Displacement Logic for Grammar

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Lecture 1

From Lambek Calculus to Displacement Calculus:
Natural Deduction

The challenge

A/the main challenge of natural grammar: syntax/semantics mismatch or *displacement*.

E.g. overt ‘movement’ such as relativisation:

(1) a. (the book) that$_i$ Mary reads e_i
$\lambda y \lambda z[(y\ z) \wedge ((\mathit{read}\ z)\ m)]$
b. the book that$_i$ John gave e_i to Mary
$\lambda y \lambda z[(y\ z) \wedge (((\mathit{give}\ z)\ m)\ j)]$

and covert ‘movement’ such as quantification:

(2) a. Mary reads every book
$\forall z[(\mathit{book}\ z) \rightarrow ((\mathit{read}\ z)\ m)]$
b. John gave every book to Mary
$\forall z[(\mathit{book}\ z) \rightarrow (((\mathit{give}\ z)\ m)\ j)]$

As we shall see now, the Lambek calculus **L** of Lambek (1958[1]) can handle the (peripheral) a. cases, but the (medial) b. cases require something like the displacement calculus **D** of Morrill, Valentín and Fadda (2011[2]).

Lambek Calculus **L** (with product unit)

Types:

$$
\begin{array}{lll}
\mathcal{F} ::= \mathcal{F}\backslash\mathcal{F} & T(A\backslash C) = T(A) \to T(C) & \text{under} \\
\mathcal{F} ::= \mathcal{F}/\mathcal{F} & T(C/B) = T(B) \to T(C) & \text{over} \\
\mathcal{F} ::= \mathcal{F}\bullet\mathcal{F} & T(A\bullet B) = T(A)\&T(B) & \text{product} \\
\mathcal{F} ::= I & T(I) = \top & \text{product unit}
\end{array}
$$

Approximate algebraic interpretation as subsets of the monoid $(L, +, 0)$ of strings over an alphabet:

$$
\begin{array}{rcl}
[[A\backslash C]] & = & \{s_2 |\ \forall s_1 \in [[A]], s_1 + s_2 \in [[C]]\} \\
[[C/B]] & = & \{s_1 |\ \forall s_2 \in [[B]], s_1 + s_2 \in [[C]]\} \\
[[A\bullet B]] & = & \{s_1 + s_2 |\ s_1 \in [[A]] \text{ and } s_2 \in [[B]]\} \\
[[I]] & = & \{0\}
\end{array}
$$

Natural Deduction for **L**

$$\dfrac{\begin{array}{c}\vdots\\ \alpha\colon A\colon \phi\end{array} \qquad \begin{array}{c}\vdots\\ \beta\colon A\backslash C\colon \psi\end{array}}{\alpha+\beta\colon C\colon (\psi\ \phi)}\ E\backslash \qquad\qquad \dfrac{\begin{array}{c}\overline{a\colon A\colon x}^{\ i}\\ \vdots\\ a+\beta\colon C\colon \psi\end{array}}{\beta\colon A\backslash C\colon \lambda x\psi}\ I\backslash^{i}$$

$$\dfrac{\begin{array}{c}\vdots\\ \alpha\colon C/B\colon \phi\end{array} \qquad \begin{array}{c}\vdots\\ \beta\colon B\colon \psi\end{array}}{\alpha+\beta\colon C\colon (\phi\ \psi)}\ E/ \qquad\qquad \dfrac{\begin{array}{c}\overline{b\colon B\colon y}^{\ i}\\ \vdots\\ \alpha+b\colon C\colon \psi\end{array}}{\alpha\colon C/B\colon \lambda x\psi}\ I/^{i}$$

$$\dfrac{\gamma\colon A{\bullet}B\colon \chi \qquad \begin{array}{c}\dfrac{}{a\colon A\colon x}\,i \quad \dfrac{}{b\colon B\colon y}\,i \\ \vdots \\ \delta(a{+}b)\colon D\colon \omega\end{array}}{\delta(\gamma)\colon D\colon \omega\{\pi_1\chi/x, \pi_2\chi/y\}}\ E{\bullet}^i$$

$$\dfrac{\begin{array}{c}\vdots \\ \alpha\colon A\colon \phi\end{array} \qquad \begin{array}{c}\vdots \\ \beta\colon B\colon \psi\end{array}}{\alpha{+}\beta\colon A{\bullet}B\colon (\phi, \psi)}\ I{\bullet}$$

$$\dfrac{\begin{array}{c}\vdots \\ \alpha\colon A\colon \phi\end{array} \qquad \begin{array}{c}\vdots \\ \beta\colon I\colon \psi\end{array}}{\alpha\colon A\colon \phi}\ EI \qquad\qquad \dfrac{}{0\colon I\colon 0}\ II$$

Example: *Mary reads the book* in **L**

$$\dfrac{\textbf{Mary}: N: m \qquad \dfrac{\textbf{reads}: (N\backslash S)/N: \mathit{read} \qquad \dfrac{\textbf{the}: N/CN: \iota \quad \textbf{book}: CN: \mathit{book}}{\textbf{the}+\textbf{book}: N: (\iota\ \mathit{book})}\, E/}{\textbf{reads}+\textbf{the}+\textbf{book}: N\backslash S: (\mathit{read}\ (\iota\ \mathit{book}))}\, E/}{\textbf{Mary}+\textbf{reads}+\textbf{the}+\textbf{book}: S: ((\mathit{read}\ (\iota\ \mathit{book}))\ m)}\, E\backslash$$

Example: *John gave the book to Mary* in **L**

$$
\dfrac{
\begin{array}{c}\textbf{John}\text{:}\\ N\text{:}\\ j\end{array}
\qquad
\dfrac{
\dfrac{
\begin{array}{c}\textbf{gives}\text{:}\\ ((N\backslash S)/PP)/N\text{:}\\ \mathit{give}\end{array}
\qquad
\begin{array}{c}\textbf{the}+\textbf{book}\text{:}\\ N\text{:}\\ (\iota\ \mathit{book})\end{array}
}{
\textbf{gives}+\textbf{the}+\textbf{book}\text{: } (N\backslash S)/PP\text{: } (\mathit{give}\ (\iota\ \mathit{book}))
}\,E/
\qquad
\dfrac{
\begin{array}{c}\textbf{to}\text{:}\\ PP/N\text{:}\\ \lambda xx\end{array}
\qquad
\begin{array}{c}\textbf{Mary}\text{:}\\ N\text{:}\\ m\end{array}
}{
\textbf{to}+\textbf{Mary}\text{: } PP\text{: } m
}\,E/
}{
\textbf{gives}+\textbf{the}+\textbf{book}+\textbf{to}+\textbf{Mary}\text{: } N\backslash S\text{: } ((\mathit{give}\ (\iota\ \mathit{book}))\ m)
}\,E/
}{
\textbf{John}+\textbf{gives}+\textbf{the}+\textbf{book}+\textbf{to}+\textbf{Mary}\text{: } S\text{: } (((\mathit{give}\ (\iota\ \mathit{book}))\ m)\ j)
}\,E\backslash
$$

Overt movement: *(the book) that Mary reads* in **L**

$$
\dfrac{
\begin{matrix}\textbf{that}: \\ (CN\backslash CN)/(S/N): \\ \lambda x\lambda y\lambda z[(y\ z)\wedge(x\ z)]\end{matrix}
\quad
\dfrac{\dfrac{\begin{matrix}\textbf{Mary}:\\ N:\\ m\end{matrix}\quad \dfrac{\begin{matrix}\textbf{reads}:\\ (N\backslash S)/N:\\ read\end{matrix}\quad \dfrac{}{\begin{matrix}b:\\ N:\\ y\end{matrix}}\,i}{\textbf{reads}+b: N\backslash S: (read\ y)}\,E/}{\textbf{Mary}+\textbf{reads}+b: S: ((read\ y)\ m)}\,E\backslash}{\textbf{Mary}+\textbf{reads}: S/N: \lambda y((read\ y)\ m)}\,I/^{i}
}{\textbf{that}+\textbf{Mary}+\textbf{reads}: S: \lambda y\lambda z[(y\ z)\wedge((read\ z)\ m)]}\,E/
$$

But medial overt movement:
(the book) that John gave to Mary in **L**

$$
\dfrac{\dfrac{\mathbf{John}: N: j \qquad \dfrac{\dfrac{\mathbf{gave}: ((N\backslash S)/PP)/N: \mathit{give} \qquad b: N: y}{\mathbf{gave}+b: (N\backslash S)/PP: (\mathit{give}\ y)}\,E/ \qquad \mathbf{to}+\mathbf{Mary}: PP: m}{\mathbf{gave}+b+\mathbf{to}+\mathbf{Mary}: N\backslash S: ((\mathit{give}\ y)\ b)}\,E/}{\mathbf{John}+\mathbf{gave}+b+\mathbf{to}+\mathbf{Mary}: S: (((\mathit{give}\ y)\ b)\ j)}\,E\backslash}{}\,{*}I/
$$

Covert movement: *Mary reads every book* in **L**

$$
\dfrac{\begin{array}{c}\textbf{Mary}+\textbf{reads}:\\ S/N:\\ \lambda y((read\ y)\ m)\end{array} \quad \dfrac{\begin{array}{c}\textbf{every}:\\ ((S/N)\backslash S)/CN\\ \lambda x\lambda y\forall z[(x\ z)\to(y\ z)]\end{array} \quad \begin{array}{c}\textbf{book}:\\ CN:\\ book\end{array}}{\textbf{every}+\textbf{book}: (S/N)\backslash S: \lambda y\forall z[(book\ z)\to(y\ z)]}\ E/}{\textbf{Mary}+\textbf{reads}+\textbf{every}+\textbf{book}: S: \forall z[(book\ z)\to((read\ z)\ m)]}\ E\backslash
$$

But medial covert movement: *John gave every book to Mary* in **L**

$$\frac{\textbf{(John) gave} \quad \textbf{every+book}: (S/N)\backslash S \quad \textbf{to+Mary}: PP}{}\,*$$

Displacement Calculus (**D**)

Discontinuous strings s are strings over an alphabet with a distinguished symbol 1 called the *separator*.

The *sort* $\sigma(s)$ of a discontinuous string s is the number of separators it contains.

We define the sets L_i of discontinuous strings of sort i by:

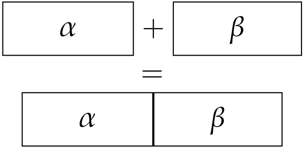
$$L_i = \{s |\ \sigma(s) = i\}$$

The continuous strings are L_0.

Operations on discontinuous strings

We consider the operations concatenation and intercalation on discontinuous strings. Concatenation is represented in (3).

(3)



concatenation $+ : L_i, L_j \rightarrow L_{i+j}$

For example, the concatenation of **Leslie**+1+**Sandy** and **and**+**Robin**+**Bill** is:

(4) **Leslie**+1+**Sandy** + **and**+**Robin**+**Bill**

=

Leslie+1+**Sandy**+**and**+**Robin**+**Bill**

Intercalation is represented in (5):

(5) $$\boxed{\quad\alpha\quad}\boxed{1}\boxed{\quad\gamma\quad} \times_k \boxed{\quad\beta\quad}$$
$$=$$
$$\boxed{\quad\alpha\quad}\boxed{\quad\beta\quad}\boxed{\quad\gamma\quad}$$

intercalation $\times_k : L_{i+1}, L_j \to L_{i+j}$

For example, the intercalation at the second separator of 1+**dogs**+1+**Whiskas**+**and**+**cats**+**Alpo** and **like** is:

(6) 1+**dogs**+1+**Whiskas**+**and**+**cats**+**Alpo** $\times_2$ **like**

=

1+**dogs**+**like**+**Whiskas**+**and**+**cats**+**Alpo**

Sorted types:

$\mathcal{F}_j ::= \mathcal{F}_i \backslash \mathcal{F}_{i+j}$	$T(A\backslash C) = T(A) \to T(C)$	under
$\mathcal{F}_i ::= \mathcal{F}_{i+j} / \mathcal{F}_j$	$T(C/B) = T(B) \to T(C)$	over
$\mathcal{F}_{i+j} ::= \mathcal{F}_i \bullet \mathcal{F}_j$	$T(A \bullet B) = T(A) \& T(B)$	product
$\mathcal{F}_0 ::= I$	$T(I) = \top$	product unit
$\mathcal{F}_j ::= \mathcal{F}_{i+1} \downarrow_k \mathcal{F}_{i+j}$	$T(A \downarrow_k C) = T(A) \to T(C)$	infix
$\mathcal{F}_{i+1} ::= \mathcal{F}_{i+j} \uparrow_k \mathcal{F}_j$	$T(C \uparrow_k B) = T(B) \to T(C)$	extract
$\mathcal{F}_{i+j} ::= \mathcal{F}_{i+1} \odot_k \mathcal{F}_j$	$T(A \odot_k B) = T(A) \& T(B)$	disc. product
$\mathcal{F}_1 ::= J$	$T(J) = \top$	disc. product unit

Sort map

$$
\begin{aligned}
s(I) &= 0\\
s(A \bullet B) &= s(A) + s(B)\\
s(C/B) &= s(C) - s(B)\\
s(A \backslash C) &= s(C) - s(A)
\end{aligned}
$$

$$
\begin{aligned}
s(J) &= 1\\
s(A \odot_k B) &= s(A) + s(B) - 1\\
s(C \uparrow_k B) &= 1 + s(C) - s(B)\\
s(A \downarrow_k C) &= 1 + s(C) - s(A)
\end{aligned}
$$

Approximate algebraic interpretation as *sort consistent* subsets of the displacement algebra $(\{L_i\}_{i\in\mathcal{N}}, +, \{\times_k\}_{k\in\mathcal{N}}, 0, 1)$ of discontinuous strings over an alphabet:

$$
\begin{array}{rcl}
[[A\backslash C]] & = & \{s_2 |\ \forall s_1 \in [[A]], s_1 + s_2 \in [[C]]\} \\
[[C/B]] & = & \{s_1 |\ \forall s_2 \in [[B]], s_1 + s_2 \in [[C]]\} \\
[[A \bullet B]] & = & \{s_1 + s_2 |\ s_1 \in [[A]] \text{ and } s_2 \in [[B]]\} \\
[[I]] & = & \{0\}
\end{array}
$$

$$
\begin{array}{rcl}
[[A\downarrow_k C]] & = & \{s_2 |\ \forall s_1 \in [[A]], s_1 \times_k s_2 \in [[C]]\} \\
[[C\uparrow_k B]] & = & \{s_1 |\ \forall s_2 \in [[B]], s_1 \times_k s_2 \in [[C]]\} \\
[[A \odot_k B]] & = & \{s_1 \times_k s_2 |\ s_1 \in [[A]] \text{ and } s_2 \in [[B]]\} \\
[[J]] & = & \{1\}
\end{array}
$$

Natural Deduction for **D**: Continuous connectives (same as **L**)

$$\dfrac{\begin{array}{cc}\vdots & \vdots \\ \alpha\colon A\colon \phi & \beta\colon A\backslash C\colon \psi\end{array}}{\alpha{+}\beta\colon C\colon (\psi\ \phi)}\ E\backslash
\qquad
\dfrac{\begin{array}{c}\overline{a\colon A\colon x}^{\ i} \\ \vdots \\ a{+}\beta\colon C\colon \psi\end{array}}{\beta\colon A\backslash C\colon \lambda x\psi}\ I\backslash^{i}$$

$$\dfrac{\begin{array}{cc}\vdots & \vdots \\ \alpha\colon C/B\colon \phi & \beta\colon B\colon \psi\end{array}}{\alpha{+}\beta\colon C\colon (\phi\ \psi)}\ E/
\qquad
\dfrac{\begin{array}{c}\overline{b\colon B\colon y}^{\ i} \\ \vdots \\ \alpha{+}b\colon C\colon \psi\end{array}}{\alpha\colon C/B\colon \lambda x\psi}\ I/^{i}$$

$$\dfrac{\gamma\colon A\bullet B\colon \chi \qquad \begin{array}{c} \overline{a\colon A\colon x}^{\,i} \quad \overline{b\colon B\colon y}^{\,i} \\ \vdots \\ \delta(a+b)\colon D\colon \omega \end{array}}{\delta(\gamma)\colon D\colon \omega\{\pi_1\chi/x, \pi_2\chi/y\}}\, E\bullet^i$$

$$\dfrac{\begin{array}{c}\vdots\\ \alpha\colon A\colon \phi\end{array} \qquad \begin{array}{c}\vdots\\ \beta\colon B\colon \psi\end{array}}{\alpha+\beta\colon A\bullet B\colon (\phi, \psi)}\, I\bullet$$

$$\dfrac{\begin{array}{c}\vdots\\ \alpha\colon A\colon \phi\end{array} \qquad \begin{array}{c}\vdots\\ \beta\colon I\colon \psi\end{array}}{\alpha\colon A\colon \phi}\, EI$$

$$\dfrac{}{0\colon I\colon 0}\, II$$

Natural Deduction for **D**: Discontinuous connectives (same as **L** also!, except with $\times_k$)

$$\dfrac{\begin{array}{c}\vdots\\ \alpha\colon A\colon \phi\end{array}\qquad\qquad \begin{array}{c}\vdots\\ \beta\colon A{\downarrow_k}C\colon \psi\end{array}}{\alpha{\times_k}\beta\colon C\colon (\psi\ \phi)}\ E{\downarrow_k}$$

$$\dfrac{\begin{array}{c}\overline{a\colon A\colon x}\ ^{i}\\ \vdots\\ a{\times_k}\beta\colon C\colon \psi\end{array}}{\beta\colon A{\downarrow_k}C\colon \lambda x\psi}\ I{\downarrow_k}^{i}$$

$$\dfrac{\begin{array}{c}\vdots\\ \alpha\colon C{\uparrow_k}B\colon \phi\end{array}\qquad\qquad \begin{array}{c}\vdots\\ \beta\colon B\colon \psi\end{array}}{\alpha{\times_k}\beta\colon C\colon (\phi\ \psi)}\ E{\uparrow_k}$$

$$\dfrac{\begin{array}{c}\overline{b\colon B\colon y}\ ^{i}\\ \vdots\\ \alpha{\times_k}b\colon C\colon \psi\end{array}}{\alpha\colon C{\uparrow_k}B\colon \lambda x\psi}\ I{\uparrow_k}^{i}$$

$$\dfrac{\gamma\colon A \odot_k B\colon \chi \qquad \begin{matrix} \dfrac{}{a\colon A\colon x}\, i \quad \dfrac{}{b\colon B\colon y}\, i \\ \vdots \\ \delta(a \times_k b)\colon D\colon \omega \end{matrix}}{\delta(\gamma)\colon D\colon \omega\{\pi_1\chi/x, \pi_2\chi/y\}}\, E{\odot_k}^i$$

$$\dfrac{\begin{matrix}\vdots \\ \alpha\colon A\colon \phi\end{matrix} \qquad \begin{matrix}\vdots \\ \beta\colon B\colon \psi\end{matrix}}{\alpha \times_k \beta\colon A \odot_k B\colon (\phi, \psi)}\, I\odot_k$$

$$\dfrac{\begin{matrix}\vdots \\ \alpha\colon A\colon \phi\end{matrix} \qquad \begin{matrix}\vdots \\ \beta\colon J\colon \psi\end{matrix}}{\alpha\colon A\colon \phi}\, EJ$$

$$\dfrac{}{1\colon I\colon 0}\, IJ$$

Overt movement: *(the book) that Mary reads* in **D**

$$\frac{}{a: N: x}\ i$$

$$\frac{\mathbf{reads}: (N\backslash S)/N: read \qquad a: N: x}{\mathbf{reads}{+}a: N\backslash S: (read\ x)}\ E/$$

$$\frac{\mathbf{Mary}: N: m \qquad \mathbf{reads}{+}a: N\backslash S: (read\ x)}{\mathbf{Mary}{+}\mathbf{reads}{+}a: S: ((read\ x)\ m)}\ E\backslash$$

$$\frac{\mathbf{Mary}{+}\mathbf{reads}{+}a: S: ((read\ x)\ m)}{\mathbf{Mary}{+}\mathbf{reads}{+}1: S{\uparrow}N: \lambda x((read\ x)\ m)}\ I{\uparrow}^{i}$$

$$\frac{}{0: I: 0}\ II$$

$$\frac{\mathbf{Mary}{+}\mathbf{reads}{+}1: S{\uparrow}N: \lambda x((read\ x)\ m) \qquad 0: I: 0}{\mathbf{Mary}{+}\mathbf{reads}: (S{\uparrow}N)\odot I: (\lambda x((read\ x)\ m), 0)}\ I\odot$$

$$\frac{\mathbf{that}: (CN\backslash CN)/((S{\uparrow}N)\odot I): \lambda x\lambda y\lambda z[(y\ z) \wedge (\pi_1 x\ y)] \qquad \mathbf{Mary}{+}\mathbf{reads}: (S{\uparrow}N)\odot I: (\lambda x((read\ x)\ m), 0)}{\mathbf{that}{+}\mathbf{Mary}{+}\mathbf{reads}: CN\backslash CN: \lambda y\lambda z[(y\ z) \wedge ((read\ z)\ m)}\ E/$$

Medial Overt movement:
(the book) that John gave to Mary in **D**

$$\dfrac{}{\mathit{a}: N: x}\ i$$

$$\dfrac{\textbf{gave}: ((N\backslash S)/PP)/N: \mathit{give} \qquad \mathit{a}: N: x}{\textbf{gave}+\mathit{a}: (N\backslash S)/PP: (\mathit{give}\ x)}\ E/$$

$$\dfrac{\textbf{gave}+\mathit{a}: (N\backslash S)/PP: (\mathit{give}\ x) \qquad \textbf{to}+\textbf{Mary}: PP: m}{\textbf{gave}+\mathit{a}+\textbf{to}+\textbf{Mary}: N\backslash S: ((\mathit{give}\ x)\ m)}\ E/$$

$$\dfrac{\textbf{John}: N: j \qquad \textbf{gave}+\mathit{a}+\textbf{to}+\textbf{Mary}: N\backslash S: ((\mathit{give}\ x)\ m)}{\textbf{John}+\textbf{gave}+\mathit{a}+\textbf{to}+\textbf{Mary}: S: (((\mathit{give}\ x)\ m)\ j)}\ E\backslash$$

$$\dfrac{\textbf{John}+\textbf{gave}+\mathit{a}+\textbf{to}+\textbf{Mary}: S: (((\mathit{give}\ x)\ m)\ j)}{\textbf{John}+\textbf{gave}+1+\textbf{to}+\textbf{Mary}: S{\uparrow}N: \lambda x(((\mathit{give}\ x)\ m)\ j)}\ I{\uparrow}$$

$$\dfrac{}{0: I: 0}\ i$$

$$\dfrac{\textbf{John}+\textbf{gave}+1+\textbf{to}+\textbf{Mary}: S{\uparrow}N: \lambda x(((\mathit{give}\ x)\ m)\ j) \qquad 0: I: 0}{\textbf{John}+\textbf{gave}+\textbf{to}+\textbf{Mary}: (S{\uparrow}N)\odot I: (\lambda x(((\mathit{give}\ x)\ m)\ j), 0)}\ I\odot$$

$$\dfrac{\textbf{that}: (CN\backslash CN)/((S{\uparrow}N)\odot I): \lambda x\lambda y\lambda z[(y\ z) \wedge (\pi_1 x\ y)] \qquad \textbf{John}+\textbf{gave}+\textbf{to}+\textbf{Mary}: (S{\uparrow}N)\odot I: (\lambda x(((\mathit{give}\ x)\ m)\ j), 0)}{\textbf{that}+\textbf{John}+\textbf{gave}+\textbf{to}+\textbf{Mary}: CN\backslash CN: \lambda y\lambda z[(y\ z) \wedge (((\mathit{give}\ z)\ m)\ j)]}\ E/$$

Covert movement: *Mary reads every book* in **D**

$$\dfrac{\begin{array}{c}\textbf{Mary}+\textbf{reads}+1:\\ S{\uparrow}N:\\ \lambda x((read\ x)\ m)\end{array} \qquad \dfrac{\begin{array}{c}\textbf{every}:\\ ((S{\uparrow}N){\downarrow}S)/CN:\\ \lambda x\lambda y\forall z[(x\ z)\to(y\ z)]\end{array} \qquad \begin{array}{c}\textbf{book}:\\ CN:\\ book\end{array}}{\textbf{every}+\textbf{book}: (S{\uparrow}N){\downarrow}S: \lambda y\forall z[(book\ z)\to(y\ z)]}\,E/}{\textbf{Mary}+\textbf{reads}+\textbf{every}+\textbf{book}: S: \forall z[(book\ z)\to(((read\ z)\ m)]}\,E{\downarrow}$$

Medial covert movement: *John gave every book to Mary* in **D**

$$\frac{\begin{array}{c}\textbf{John}+\textbf{gave}+1+\textbf{to}+\textbf{Mary}:\\ S{\uparrow}N:\\ \lambda x(((give\ x)\ m)\ j)\end{array} \qquad \begin{array}{c}\textbf{every}+\textbf{book}:\\ (S{\uparrow}N){\downarrow}S:\\ \lambda y\forall z[(book\ z)\rightarrow(y\ z)]\end{array}}{\textbf{John}+\textbf{gave}+\textbf{every}+\textbf{book}+\textbf{to}+\textbf{Mary}: S: \forall z[(book\ z)\rightarrow(((give\ z)\ m)\ j)]}\ E{\downarrow}$$

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