GLYN MORRILL AND BOB CARPENTER

COMPOSITIONALITY, IMPLICATIONAL LOGICS, AND THEORIES OF GRAMMAR*

Consider the following ungrammatical sentences:

(1) *John says
(2) *John laughed Mary

Example (1) can be described as a case of 'missing-word' ungrammaticality – the verb's complement is missing; example (2) can be described as a case of 'redundant-word' ungrammaticality – the second proper name is superfluous. In this paper we discuss how a principle of compositionality, i.e., a regime for building up meanings of expressions out of the meanings of their parts, can rule out such ungrammaticality, independently of a theory of syntax.

The principle of compositionality (see e.g., Janssen 1983, Chapter I; Partee 1984) usually takes the following form:

(3) *Strong Compositionality*
The meaning of an expression is a function of the meanings of its immediate syntactic subexpressions, and their mode of combination.

For this to be contentful it seems necessary to understand 'function' deterministically in the sense that given submeanings and a mode of combination, there can be only one result meaning. Thus we can associate with each 'mode of combination' (rule) a mathematical function which maps the meanings of subexpressions into the meanings of the expressions formed by the combination. Under strong compositionality, all (non-lexical) ambiguity is formalised as syntactic ambiguity; this is the characteristic feature of Montague semantics (Montague 1970).

According to strong compositionality, the meaning of a sentence is a function of the meanings of its immediate subexpressions and their mode of combination, which are in turn functions of the meanings of their immediate subexpressions and their modes of combination, and so on. It

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follows that ultimately the meaning of a sentence is uniquely determined by the meanings of its words, and the configured modes of combination by which it is built – the 'derivation'. Thus we can associate with each derivation a mathematical function which maps the meanings of words into the meanings of the sentences formed by the derivation, so that strong compositionality has the following corollary:

(4) *Weak Compositionality*
The meaning of a sentence is a function of the meanings of its words, and their configured modes of combination.

Weak compositionality is not committed to the association of meanings with intermediate expressions, or to the keying of semantic analysis on syntactic structure. In what follows we construe compositionality in the weak sense; the claims we make about it have accordingly wider applicability.

We shall propose a refined version of weak compositionality in which the class of functions available to map word meanings into sentence meanings is delimited, and we shall show how the principle ensures effects like those of Lexical-Functional Grammar's completeness and coherence conditions, and Government-Binding's θ-criterion. We do this by exploiting the fact that in order for one of the functions available to be a mapping from the meanings of some words into the meaning of a sentence, there must be available a function of a type mapping from the types of the word meanings into the type of the sentence meanings. Not all function types will be available given our specific formulations of compositionality; by construing types as formulae of implicational logic, we show that typehood is equivalent to theoremhood, and by proving non-validity, we prove that certain kinds of ill-formed sentences could never be generated by grammars respecting what we call λI-compositionality.

We identify classes of functions by reference to the pure typed λ-Calculus and Combinatory Logic. 'Pure' means that we have no constants; functions of 'type' $A \to B$ map from objects of type A into objects of type B. A non-empty set Δ of *basic types* defines a set of *types* as follows (here and throughout the classes defined are the smallest ones satisfying the specified conditions):

(5)a If $A \in \Delta$
then A is a type

b If A and B are types
then $A \to B$ is a type

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We shall propose a refined version of weak compositionality in which the class of functions available to map word meanings into sentence meanings is delimited, and we shall show how the principle ensures effects like those of Lexical-Functional Grammar's completeness and coherence conditions, and Government-Binding's θ-criterion. We do this by exploiting the fact that in order for one of the functions available to be a mapping from the meanings of some words into the meaning of a sentence, there must be available a function of a type mapping from the types of the word meanings into the type of the sentence meanings. Not all function types will be available given our specific formulations of compositionality; by construing types as formulae of implicational logic, we show that typehood is equivalent to theoremhood, and by proving non-validity, we prove that certain kinds of ill-formed sentences could never be generated by grammars respecting what we call λI-compositionality.

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of noun phrases, sentences, and sentences with complementizers. Then the set of types will include $NP \to S$, $SP \to (NP \to S)$, $NP \to ((NP \to S) \to S)$, and $NP \to (NP \to ((NP \to S) \to S))$. The arrow '$\to$' will be used right-associatively so that, for example, this last formula may be written $NP \to NP \to (NP \to S) \to S$. Given an infinite set Var_A of variables for each type A, the set of λK-*terms* is defined by:

(6)a If $v_A \in Var_A$
then v_A is a λK-term of type A

b If ϕ is a λK-term of type $A \to B$ and ψ is a λK-term of type A
then $\phi\psi$ is a λK-term of type B

c If $v_A \in Var_A$ and ϕ is a λK-term of type B
then $\lambda v_A \phi$ is a λK-term of type $A \to B$

A λK-term without any free variables is said to be *closed*. Assuming the standard functional interpretation, we will call the functions definable by closed λK-terms the λK-*functions*. Then one version of weak compositionality is:

(7) λK-*Compositionality*
The meanings of sentences are λK-functions of the meanings of their words.

The λK-functions are closed under permutation in the sense that if there is a function mapping certain arguments into a certain result, then there is a function mapping any permutation of those arguments into the same result: the different functions are defined by terms in which the λ-bindings appear in different orders. We are free, then, to adopt the convention that the functions mapping the meanings of words into the meanings of the sentences they form apply to the meanings of the words in left-to-right order. Words with meanings of types $A_1, A_2, \ldots$ can form a sentence, with meaning of type S, λK-*compositionally* only if $A_1 \to A_2 \to \ldots \to S$ is a λK-*type*, i.e., the type of some λK-function: if this is not a λK-type, then the expression as a whole cannot be assigned a meaning of type S by any λK-function of the meanings of its words.[1] For example, for it to be possible for words of type NP and $SP \to NP \to S$ to form a sentence, $NP \to (SP \to NP \to S) \to S$ must be a λK-type. Likewise, for it to be possible for words of type NP, $NP \to S$, and NP to form a sentence, $NP \to (NP \to S) \to NP \to S$ must be a λK-type.

In order to determine whether a type is a λK-type, we take advantage of the fact that a function is definable by a closed λK-term if and only if it is definable by a Combinatory Logic (CL) term, as follows:[2]

(8)a If A, B, and C are types,
$\mathbf{I}_{A\to A}(\equiv\lambda x_A[x_A])$
is a CL-term of type $A\to A$
$\mathbf{B}_{(A\to B)\to(C\to A)\to C\to B}(\equiv\lambda x_{A\to B}\lambda y_{C\to A}\lambda z_C[x_{A\to B}(y_{C\to A}z_C)])$
is a CL-term of type $(A\to B)\to(C\to A)\to C\to B$
$\mathbf{C}_{(A\to B\to C)\to B\to A\to C}(\equiv\lambda x_{A\to B\to C}\lambda y_B\lambda z_A[x_{A\to B\to C}z_Ay_B])$
is a CL-term of type $(A\to B\to C)\to B\to A\to C$
$\mathbf{W}_{(A\to A\to B)\to A\to B}(\equiv\lambda x_{A\to A\to B}\lambda y_A[x_{A\to A\to B}y_Ay_A])$
is a CL-term of type $(A\to A\to B)\to A\to B$
$\mathbf{K}_{A\to B\to A}(\equiv\lambda x_A\lambda y_B[x_A])$
is a CL-term of type $A\to B\to A$

b If ϕ is a CL-term of type $A\to B$ and ψ is a CL-term of type A then $\phi\psi$ is a CL-term of type B

Thus the λK-types are those types which are derivable from the axiom schemata (9a), corresponding to the combinators in (8a), and the *modus ponens* rule (9b), corresponding to the application in (8b).

(9)a $A\to A$
$(A\to B)\to(C\to A)\to C\to B$
$(A\to B\to C)\to B\to A\to C$
$(A\to A\to B)\to A\to B$
$A\to B\to A$

b $A\to B, A\vdash B$

Viewing '$\to$' as implication, (9) provides an axiomatisation of Heyting's implicational system, the implicational intuitionistic logic which Anderson and Belnap (1975) call $H_\to$.[3] So we know that a type A is a λK-type if and only if, regarded as an implicational formula, it is a theorem of $H_\to$.[4]

[2] See Curry and Feys (1958) or Barendregt (1981) for proofs of the equivalence. Often, CL definitions are given using the substitution combinator $\mathbf{S}$:

(i) $\mathbf{S}_{(A\to B\to C)\to(A\to B)\to A\to C}(\equiv\lambda x_{A\to B\to C}\lambda y_{A\to B}\lambda z_A[x_{A\to B\to C}z_A(y_{A\to B}z_A)])$

$\{\mathbf{S}, \mathbf{K}\}$ is equivalent to $\{\mathbf{I}, \mathbf{B}, \mathbf{C}, \mathbf{W}, \mathbf{K}\}$ as in the main text, which is in fact equivalent to $\{\mathbf{B}, \mathbf{C}, \mathbf{W}, \mathbf{K}\}$. The formulation given is convenient for our purposes.

[3] Their axiomatisation on p. 10 corresponds to $\{\mathbf{S}, \mathbf{K}, \mathbf{I}\}$ which is equivalent to $\{\mathbf{S}, \mathbf{K}\}$, $\{\mathbf{I}, \mathbf{B}, \mathbf{C}, \mathbf{W}, \mathbf{K}\}$, and $\{\mathbf{B}, \mathbf{C}, \mathbf{W}, \mathbf{K}\}$.

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Viewing '$\to$' as implication, (9) provides an axiomatisation of Heyting's implicational system, the implicational intuitionistic logic which Anderson and Belnap (1975) call $H_{\to}$.[3] So we know that a type A is a λK-type if and only if, regarded as an implicational formula, it is a theorem of $H_{\to}$.[4]

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(i) $((A \to B) \to A) \to A$

For example, assuming that the meaning of *John* is of type *NP*, and that the meaning of *says* is of type $SP \to NP \to S$, the string in (10a) could be generated λK-compositionally as a sentence only if (10b) is a theorem of $H_{\to}$.

(10)a *John says

b $NP \to (SP \to NP \to S) \to S$

We will prove that (10b) is not a theorem of $H_{\to}$ by exhibiting a counter-model.

Given a set P of proposition symbols, we define models for implicational logics in the manner prescribed by Urquhart (1972). A model for $H_{\to}$ is a quadruple $M = \langle L, \cup, \bot, \nu\rangle$ where $\langle L, \cup, \bot\rangle$ is a join semi-lattice with bottom element $\bot$,[5] and the valuation function ν is a function mapping from L into subsets of P, meeting the following condition:

(11) *Hereditary Condition*
For every $p \in P$ and all $i, j \in L$, if $p \in \nu(i)$ then $p \in \nu(i \cup j)$.

We refer to the elements of L as indices, and to $\cup$ as the least upper bound operation. Intuitively, the indices are information states, and least upper bound is the operation of combining information. The set of proposition symbols associated with an index by ν corresponds to the set of basic propositions which are true at the index. The hereditary condition entails that the set of true propositions increases monotonically as we move up the lattice.

For a model M we define a satisfaction relation $\models_M$ between indices and formulae by:

(12)a For every $p \in P$ and every $i \in L$, $i \models_M p$ if and only if $p \in \nu(i)$

b For all formulae ϕ, ψ, and every $i \in L$, $i \models_M \phi \to \psi$ if and only if for every $j \in L$, $j \models_M \phi$ only if $i \cup j \models_M \psi$

Thus an implicational formula is satisfied on the basis of the information at an index i if and only if for every index j which satisfies the antecedent, the consequent is satisfied by the information obtained by putting together that at i and j. A formula ϕ is *valid with respect to a model M* if and only

[5] A join semi-lattice $\langle L, \cup, \bot\rangle$ consists of a set L, with a distinguished element $\bot$, over which a binary operation $\cup$ is defined such that for all $i, j, k \in L$

(i) $i \cup i = i$

(ii) $i \cup j = j \cup i$

if it is satisfied at the bottom index. A formula is *valid* if and only if it is valid in every model.

Given these definitions we can now show that (10b) is not valid by exhibiting a counter-model – a model in which it is not valid. Consider the following model:

(13) $\langle\{\bot\}, \cup, \bot, \nu\rangle$
where $\nu(\bot) = \{NP\}$

A proof that this is a counter-model runs as follows. We are required to show

(14) $\bot \not\models NP \rightarrow (SP \rightarrow NP \rightarrow S) \rightarrow S$

Since $\bot \models NP$, (14) holds if

(15) $\bot \not\models (SP \rightarrow NP \rightarrow S) \rightarrow S$

And (15) holds if $\bot \not\models$ S (which is true by assumption) and

(16) $\bot \models SP \rightarrow NP \rightarrow S$

But (16) is true since no member of $\{\bot\}$ satisfies *SP*. Hence (14) is true and (10b) is not a theorem of $H_{\rightarrow}$ and not the type of any λK-function. So assuming the given assignment of word meanings to types, (10a) could not be generated by any λK-compositional grammar.

Within Lexical-Functional Grammar (LFG), sentences like (10a) in which an argument is missing are excluded by the completeness condition and within Government-Binding (GB), they are excluded by the θ-criterion (in conjunction with the projection principle, etc.):

(17) *Completeness*
An *f*-structure is *locally complete* if and only if it contains all the governable grammatical functions that its predicate governs. An *f*-structure is *complete* if and only if it and all its subsidiary *f*-structures are locally complete. (Kaplan and Bresnan 1982, pp. 211–12)

(18) *θ-Criterion*
Each argument bears one and only one θ-role, and each θ-role is assigned to one and only one argument. (Chomsky 1981, p. 36)

Completeness in LFG requires that *f*-structures contain the grammatical functions governed by predicates, for example they must contain the grammatical functions fulfilled by complements for which a verb is subcategorized. Completeness excludes sentences like **John says* because the f-if it is satisfied at the bottom index. A formula is *valid* if and only if it is valid in every model.

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structure would not contain the grammatical function governed by the predicate 'say'. GB's θ-criterion requires that a verb's θ-roles stand in a one-to-one relation with arguments present. The θ-criterion excludes sentences such as **John says* because the θ-role that should be filled by a sentential complement would not be assigned to any argument. The θ-criterion also excludes sentences like (19a) which contain a redundant argument. In LFG this is done by the coherence condition (20).

(19)a *John laughed Mary
b $NP \rightarrow (NP \rightarrow S) \rightarrow NP \rightarrow S$

(20) *Coherence*
An *f*-structure is *locally coherent* if and only if all the governable grammatical functions that it contains are governed by a local predicate. An *f*-structure is *coherent* if and only if it and all its subsidiary *f*-structures are locally coherent. (Kaplan and Bresnan 1982, p. 212)

Coherence excludes (19a) because the grammatical function fulfilled by *Mary* will not be governed, and the θ-criterion excludes the sentence because *Mary* will be assigned no θ-role. However λK-compositionality does not exclude such a sentence; for example the following λK-term designates a function of the requisite type $NP \rightarrow (NP \rightarrow S) \rightarrow NP \rightarrow S$:

(21) $\lambda x_{NP} \lambda y_{NP \rightarrow S} \lambda z_{NP}[y_{NP \rightarrow S} x_{NP}]$

What is distinctive about this function is that it engenders vacuous abstraction: z_{NP} does not appear in the body of the λK-term. We suggest that universal grammar does not admit vacuous functional abstraction.[6]

It would be odd for a grammar to afford vacuous abstraction: its significance would be that on occasion the meanings of words do not contribute to the meanings of the sentences in which they appear. Such redundancy of expression would be an unexpected feature in a system which evolved to facilitate communication. Potential counterexamples to our hypothesis include dummy subjects:

(22)a It seems that Mary left
b There is a party

However it is not the case that such examples can only be analysed by

[6] In relation to truth-functional connectives, Gazdar and Pullum (1976) suggest that every conjunct in a coordinate sentence must be potentially relevant to determining the truth value

vacuous abstraction; for example Sag (1982) provides an analysis in which *it* and *there* have the identity function as their lexical semantics.

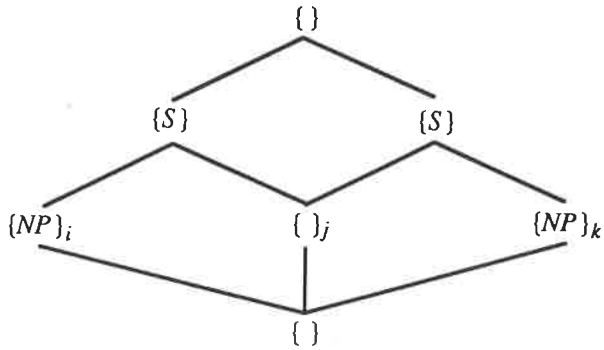
The suggestion, then, is a refined version of weak compositionality making reference to just the *λI-functions*, the functions definable by closed λK-terms without vacuous abstraction:

(23) *λI-Compositionality*
The meanings of sentences are λI-functions of the meanings of their words.

The combinator corresponding to vacuous abstraction is **K**. The functions definable by CL-terms as indicated earlier, except without **K**, are thc λI-functions.[7] The axioms corresponding to these remaining combinators, with *modus ponens*, are those of Church's weak theory of implication, the implicational relevance logic which Anderson and Belnap call $R_{\rightarrow}$.[8]

We can prove that (19b) is not a theorem of $R_{\rightarrow}$, and hence prove that sentences like (19a) cannot be generated λI-compositionally. A model for $R_{\rightarrow}$ is just like a model for $H_{\rightarrow}$ except that v is not required to meet the hereditary condition. A counter-model for (19b) is represented by the following diagram:

(24)



Each node corresponds to an index and shows its associated image under v. The least upper bound of any two indices is the lowest index that dominates them both; for convenience of reference some of the nodes have been labeled with subscripts. To prove that (24) is a counter-model we need to show that

(25) $\perp \nVdash NP \rightarrow (NP \rightarrow S) \rightarrow NP \rightarrow S$

This is true if for some index x, the antecedent is satisfied at x while the

[7] Again, See Curry and Feys (1958) or Barendregt (1981).
[8] See especially their axiomatisation $R_{\rightarrow 1}$ of $R_{\rightarrow}$ on p. 88.

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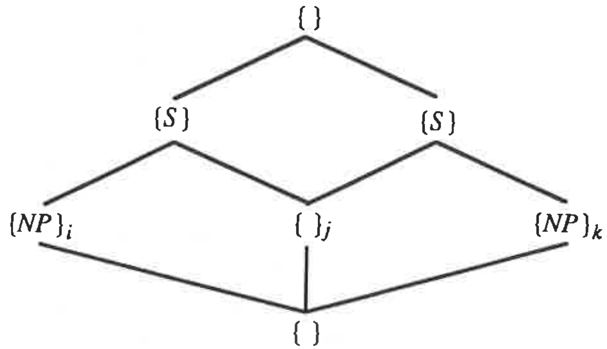

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consequent is not satisfied at $x \cup \perp$ $(=x)$. In particular, (25) is true if

(26) $i \vDash NP$ and $i \vDash (NP \to S) \to NP \to S$

Since $i \vDash NP$ (by assumption), the value of (26) coincides with its right hand conjunct, which holds if there is an index which satisfies $NP \to S$ but whose least upper bound with i fails to satisfy $NP \to S$, as in

(27) $j \vDash NP \to S$ and $i \cup j \nvDash NP \to S$

But the left hand conjunct of (27) is true because it *is* the case that for every index satisfying NP, the least upper bound of that index and j satisfies S. The right hand side is true because while the index l satisfies NP, the least upper bound of this index and $k \cup j$ does not satisfy S. This completes the proof.

In this paper we have attempted to provide evidence that the perspective of compositionality offers a sound and useful linguistic methodology. Operating according to this methodology, we have shown how a specific hypothesis, λI-compositionality, captures missing-word and redundant-word ungrammaticality. Under type-driven interpretation (such as forms the basis of the Semantic Interpretation Schema of Generalised Phrase Structure Grammar; see Gazdar, Klein, Pullum, and Sag 1985), the interpretation of combination is determined on the basis of the types of the daughter meanings. In its simplest form this is limited to functional application, and certainly falls within the regime of λI-compositionality. In the context of Combinatory Categorial Grammar, Steedman (1988) claims that the interpretation of combination must be extended to include functional composition **B** and substitution **S**, but not **K**, so that these proposals also adhere to λI-compositionality.

In fact λI-compositionality may be too strong a claim in that it admits only pure functions: the analysis of bare plurals might require the non-lexical introduction of quantification, and *that*-less relative clauses may or may not require non-lexical introduction of a conjunction operation to define the restriction of the head noun. λI-compositionality may also be too weak a claim in that it affords the full power of functional abstraction: it remains an interesting question whether compositionality can be narrowed down to a smaller class of functions. However we take it as encouraging for the methodological perspective of compositionality that even the rather crude hypothesis λI-compositionality marries up with proposals made in the context of phrase structure grammar and categorial grammar, and brings with it certain desirable effects like those of LFG's completeness and coherence conditions, and GB's θ-criterion.

References

Anderson, A. R. and N. D. Belnap: 1975, *Entailment*, Volume 1, Princeton University Press, Princeton, New Jersey.

Barendregt, H. P.: 1981, *The Lambda Calculus: Its Syntax and Semantics*, North-Holland, Amsterdam.

Chomsky, N.: 1981, *Lectures on Government and Binding*, Foris, Dordrecht.

Curry, H. B. and R. Feys: 1958, *Combinatory logic*, Volume 1, North-Holland, Amsterdam.

Gazdar, G. and G. K. Pullum: 1976, 'Truth-Functional Connectives in Natural Language', in S. S. Mufwene, C. A. Walker and S. B Steever (eds.), *Papers from the 12th Regional Meeting of the Chicago Linguistics Society*, Chicago, Illinois.

Gazdar, G., E. Klein, G. K. Pullum and I. Sag: 1985, *Generalized Phrase Structure Grammar*, Basil Blackwell, London.

Janssen, T. M. V.: 1983, *Foundations and Applications of Montague Grammar*, Ph.D. dissertation, Mathematisch Centrum, Universiteit van Amsterdam.

Kaplan, R. M. and J. Bresnan: 1982, 'Lexical-Functional Grammar: a Formal System for Grammatical Representation', in J. Bresnan (ed.), *The Mental Representation of Grammatical Relations*, MIT Press, Cambridge, Massachusetts.

Montague, R.: 1970, 'Universal Grammar', *Theoria* **36**, 373–98, reprinted in R. H. Thomason (ed.): 1974, *Formal Philosophy: Selected Papers of Richard Montague*, pp. 222–246, Yale University Press, New Haven.

Partee, B. H.: 1984, 'Compositionality', in F. Landman and F. Veltman (eds.), *Varieties of Formal Semantics: Proceedings of The Fourth Amsterdam Colloquium, 1982*, Foris, Dordrecht.

Sag, I.: 1982, 'A Semantic Theory of "NP-Movement" Dependencies', in P. Jacobson and G. K. Pullum (eds.), *The Nature of Syntactic Representation*, D. Reidel, Dordrecht, pp. 427–66.

Steedman, M.: 1988, 'Combinators and Grammars', in R. Oehrle, E. Bach and D. Wheeler (eds.), *Categorial Grammars and Natural Language Structures*, D. Reidel, Dordrecht. Paper presented at the Conference on Categorial Grammar, Tucson, June 1985.

Urquhart, A.: 1972, 'Semantics for Relevant Logics', *Journal of Symbolic Logic* **37**, 159–169.

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