commutative linear logic, exchange is also dropped, so that assumptions are treated as lists. Then different implications are distinguished according to their directionality, as in the Lambek calculus (cf. Girard 1989 II.6.γ).

The calculus of Lambek (1958) actually carries a condition that a sole assumption cannot be conditionalised, so the sequent system contains no theorems with empty antecedent lists. If empty antecedent lists were allowed, the linguistic significance would be assignment of types to the empty string; for example, / introduction on any sole assumption X derives the empty string as of type X/X. Then an intensifier *extremely* (as in *extremely tall man*), assumed to be of type (N/N)/(N/N), could combine with the empty string to form an adjective, yielding e.g. **the extremely man*.

(20) *



When the distinction needs to be made, the Lambek calculus with the non-empty antecedent condition will be called L^+; that without the condition, L^*. The Lambek-van Benthem calculus will be called LP, and more specifically the systems with and without the non-empty antecedent

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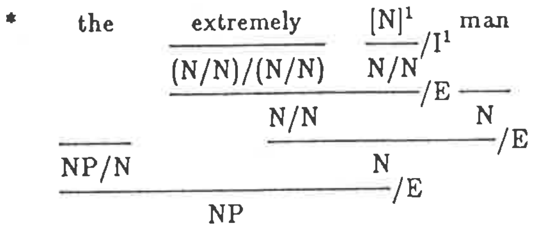

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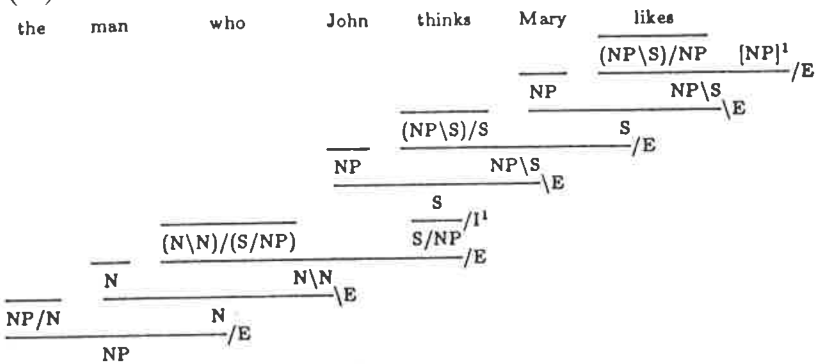
In several works, categorial grammars with some degree of conditionalisation have been used to develop accounts of discontinuity phenomena such as extraction, coordination, and anaphora (see e.g. Ades and Steedman 1982; Steedman 1985, 1987; Szabolcsi 1983, 1987; Morrill 1988; Moortgat 1988). Ades and Steedman (1982) show how a categorial grammar can provide the necessary mediation between the extraction site and a filler in left extraction such as relativisation. The essence of their account is already reproduced in (10) which has the lambda encoding (21).

(21) **the ' (man ' (who ' λx[Mary ' (likes ' x)]))**

This account features the unboundedness of relativisation:

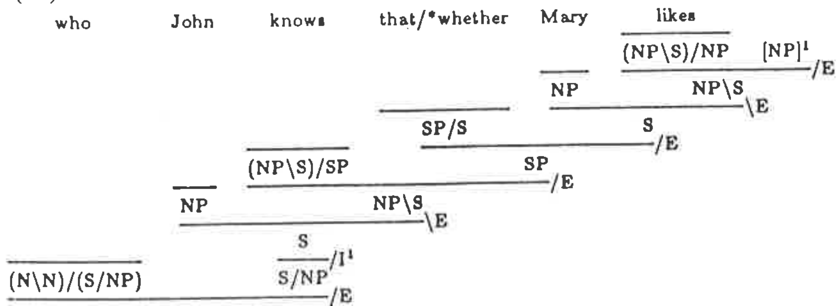
(22)


(23) **the ' (man ' (who ' λx[John ' (thinks ' (Mary ' (likes ' x)))]))**

However, the grammar does not capture the contrast in (24).

(24) the man who John knows that/*whether Mary likes

These examples illustrate Chomsky's *wh*-island constraint, but a straightforward type assignment makes no distinction:

(25)


Subject relative pronouns may be assigned type (N\N)/(NP\S), but then the grammar will also fail to exclude violations of the *wh*-island constraint such as (26).

(26) *the man who John likes the woman who loves

The approach to relativisation above also undergenerates: the conditions on conditionalisation mean that extraction sites must be peripheral and e.g. *the man who John saw yesterday* is not obtained. This, and many other issues in relativisation will not be addressed; the specific concern here is to begin sensitisation of relativisation to *wh*-islands.

Szabolcsi (1987) observes that assignment of type ((NP\S)/NP)\(NP\S) to reflexives can yield a characterisation of reflexivisation:

(27)

$$\cfrac{\cfrac{\text{John}}{NP} \quad \cfrac{\cfrac{\text{likes}}{(NP\backslash S)/NP} \quad \cfrac{\text{himself}}{((NP\backslash S)/NP)\backslash(NP\backslash S)}}{NP\backslash S}\,\backslash E}{S}\,\backslash E$$

Szabolcsi notes that the reflexive should have the semantics of the combinator **W** (lambda term $\lambda x \lambda y[xyy]$) which takes a function and an argument and applies the function twice to the argument; Desclés, Guentchéva and Shaumyan (1986) also make this proposal. As a directional lambda term, **himself** will be $[y\,`\,(x\,'\,y)]y\lambda x\lambda$ so $y\,`\,(x$ ‘ **himself**) reduces to $y\,`\,(x\,'\,y)$ and the functional term derivation/meaning representation **John** ‘ (**likes** ‘ **himself**) for (27) simplifies to **John** ‘ (**likes** ’ **John**).

Conditionalisation allows for less trivial cases:

(28)

$$\cfrac{\cfrac{\text{John}}{NP} \quad \cfrac{\cfrac{\cfrac{\cfrac{\text{votes}}{(NP\backslash S)/PP} \quad \cfrac{\cfrac{\text{for}}{PP/NP} \quad [NP]^1}{PP}\,/E}{NP\backslash S}\,/E}{(NP\backslash S)/NP}\,/I^1 \quad \cfrac{\text{himself}}{((NP\backslash S)/NP)\backslash(NP\backslash S)}}{NP\backslash S}\,\backslash E}{S}\,\backslash E$$

(29) **John** ‘ (λx[**votes** ’ (**for** ’ x)] ‘ **himself**) $\triangleright$ **John** ‘ (**votes** ’ (**for** ’ **John**))

However, the current grammar also allows a reflexive to take an antece-

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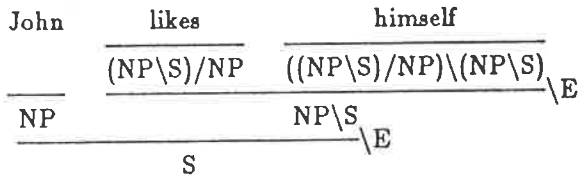

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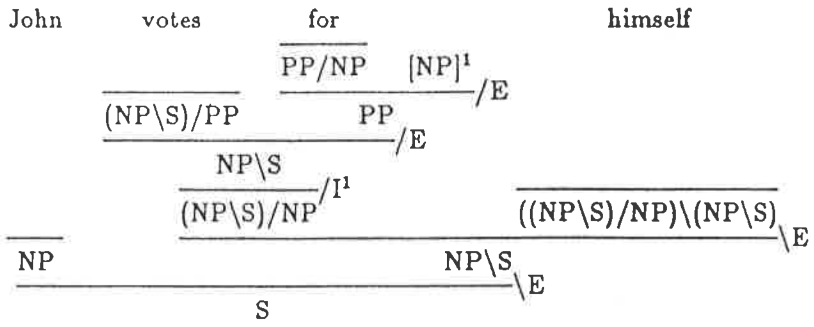
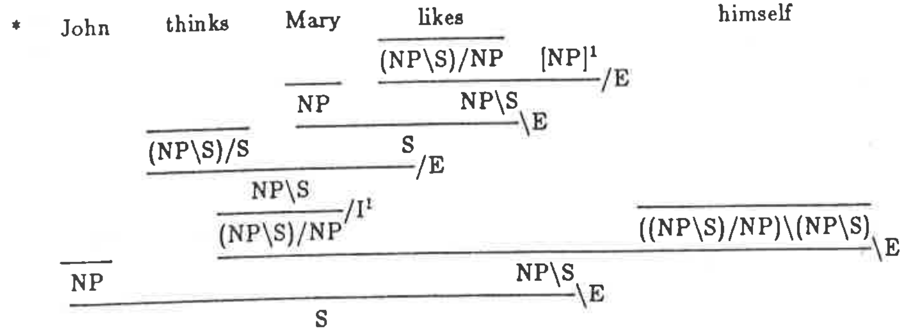


(29) **John** ‘ (λx[**votes** ’ (**for** ’ x)] ‘ **himself**) ▷ **John** ‘ (**votes** ’ (**for** ’ **John**))

dent from a superordinate clause:

(30)



(31) **John** ‘ (λx[**thinks** ’ (**Mary** ‘ (**likes** ’ x))] ‘ **himself**) ▷
John ‘ (**thinks** ’ (**Mary** ‘ (**likes** ’ **John**)))

Thus the grammar fails to capture the fact that the reflexive cannot take an antecedent from outside the tensed clause in which it occurs.

This paper will not address object-antecedent reflexivisation:

(32) Mary questions John about himself.

Although object-antecedent and subject-antecedent reflexives have the same forms in English, in e.g. Dutch, Icelandic, and Norwegian they are different (Popowich 1988). This indicates that subject oriented anaphora and object oriented anaphora do not constitute a single phenomenon; the latter is not discussed here and in fact two-complement verbs will be eschewed altogether: these areas are addressed in Szabolcsi (1987) but it is reflexivisation as an example of boundedness that is the present focus.

For similar reasons, nor will picture noun reflexives be addressed:

(33)a. John thinks Mary likes the picture of herself/*himself.
b. John likes Mary's picture of herself/*himself.

The standard view on English reflexives is that they must have a ‘local’, c-commanding antecedent. Pollard and Sag (1989) challenge this position. Although data such as (33) suggests that picture noun anaphors fit the scheme, further data suggests that they behave rather differently. Examples such as (34) have been taken to motivate expansion of local domains.

(34)a. John claims that the picture of himself disappeared.

Yet the antecedent of a picture noun reflexive need not even c-command the anaphor:

(35) The fact that there is a picture of himself hanging in the post office is believed (by Mary) to be disturbing Tom. (Jackendoff 1972, 4.123)

And Pollard and Sag observe that picture noun reflexives can go without any intrasentential antecedent at all:

(36)a. John was furious. The picture of himself in the museum had been mutilated.

b. John was devastated by the loss of his entire family. Now there was only himself.

The aim below is to produce an account of the boundedness of (subject antecedent) non-picture noun reflexives, and will not consider picture noun reflexives, these not being generally subject to locality conditions.

2. Intensional Categorial Grammar

The central idea of intensionality is that an expression may have different denotations depending on the possible world, point of reference, or INDEX with respect to which it is evaluated, and that the interpretation of an expression at some index may depend on the interpretation of subexpressions at other indices. Since the contribution of a subexpression to the interpretation of an expression in which it occurs may potentially depend on the interpretation of the subexpression at various indices, a compositional semantics requires that the meaning of an expression include its intension, i.e. the generalisation of its extensions across indices.

In Gallin's (1975) two-sorted theory of types Ty2, there is added to the sort e of individuals (and the sort t of truth values), a sort s of indices. Then the set of types, which are non-directional, are defined as follows (an arrow type-forming connective is used):

(37)a. e, t, and s are types.

b. If X and Y are types
then $X \rightarrow Y$ is a type.

In an intensional lambda calculus so typed, variables and terms may be of type s so that explicit reference to indices can be made. In Montague's

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(38)a. e and t are types.
b. If X and Y are types
then $X \rightarrow Y$ is a type.
c. If X is a type
then $s \rightarrow X$ is a type.

In IL, s is not a type, and no type has s as range. The idea is that no expression ever denotes or maps into an index: rather, interpretation takes place relative to indices. In an intensional lambda calculus typed in this way there is no explicit reference to indices; instead they appear as semantic parameters. Although type $s \rightarrow X$ can still be interpreted as the mappings from a domain of indices into the domain of type X, the uniform representation of extensional and intensional types fails to express the fact that in IL they exist on different levels. In the case of a *directional* type system, attempting to collapse extensional and intensional types forces a specification of directionality for indices: X/s or $s\backslash X$. But interpretation takes place *relative* to indices: there is no component of directionality involved since no expression denotes an index and therefore there is no application to an expression denoting an index.

So far as well-formed Montagovian intensional types are concerned, $s \rightarrow$ is an atomic 1-place intensional type constructor, because s alone is not a type. If it is represented as such, instead of as a composite of index domain and type connective, the distinction between intensional and extensional types is established, and directionality properties of the latter are not forced onto the former. In view of the analysis below where intensional types have the logic of universal modality, the unary operator will be written $\Box$. The clause (39) is added to the definition (2) of types for extensional categorial grammar:

(39) If X is a type
then $\Box X$ is a type.

The box connective will be assumed to bind more tightly than the binary slashes.

There is a sense in which intensionality can be trivially accommodated in a grammar by intensionalising on a massive scale. For instance, each primitive extensional type could be intensionalised so that say (NP\S)/NP, the type of a transitive verb like *likes*, becomes ($\Box$NP\$\Box$S)/$\Box$NP. Such an approach suffices for compositionality, but in doing everything, it does very little, because the redundant intensionalisation collapses extensional-

Benthem (1988b) contrasts such 'global' strategies with 'local' strategies, in which intensionality is introduced as and how it is empirically motivated.

Of similar character to such heavy-handed intensionalisation is the Motagovian strategy of 'generalising to the worst case'. Thus, rule-to-rule compositionality appears to require intensional and extensional transitive verbs to share the same type, since they share distribution, and this must be the intensional one because the extensional type will not yield the intensional readings. But then something further, usually meaning postulates, is required to eliminate the intensional readings otherwise predicted for extensional verbs under their intensional type-assignment. Partee and Rooth (1983) and Groenendijk and Stokhof (1984) argue on the basis of coordination and interrogation respectively that the worst case approach is in fact unworkable.

A priori, a strategy of MINIMAL TYPE ASSIGNMENT is the default. It is only for theory-internal reasons that alternatives might be entertained. We must thus assume here that minimal type assignment is the desideratum, without further discussion; for this the reader may consult, in addition to the references above, Partee (1986), van Benthem (1986), Groenendijk and Stokhof (1987), and Hendriks (1987). Some minimal intensionalisations are as follows:

(40)

for	:=	$\Box(PP/NP)$
John, Mary	:=	$\Box NP$
likes	:=	$\Box((NP\backslash S)/NP)$
man	:=	$\Box N$
the	:=	$\Box(NP/N)$
thinks	:=	$\Box((NP\backslash S)/\Box S)$
votes	:=	$\Box((NP\backslash S)/PP)$

The general strategy is to say that elements have extensions of their former extensional type, relative to indices. But in the case of the sentence-embedding verb, the sentential argument is of type $\Box S$, indicating that the verb expresses a relation between individuals and propositions, not individuals and truth values.

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spondence forms the basis for type theory, which extends the relation between logic and computation to include other connectives and operations. For example, conjunction introduction and elimination turn out to correspond to pairing and projection respectively, The proposal here is to interpret introduction and elimination for universal modality as intension and extension.The natural deduction rules for S4 modality are as follows:

$$\text{(41)a.}\quad \dfrac{\begin{array}{c}\vdots\\ X\end{array}}{\Box X}\,\Box\text{I} \qquad\qquad \text{b.}\quad \dfrac{\begin{array}{c}\vdots\\ \Box X\end{array}}{X}\,\Box\text{E}$$

Rule (41a) is subject to the condition that every path from the root to an undischarged assumption contains a licencing modal type, i.e. a type of the form $\Box Y$ which does not depend on a discharged assumption. The condition amounts to compositionality for intensional readings: if there is a derivation of an expression of meaning α in which the component parts are intensional, then that expression can be derived with meaning $\hat{}\alpha$. Rule (41b) states that an intensional reading is sufficient to determine an extensional one: if there is a derivation of an expression with an intensional meaning α, there is a derivation of that expression with its extensional reading $\check{}\alpha$.

As for implication, feeding the introduction rule into the elimination rule results in a reducible deduction.

$$\text{(42)}\quad \dfrac{\dfrac{\begin{array}{c}\vdots\\ X\end{array}}{\Box X}\,\Box\text{I}}{X}\,\Box\text{E} \quad \rhd \quad \begin{array}{c}\vdots\\ X\end{array}$$

While the reduction for implication was beta-reduction in lambda calculus, this reduction for universal modality is the down-up cancellation of intensional applicative calculi.

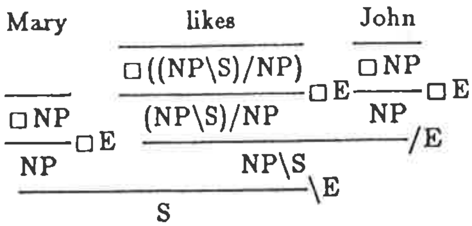
$$\text{(43)}\quad \check{}\hat{}\alpha \rhd \alpha$$

The sequent rules for S4 universal modality are as follows:

$$\text{(44)}\quad \begin{array}{c}\Box\Gamma \Rightarrow \Box X \quad [\Box R]\\[1ex] \Box\Gamma \Rightarrow X\end{array} \qquad\qquad \begin{array}{c}\Gamma(\Box X) \Rightarrow Y \quad [\Box L]\\[1ex] \Gamma(X) \Rightarrow Y\end{array}$$

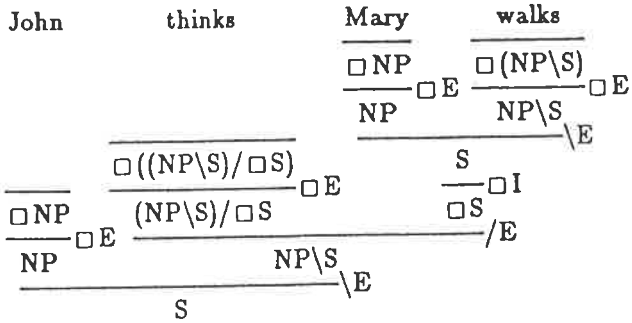
The $\Box$ prefix to a sequence indicates that each type must be modal. Intuitionistic S4 has cut elimination, and the Lambek calculus with S4

The derivation (45) is encoded (46); the unary extension and intension operators are assumed to bind more tightly than the binary applications.

(45) 

(46) ˇ**Mary** ‘ (ˇ**likes** ’ ˇ**John**)

The intensional logic meaning representation obtained here differs from that which would be obtained in Montague's IL in the following respect. In IL, constants of a given type are assigned meanings in the corresponding intensional domain, and the denotation of a constant is defined to be the result of applying this intension to the index of evaluation. Here, as in Ty2, a constant is assumed to take a meaning in its actual type domain, and if this is intensional, there must be explicit extensionalisation to obtain the denotation at the index of evaluation. In the case of Ty2 there would be explicit application of the constant to an index to obtain the denotation at that index: $^\vee\alpha$ here would translate as $\alpha'(i)$ in Ty2, where α' is the translation of α and i is the index of evaluation; $^\wedge\alpha$ would translate as $\lambda i\alpha'$ in Ty2. The present proposal only allows intensionalisation if at some level of derivation every component is intensional. The derivation (47) is encoded (48); the □I is permitted since every lexical type on which the subordinate S depends is modal.

(47) 

(48) ˇ**John** ‘ (ˇ**thinks** ’ ^(ˇ**Mary** ‘ ˇ**walks**))

As a further example, *Mary thinks John votes for the man* is now derived thus:

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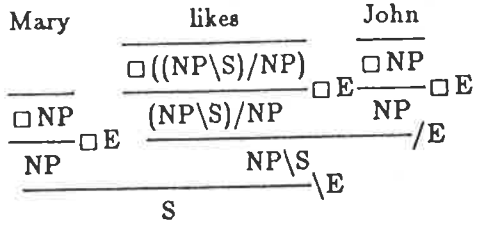
(45)


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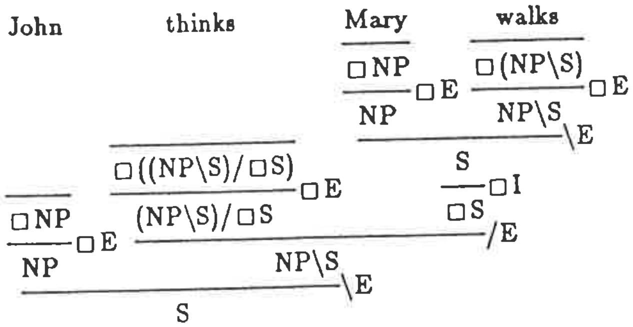

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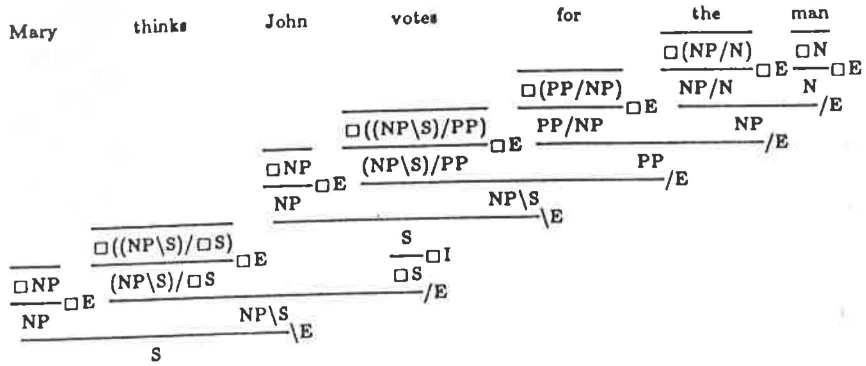
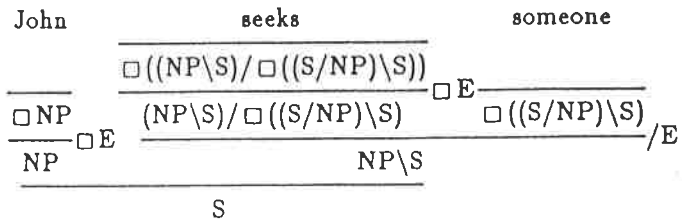


(50) **ˇMary ‘ (ˇthinks ’ ^(ˇJohn ‘ (ˇvotes ’ (ˇfor ’ (ˇthe ’ ˇman)))))**

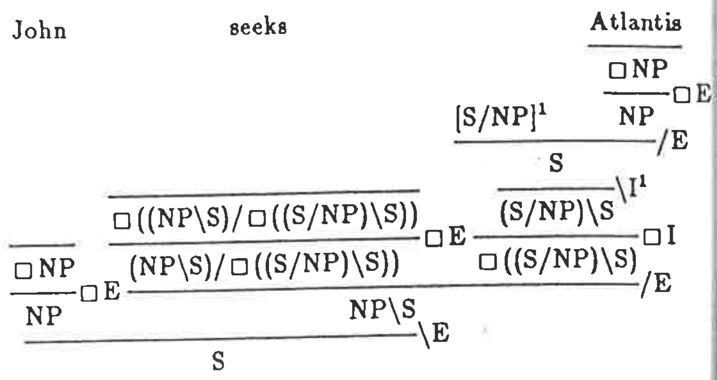
So far as intensional object verbs are concerned, Dowty, Wall and Peters (1981; n. 14 p. 250) observe that an object argument type $\Box((e \to t) \to t)$ is adequate for nonspecificity, e.g. the reading of *John seeks someone* where finding anyone would fulfill John's quest. The intensionality is required to distinguish between *John seeks a unicorn* and *John seeks a centaur* at an index where there is neither such creature.

(51)


(52) ˇJohn ‘ (ˇseeks ’ someone)

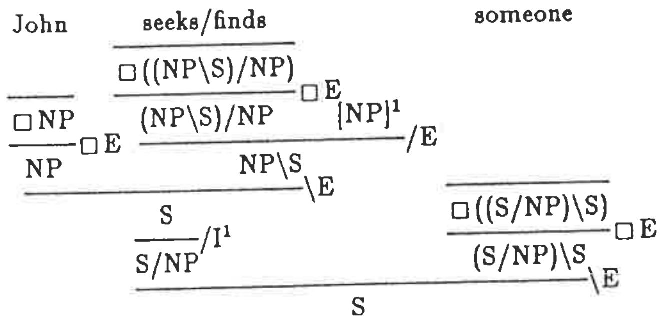
This also accommodates definite descriptions and names without referent at the index of evaluation. Assuming there is some null extension at the non-referring indices, the extension of a name will belong to various sets at referring indices, and no sets at non-referring indices. Note that in the following the □I is permitted because the only undischarged assumption

(53)



(54) ˇ**John** ‘(ˇ**seeks** ’ ˆλx[x ’ ˇ**Atlantis**])

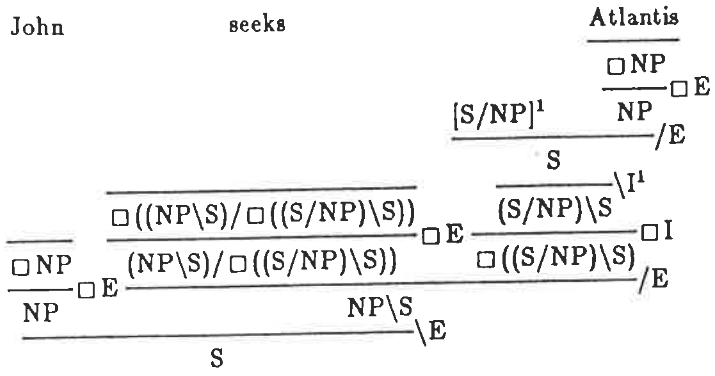
For specific readings of such verbs, i.e. readings entailing the existen of an object entity, it might be hoped that the laws of type combinati would yield the extensional reading from the same type that offers t intensional one. However this appears to require the fulfillment of essentially intensional role by an extensional element and should not possible (the proposal of Morrill 1989a is thus in error). For exam $e \Rightarrow \Box((e \rightarrow t) \rightarrow t)$ is not valid. It seems necessary then to assume for intensional object transitive verb an additional ordinary extensional lexi entry of type □((NP\S)/NP); this will deliver the specific reading of *Jo seeks someone* in the same way as in *John finds someone*.



(55) λx[ˇJohn ‘ (ˇseeks/finds ’ x)] ‘ ˇsomeone

Once the types by which expressions are classified are made sensitiv intensionality, different intensional type assignments to relative prono reflexives, sentence-embedding elements, etc. will result in different

(53)



(54) $^{\vee}$**John** ‘ ($^{\vee}$**seeks** ’ $^{\wedge}\lambda x[x$ ’ $^{\vee}$**Atlantis**])

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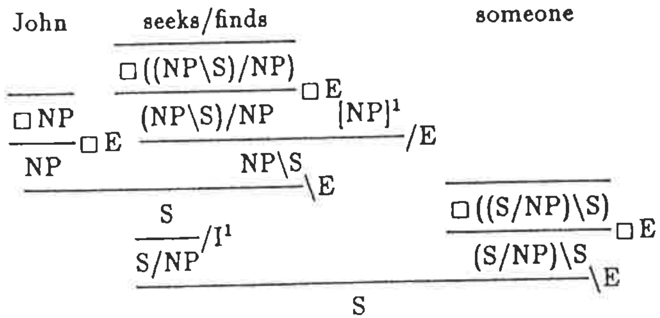
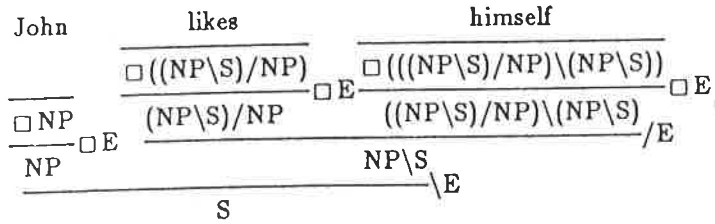


(55) $\lambda x[^{\vee}$**John** ‘ ($^{\vee}$**seeks/finds** ’ x)] ‘ $^{\vee}$**someone**

Once the types by which expressions are classified are made sensitive to intensionality, different intensional type assignments to relative pronouns,

pressions being well-formed or not. In Section 1 the extensional treatment of reflexives failed to observe clause boundaries, because there was no distinction in the types of a sentence missing an object in its own clause or missing an object in an embedded clause. In the present intensional grammar, assignment of type $\Box(((NP\backslash S)/NP)\backslash(NP\backslash S))$ to reflexives will render reflexivisation clause-bounded. A minimal example is derived as follows:

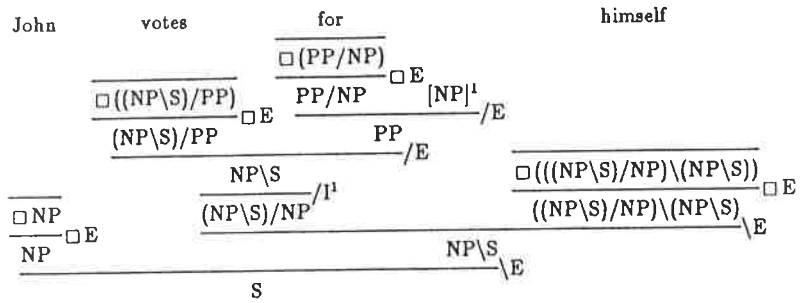
(56)



(57) $^{\vee}$**John** ‘ ($^{\vee}$**likes** ‘ $^{\vee}$**himself**)

A more complex example is still possible:

(58)



(59) $^{\vee}$**John** ‘ (λx[$^{\vee}$**votes** ’ ($^{\vee}$**for** ’ x)] ‘ $^{\vee}$**himself**)

However, reflexivisation across clause boundaries is not obtained. Consider (60), where types include gender in order to block the clause-local reading.

(60)

* Bill	thinks	Mary	likes	himself
$\Box NPm$	$\Box((NPm\backslash S)/\Box S)$	$\Box NPf$	$\Box((NPf\backslash S)/NPm)$	$\Box(((NPm\backslash S)/NPm)\backslash(NPm\backslash S))$

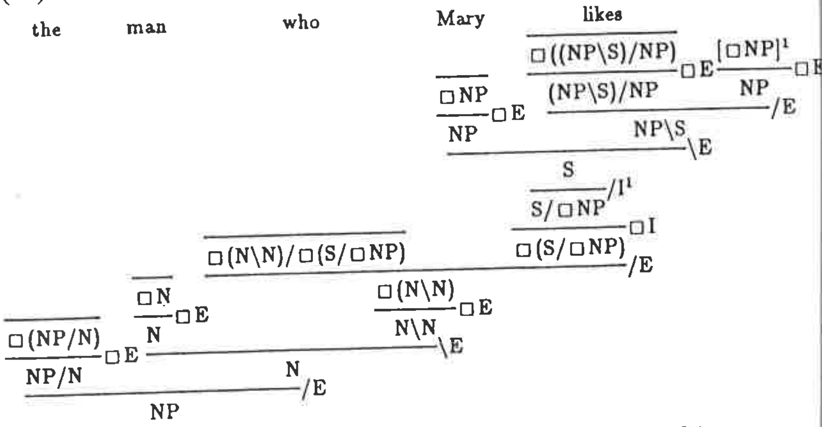
There is no derivation to S. This can be understood intuitively as follows. In order to analyse the embedded sentence as intensional (as dictated by otherwise the path to this leaf would not contain a modal type, preventing □I. But then conditionalising this assumption yields (NPm\S)/□NPm, instead of (NPm\S)/NPm which is the argument type of *himself*. More concretely, using the cut-free sequent formulation as a decision procedure for LS4 would show that (61) is not a theorem.

(61)
□NPm □((NPm\S)/□S) □NPf □((NPf\S)/NPm) □(((NPm\S)/NPm)\(NPm\S)) ⇒ S

Thus the intensional grammar fragment respects clause-boundedness of reflexivisation by forbidding reflexivisation to pass out of an intensional domain. In the pragmatically awkward *John seeks himself* it must be assumed that the extensional type of ordinary transitive verbs is operative.

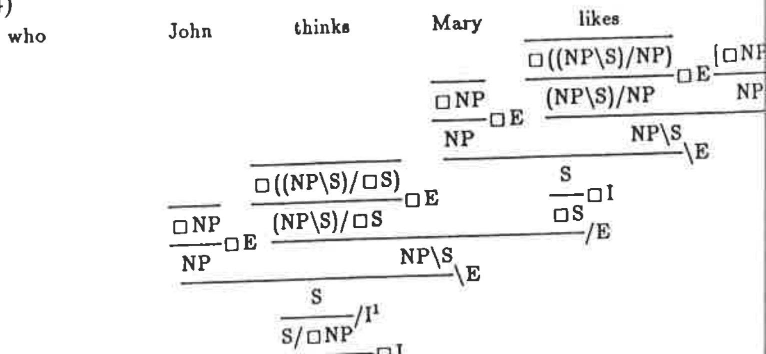
Assignment of a type □(N\N)/□(S/□NP) to an object relative pronoun will allow both local and long-distance relativisation.

(62)



(63) ˇthe ‘ (ˇman ‘ ˇ(who ’ ˆλx[ˇMary ‘ (ˇlikes ’ ˇx)]))

(64)



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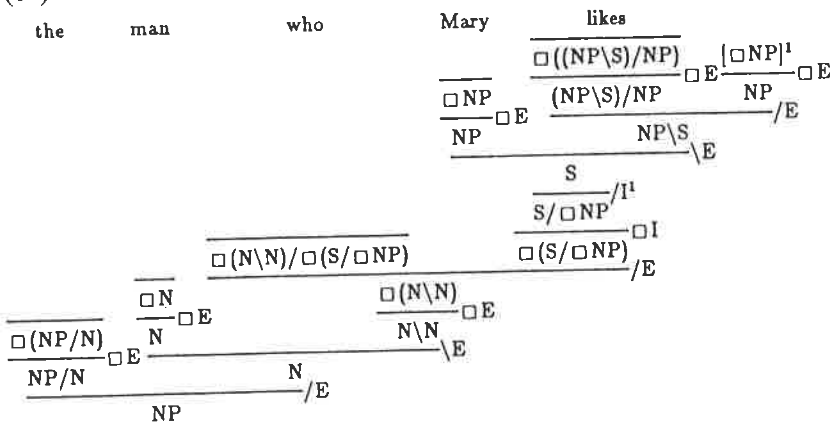

(63) ˇthe ‘ (ˇman ‘ ˇ(who ’ ˆλx[ˇMary ‘ (ˇlikes ’ ˇx)]))

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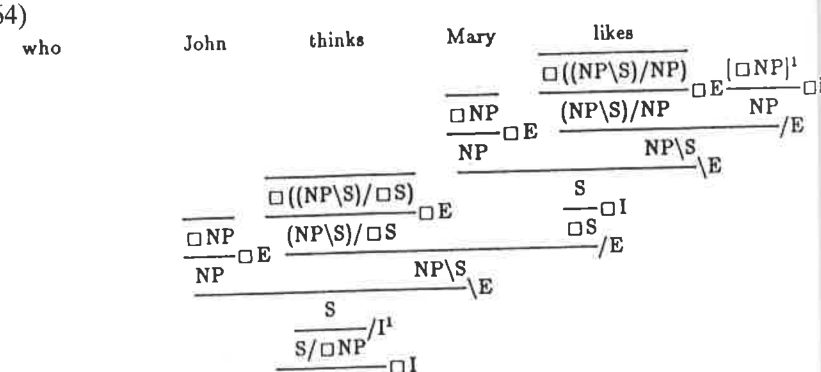


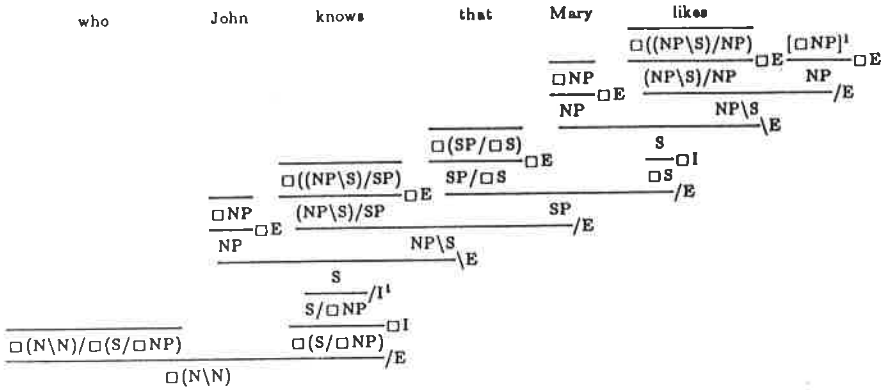
(65) who ’ ˆλx[ˇJohn ‘ (ˇthinks ’ ˆ(ˇMary ‘ (ˇlikes ’ ˇx)))]

Recall however the following contrast:

(66) the man who John knows that/*whether Mary likes

The difference must be obtained in the complementiser type assignment. Assuming the embedded sentence is intensional the following two options are available. A complementiser could have an intensional type □(SP/□S) where the embedded intensional domain projected from the lexical type is nested within the superordinate one, or an extensional type □SP/□S where the embedded intensional domain is juxtaposed with the superordinate one. The effect of assigning a relative pronoun type □(N\N)/□(S/□NP) and complementisers *that* and *whether* types □(SP/□S) and □SP/□S respectively will allow extraction past only the former. Extraction past the latter would only be possible if a relative pronoun sought an argument such as □S/□NP instead of □(S/□NP). Relativisation out of a *that*-embedded clause is illustrated in (67).

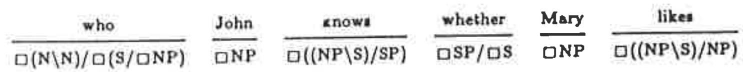
(67)



(68) who ’ ˆλx[ˇJohn ‘ (ˇknows ’ (ˇthat ’ ˆ(ˇMary ‘ (ˇlikes ’ ˇx))))]

When the superordinate □I takes place in (67), the □S of the embedded sentence cannot be the licencing type for the elements of the subordinate sentence because it depends on a discharged assumption. However, since all the lexical types are modal, these licence the intensionalisation. But consider (69).

(69) *



will mean that the path from it is not licenced for $\Box$I. Use of the cut-free sequent decision procedure will verify that (70) is not a theorem

(70)

$$\Box(N\backslash N)/\Box(S/\Box NP)\ \Box NP\ \Box((NP\backslash S)/SP)\ \Box SP/\Box S\ \Box NP\ \Box((NP\backslash S)/NP) \Rightarrow \Box(N\backslash N)$$

Note that relative pronouns themselves have been assigned extensional types, so that *wh*-island violations such as (71) will not be obtained either.

(71) *the man who John likes the woman who loves

This concludes the sketch of how intensional domains may provide an appropriate term of reference for boundedness facts. The rigidity of the above account with respect to word order is perhaps the most obvious issue to be addressed. A proposal here is to manage order in a non-commutative calculus by adding operators controlling permutation – EXCHANGE EXPONENTIALS – just as linear logic adds contract and weaken exponentials to control contraction and weakening; in fact the latter, and a whole additional range of connectives meeting the Curry–Howard isomorphism, are also relevant to grammar (Morrill 1989b, 1990). Aside from natural language applications, the technical proposal of this paper has been that there is a correspondence between S4 universal modality and the operations of intension and extension, which is of interest in its own right. This may be expected to find other applications, e.g. in the intensional analysis of programming languages (Janssen 1983). Morrill (1989a) presented intensionalisation according to the minimal modal logic K, which forms a strict subset of S4, and the appendix to this paper shows the equivalence of that formulation with an independent proposal for intensional categorial grammar.

APPENDIX

The sequent rule for the minimal modal logic K is as follows:

(72)
$$\frac{X_1 \ldots X_n \Rightarrow X_0}{\Box X_1 \ldots \Box X_n \Rightarrow \Box X_0}\ [\Box LR]$$

Morrill (1989a) presents categorial grammar with this logic of intensional types. Independently, Prijatelj (1989) proposes intensionalisation by addition to the Lambek–van Benthem calculus (73) a rule of contractio will mean that the path from it is not licenced for □I. Use of the cut-free sequent decision procedure will verify that (70) is not a theorem

(70)

$\Box(N\backslash N)/\Box(S/\Box NP)\ \Box NP\ \Box((NP\backslash S)/SP)\ \Box SP/\Box S\ \Box NP\ \Box((NP\backslash S)/NP) \Rightarrow \Box(N\backslash N)$

Note that relative pronouns themselves have been assigned extensional types, so that *wh*-island violations such as (71) will not be obtained either.

(71) *the man who John likes the woman who loves

This concludes the sketch of how intensional domains may provide an appropriate term of reference for boundedness facts. The rigidity of the above account with respect to word order is perhaps the most obvious issue to be addressed. A proposal here is to manage order in a non-commutative calculus by adding operators controlling permutation – EXCHANGE EXPONENTIALS – just as linear logic adds contract and weaken exponentials to control contraction and weakening; in fact the latter, and a whole additional range of connectives meeting the Curry–Howard isomorphism, are also relevant to grammar (Morrill 1989b, 1990). Aside from natural language applications, the technical proposal of this paper has been that there is a correspondence between S4 universal modality and the operations of intension and extension, which is of interest in its own right. This may be expected to find other applications, e.g. in the intensional analysis of programming languages (Janssen 1983). Morrill (1989a) presented intensionalisation according to the minimal modal logic K, which forms a strict subset of S4, and the appendix to this paper shows the equivalence of that formulation with an independent proposal for intensional categorial grammar.

APPENDIX

The sequent rule for the minimal modal logic K is as follows:

(72) $\Box X_1 \ldots \Box X_n \Rightarrow \Box X_0$ [□LR]

$X_1 \ldots X_n \Rightarrow X_0$

Morrill (1989a) presents categorial grammar with this logic of intensional types. Independently, Prijatelj (1989) proposes intensionalisation by addition to the Lambek–van Benthem calculus (73) a rule of contraction

(73) $X \Rightarrow X$

$\Delta(\Gamma) \Rightarrow Y$ [CUT]

$\Gamma \Rightarrow X$

$\Delta(X) \Rightarrow Y$

$\Gamma \Rightarrow X \rightarrow Y$ [→R]

$\Gamma X \Rightarrow Y$

$\Gamma(Y \rightarrow X\Delta) \Rightarrow Z$ [→L]

$\Delta \Rightarrow Y$

$\Gamma(X) \Rightarrow Z$

$\Gamma(X\ Y) \Rightarrow Z$ [P]

$\Gamma(Y\ X) \Rightarrow Z$

(74) $X_1 \ldots X_n\ s \Rightarrow X_0$ [C_s]

$X_1 \ldots X_n\ s\ s \Rightarrow X_0$

This system, obtained by augmenting LP with C_s will be called LPC_s. It may be compared to the system LPK_s obtained by augmenting LP with the K sequent rule, reverting to the notation $s \rightarrow X$ for $\Box X$ (cf. van Benthem 1986, p. 149):

(75) $s \rightarrow X_1 \ldots s \rightarrow X_n \Rightarrow s \rightarrow X_0$ [K_s]

$X_1 \ldots X_n \Rightarrow X_0$

In the case of the implicational fragment, a distinction was noted between systems allowing and disallowing empty left-hand sides. In the context of modal logic, a distinction is made between NORMAL systems, which allow the left-hand side of the modal sequent rule to be empty, and REGULAR systems, in which the left-hand side must be non-empty (Bull and Segerberg 1984). We will continue to use * and $^+$ to indicate the systems which allow and disallow empty left-hand sides. Johan van Benthem (p.c.) has

THEOREM. A sequent is a theorem of LPC^*_s if and only if it is a theor
of LPK^*_s

Proof. First we show that K_s is derivable in LPC^*_s:

(76)

$$s \rightarrow X_1 \quad \ldots \quad s \rightarrow X_n \Rightarrow s \rightarrow X_0 \qquad [\rightarrow R]$$

$$s \rightarrow X_1 \quad \ldots \quad s \rightarrow X_n \;\; s \Rightarrow X_0 \qquad [C_s]$$

$$\vdots \qquad n - 1 \text{ times}$$

$$s \rightarrow X_1 \quad \ldots \quad s \rightarrow X_n \;\; s \ldots s \Rightarrow X_0 \qquad [P]$$

$$\vdots \qquad \textstyle\sum_{i=0}^{n} i \text{ tim}$$

$$s \rightarrow X_1 \; s \quad \ldots \quad s \rightarrow X_n \; s \Rightarrow X_0 \qquad [\rightarrow L]$$

$$\vdots \qquad n \text{ times}$$

$$s \Rightarrow s \qquad [Ax]$$

$$X_1 \quad \ldots \quad X_n \Rightarrow X_0 \qquad \textit{Ass.}$$

Second, it is shown that C_s is derivable in LPK^*_s. The following auxil
rule, derived in (78), is used.

(77) $\Gamma\ \Delta \Rightarrow X$ [MP]

$$\Gamma \Rightarrow Y \rightarrow X$$

$$\Delta \Rightarrow Y$$

(78) $\Gamma\ \Delta \Rightarrow X$ [CUT]

$$\Gamma \Rightarrow Y \rightarrow X \qquad \text{Ass.}$$

$$Y \rightarrow X\ \Delta \Rightarrow X \qquad [\text{CUT}]$$

$$\Delta \Rightarrow Y \qquad \text{Ass.}$$

$$Y \rightarrow X\ Y \Rightarrow X \qquad [\rightarrow L]$$

$$Y \Rightarrow Y \qquad [\text{Ax}]$$

THEOREM. A sequent is a theorem of LPC^*_s if and only if it is a theorem of LPK^*_s

Proof. First we show that K_s is derivable in LPC^*_s:

(76)

$s \to X_1 \ \ldots \ s \to X_n \Rightarrow s \to X_0$ [→ R]

$\quad s \to X_1 \ \ldots \ s \to X_n \ s \Rightarrow X_0$ [C_s]

$\quad\quad \vdots$ $n - 1$ times

$\quad\quad s \to X_1 \ \ldots \ s \to X_n \ s \ldots s \Rightarrow X_0$ [P]

$\quad\quad\quad \vdots$ $\sum_{i=0}^{n} i$ times

$\quad\quad\quad s \to X_1 \ s \ \ldots \ s \to X_n \ s \Rightarrow X_0$ [→ L]

$\quad\quad\quad\quad \vdots$ n times

$\quad\quad\quad\quad s \Rightarrow s$ [Ax]

$\quad\quad\quad\quad X_1 \ \ldots \ X_n \Rightarrow X_0$ Ass.

Second, it is shown that C_s is derivable in LPK^*_s. The following auxiliary rule, derived in (78), is used.

(77) $\Gamma \ \Delta \Rightarrow X$ [MP]

$\quad \Gamma \Rightarrow Y \to X$

$\quad \Delta \Rightarrow Y$

(78) $\Gamma \ \Delta \Rightarrow X$ [CUT]

$\quad \Gamma \Rightarrow Y \to X$ Ass.

$\quad Y \to X \ \Delta \Rightarrow X$ [CUT]

$\quad\quad \Delta \Rightarrow Y$ Ass.

$\quad\quad Y \to X \ Y \Rightarrow X$ [→ L]

$\quad\quad\quad Y \Rightarrow Y$ [Ax]

The output scheme of C_s is derivable in LPK_s on assumption of its input scheme:

(79)

$X_1 \ \ldots \ X_n \ s \Rightarrow X_0$ [MP]

$\quad X_1 \ \ldots \ X_n \Rightarrow s \to X_0$ [CUT]

$\quad\quad X_1 \ \ldots \ X_n \Rightarrow s \to (s \to X_0)$ [→ R]

$\quad\quad\quad X_1 \ \ldots \ X_n \ s \Rightarrow s \Rightarrow X_0$ [→ R]

$\quad\quad\quad\quad X_1 \ \ldots \ X_n \ s \ s \Rightarrow X_0$ Ass.

$\quad\quad s \to (s \to X_0) \Rightarrow s \to X_0$ [MP]

$\quad\quad\quad s \to (s \to X_0) \Rightarrow (s \to s) \to (s \to X_0)$ [→ R]

$\quad\quad\quad\quad s \to (s \to X_0) \ s \to s \Rightarrow s \to X_0$ [K_s]

$\quad\quad\quad\quad\quad s \to X_0 \ s \Rightarrow X_0$ [→ L]

$\quad\quad\quad\quad\quad\quad s \Rightarrow s$ [Ax]

$\quad\quad\quad\quad\quad\quad X_0 \Rightarrow X_0$ [Ax]

$\quad\quad\quad \Rightarrow s \to s$ [→ R]

$\quad\quad\quad\quad s \Rightarrow s$ [Ax]

$\quad s \Rightarrow s$ [Ax]

The respect in which K differs from S4 follows from the absence in the former of the following axioms, T and 4 respectively:

(80)a. $\Box X \to X$

b. $\Box X \to \Box\Box X$

Nevertheless these seem to be appropriate to Montague's conception of intention. T indicates that an intension determines its extension, and 4 indicates that an intension determines its own intension: this is defined to

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