Mathematical Logic and Linguistics

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BGSMath Course
Class 11

Syntactic and Semantic Analyses: Relativisation

Relativisation is an unbounded dependency phenomenon: the distance between a relative pronoun and its extraction site can be indefinitely long:

(1) a. the man that$_i$ I know t_i
b. the man that$_i$ you know I know t_i
c. the man that$_i$ I know you know I know t_i
⋮

The treatment of relativisation in categorial grammar by means of assignment of higher-order functors to relative pronouns is well-established since Ades and Steedman (1982[1]) and yields the unboundedness property through associative assembly of the body of a relative clause.

However, although relativisation is unbounded it is not unconstrained. Various ‘islands’ can inhibit or block relativisation: weak islands such as subjects and adverbial phrases, from which extraction is mildly unacceptable, and strong islands such as coordinate structures and relative clauses themselves, from which extraction is completely unacceptable:

(2) a. ?man who$_i$ the friend of t_i laughed
b. ?paper which$_i$ John laughed before reading t_i

(3) a. *man who$_i$ John laughed and Mary likes t_i
b. *man who$_i$ John likes the woman that loves t_i

Such ‘structural inhibition’ represents a challenge to categorial grammar and all approaches to grammar.

Furthermore, relativisation can also comprise ‘parasitic extraction’ in which a relative pronoun binds more than one extraction site (Taraldsen 1979[6]; Engdahl 1983[4]; Sag 1983[5]). There is a *single* ‘host’ gap which is not in an island, and according to the received wisdom, and according with the terminology ‘parasitic’, this may license a ‘parasitic’ gap in (any number of immediate weak) islands:

(4) a. *the slave who$_i$ John sold t_i t_i
b. *the slave who$_i$ John sold t_i to t_i

(5) a. the man who$_i$ the friends of t_i admire t_i
b. the paper which$_i$ John filed t_i without reading t_i
c. the paper which$_i$ the editor of t_i filed t_i without reading t_i

In addition, we observe here that these parasitic gaps may in turn function as host gaps licensing further parasitic gaps in (weak) subislands, and so on recursively:

(6) a. man who$_i$ the fact that the friends of t_i admire t_i surprises t_i
b. man who$_i$ the fact that the friends of t_i admire t_i without praising t_i offends t_i without surprising t_i

Such ‘structural facilitation’ represents a further challenge to categorial grammar and all approaches to grammar.

Framework

The formalism used comprises the following connectives:

	cont. mult.	disc. mult.	add.	qu.	norm. mod.	brack. mod.	exp.	lim. contr. & weak.
primary	$/$ $\backslash$ $\bullet$ I	$\uparrow$ $\downarrow$ $\odot$ J	$\&$ $\oplus$	$\bigwedge$ $\bigvee$	$\Box$ $\Diamond$	$[\,]^{-1}$ $\langle\rangle$	$!$ $?$	$\mid$ W
sem. inactive variants	$\bullet\!\!-$ $-\!\!\circ$ $\circ\!\!-$ $-\!\!\bullet$ ◐ ◑	$\overset{\bullet}{\mid}$ $\underset{\circ}{\mid}$ $\overset{\circ}{\mid}$ $\underset{\bullet}{\mid}$ ◓ ◒	$\sqcap$ $\sqcup$	$\forall$ $\exists$	$\blacksquare$ $\blacklozenge$			
det. synth.	$\triangleleft^{-1}$ $\triangleright^{-1}$ $\triangleleft$ $\triangleright$	$\check{}$ $\hat{}$						
nondet. synth.	$\div$ $\times$	$\Uparrow$ $\Downarrow$ $\circledcirc$						

Initial examples

The first example is as follows:

(7) [**john**]+**walks** : Sf

Note that in our syntactical form the subject is a bracketed domain, and this will always be the case — implementing that subjects are weak islands. Lookup in our lexicon yields the following semantically labelled sequent:

(8) $[\blacksquare Nt(s(m)) : j], \square(\langle\rangle \exists g Nt(s(g)) \backslash Sf) :$
$\hat{}\lambda A(Pres\ (\check{}walk\ A)) \ \Rightarrow \ Sf$

The derivation is as follows:

$$(9)\quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\boxed{Nt(s(m))} \;\Rightarrow\; Nt(s(m))}}{\boxed{\blacksquare Nt(s(m))} \;\Rightarrow\; Nt(s(m))}\,\blacksquare L}{\blacksquare Nt(s(m)) \;\Rightarrow\; \boxed{\exists g Nt(s(g))}}\,\exists R}{[\blacksquare Nt(s(m))] \;\Rightarrow\; \boxed{\langle\rangle\exists g Nt(s(g))}}\,\langle\rangle R \qquad \dfrac{}{\boxed{Sf} \;\Rightarrow\; Sf}}{[\blacksquare Nt(s(m))], \boxed{\langle\rangle\exists g Nt(s(g))\backslash Sf} \;\Rightarrow\; Sf}\,\backslash L}{[\blacksquare Nt(s(m))], \boxed{\square(\langle\rangle\exists g Nt(s(g))\backslash Sf)} \;\Rightarrow\; Sf}\,\square L$$

The flow of information in the semantic reading of derivations can be illustrated for the case in hand as follows. First, variables for the antecedent semantics are added in the endsequent:

(10) $[\blacksquare Nt(s(m)) : x], \Box(\langle\rangle\exists g Nt(s(g))\backslash Sf) : y \ \Rightarrow\ Sf$

Reading bottom-up, at the lowest inference step ($\Box$L) the verb semantics is replaced by the extension *z* and the subject semantics *x* is carried over:

(11) $$\frac{[\blacksquare Nt(s(m)) : x], \boxed{\langle\rangle\exists g Nt(s(g))\backslash Sf} : z \ \Rightarrow\ Sf}{[\blacksquare Nt(s(m)) : x], \Box(\langle\rangle\exists g Nt(s(g))\backslash Sf) : y \ \Rightarrow\ Sf}\,\Box L$$

At the second inference we propagate the subject semantics on the argument branch:

$$(12)\quad \cfrac{\cfrac{[\blacksquare Nt(s(m)) : x] \Rightarrow \boxed{\langle\rangle\exists gNt(s(g))} \qquad \boxed{Sf} \Rightarrow Sf}{[\blacksquare Nt(s(m)) : x], \boxed{\langle\rangle\exists gNt(s(g))\backslash Sf} : z \Rightarrow Sf}\,\backslash L}{[\blacksquare Nt(s(m)) : x], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf) : y \Rightarrow Sf}\,\Box L$$

The next three inferences involve semantically transparent copying of the antecedent semantics:

$$(13)\quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\boxed{Nt(s(m))} : \mathrm{x} \;\Rightarrow\; Nt(s(m))}}{\boxed{\blacksquare Nt(s(m)) : x} \;\Rightarrow\; Nt(s(m))}\,\blacksquare L}{\blacksquare Nt(s(m)) : x \;\Rightarrow\; \boxed{\exists g Nt(s(g))}}\,\exists R}{[\blacksquare Nt(s(m)) : x] \;\Rightarrow\; \boxed{\langle\rangle\exists g Nt(s(g))}}\,\langle\rangle R \qquad \dfrac{}{\boxed{Sf} \;\Rightarrow\; Sf}}{[\blacksquare Nt(s(m)) : x], \boxed{\langle\rangle\exists g Nt(s(g))\backslash Sf} : z \;\Rightarrow\; Sf}\,\backslash L}{[\blacksquare Nt(s(m)) : x], \Box(\langle\rangle\exists g Nt(s(g))\backslash Sf) : y \;\Rightarrow\; Sf}\,\Box L$$

At the identity axiom the antecedent semantics is copied to the succedent:

(14)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\boxed{Nt(s(m)) : x} \;\Rightarrow\; Nt(s(m)) : x}}{\boxed{\blacksquare Nt(s(m)) : x} \;\Rightarrow\; Nt(s(m))}\,\blacksquare L}{\blacksquare Nt(s(m)) : x \;\Rightarrow\; \boxed{\exists g Nt(s(g))}}\,\exists R}{[\blacksquare Nt(s(m)) : x] \;\Rightarrow\; \boxed{\langle\rangle \exists g Nt(s(g))}}\,\langle\rangle R \qquad \dfrac{}{\boxed{Sf} \;\Rightarrow\; Sf}}{[\blacksquare Nt(s(m)) : x], \boxed{\langle\rangle \exists g Nt(s(g)) \backslash Sf} : z \;\Rightarrow\; Sf}\,\backslash L}{[\blacksquare Nt(s(m)) : x], \Box(\langle\rangle \exists g Nt(s(g)) \backslash Sf) : y \;\Rightarrow\; Sf}\,\Box L$$

In a following phase the succedent semantics is copied from premises to conclusions as far as the root of the argument branch:

(15)
$$\dfrac{\dfrac{\dfrac{\dfrac{\overline{\boxed{Nt(s(m)):x} \Rightarrow Nt(s(m)):x}}{\boxed{\blacksquare Nt(s(m)):x} \Rightarrow Nt(s(m)):x}\,\blacksquare L}{\blacksquare Nt(s(m)):x \Rightarrow \boxed{\exists g Nt(s(g))}:x}\,\exists R}{[\blacksquare Nt(s(m)):x] \Rightarrow \boxed{\langle\rangle\exists g Nt(s(g)):x}}\,\langle\rangle R \qquad \overline{\boxed{Sf} \Rightarrow Sf}}{\dfrac{[\blacksquare Nt(s(m)):x], \boxed{\langle\rangle\exists g Nt(s(g))\backslash Sf}:z \Rightarrow Sf}{[\blacksquare Nt(s(m)):x], \square(\langle\rangle\exists g Nt(s(g))\backslash Sf):y \Rightarrow Sf}\,\square L}\,\backslash L$$

Now the functor value semantics in the antecedent of the value branch is labelled with a new variable *w*:

(16)

$$
\cfrac{
  \cfrac{
    \cfrac{
      \cfrac{
        \cfrac{\overline{\boxed{Nt(s(m)) : x} \ \Rightarrow\ Nt(s(m)) : x}}{\boxed{\blacksquare Nt(s(m)) : x} \ \Rightarrow\ Nt(s(m)) : x}\ \blacksquare L
      }{\blacksquare Nt(s(m)) : x \ \Rightarrow\ \boxed{\exists g Nt(s(g))} : x}\ \exists R
    }{[\blacksquare Nt(s(m)) : x] \ \Rightarrow\ \boxed{\langle\rangle\exists g Nt(s(g)) : x}}\ \langle\rangle R
    \qquad
    \overline{\boxed{Sf : w} \ \Rightarrow\ Sf}
  }{[\blacksquare Nt(s(m)) : x], \boxed{\langle\rangle\exists g Nt(s(g))\backslash Sf} : z \ \Rightarrow\ Sf}\ \backslash L
}{[\blacksquare Nt(s(m)) : x], \Box(\langle\rangle\exists g Nt(s(g))\backslash Sf) : y \ \Rightarrow\ Sf}\ \Box L
$$

At the id axiom this semantics is copied from antecedent to succedent:

(17)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\boxed{Nt(s(m)):x} \;\Rightarrow\; Nt(s(m)):x}}{\boxed{\blacksquare Nt(s(m)):j} \;\Rightarrow\; Nt(s(m)):j}\,\blacksquare L}{\blacksquare Nt(s(m)):x \;\Rightarrow\; \boxed{\exists g Nt(s(g))}:x}\,\exists R}{[\blacksquare Nt(s(m)):x] \;\Rightarrow\; \boxed{\langle\rangle\exists g Nt(s(g)):x}}\,\langle\rangle R \qquad \dfrac{}{\boxed{Sf:w} \;\Rightarrow\; Sf:w}}{[\blacksquare Nt(s(m)):x], \boxed{\langle\rangle\exists g Nt(s(g))\backslash Sf}:z \;\Rightarrow\; Sf}\,\backslash L}{[\blacksquare Nt(s(m)):x], \Box(\langle\rangle\exists g Nt(s(g))\backslash Sf):y \;\Rightarrow\; Sf}\,\Box L$$

In the $\backslash L$ conclusion succedent the semantics of the major premise is subject to the substitution of w by the functional application of the functor z to the argument x:

(18)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\boxed{Nt(s(m)) : x} \ \Rightarrow \ Nt(s(m)) : x}}{\boxed{\blacksquare Nt(s(m)) : x} \ \Rightarrow \ Nt(s(m)) : x}\,\blacksquare L}{\blacksquare Nt(s(m)) : x \ \Rightarrow \ \boxed{\exists g Nt(s(g))} : x}\,\exists R}{[\blacksquare Nt(s(m)) : x] \ \Rightarrow \ \boxed{\langle\rangle \exists g Nt(s(g)) : x}}\,\langle\rangle R \qquad \dfrac{}{\boxed{Sf : w} \ \Rightarrow \ Sf : w}}{[\blacksquare Nt(s(m)) : x], \boxed{\langle\rangle \exists g Nt(s(g)) \backslash Sf} : z \ \Rightarrow \ Sf : w\{(z\ x)/w\} = (z\ x)}\,\backslash L$$

$$\dfrac{[\blacksquare Nt(s(m)) : x], \boxed{\langle\rangle \exists g Nt(s(g)) \backslash Sf} : z \ \Rightarrow \ Sf : w\{(z\ x)/w\} = (z\ x)}{[\blacksquare Nt(s(m)) : x], \Box(\langle\rangle \exists g Nt(s(g)) \backslash Sf) : y \ \Rightarrow \ Sf}\,\Box L$$

And thence to the conclusion of the endsequent:

(19)

$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{}{\boxed{Nt(s(m)):x} \;\Rightarrow\; Nt(s(m)):x}}{\boxed{\blacksquare Nt(s(m)):x} \;\Rightarrow\; Nt(s(m)):x}\,\blacksquare L}{\blacksquare Nt(s(m)):x \;\Rightarrow\; \boxed{\exists g Nt(s(g))}:x}\,\exists R}{[\blacksquare Nt(s(m)):x] \;\Rightarrow\; \boxed{\langle\rangle\exists g Nt(s(g)):x}}\,\langle\rangle R \qquad \cfrac{}{\boxed{Sf:w} \;\Rightarrow\; Sf:w}}{[\blacksquare Nt(s(m)):x], \boxed{\langle\rangle\exists g Nt(s(g))\backslash Sf}:z \;\Rightarrow\; Sf:(z\ x)}\,\backslash L}{[\blacksquare Nt(s(m)):x], \Box(\langle\rangle\exists g Nt(s(g))\backslash Sf):y \;\Rightarrow\; Sf:(z\ x)\{{\check{}}y/z\} = ({\check{}}y\ x)}\,\Box L$$

Now we can substitute in the lexical semantics *j* for *John* (*x*) and (*y*) and evaluate:

(20) $(\check{}\hat{}\lambda A(\mathit{Pres}\ (\check{}\mathit{walk}\ A))\ j) =$
$(\lambda A(\mathit{Pres}\ (\check{}\mathit{walk}\ A))\ j) =$
$(\mathit{Pres}\ (\check{}\mathit{walk}\ j))$

By way of a second example, the following is a simple transitive sentence:

(21) [**john**]$+$**loves**$+$**mary** : Sf

Lexical lookup yields:

(22) $[\blacksquare Nt(s(m)) : j], \square(((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa) :$
$\hat{}\lambda A\lambda B(Pres\ ((\check{}love\ A)\ B)), \blacksquare Nt(s(f)) : m \ \Rightarrow\ Sf$

There is the derivation:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\boxed{Nt(s(f))} \Rightarrow Nt(s(f))}}{\boxed{\blacksquare Nt(s(f))} \Rightarrow Nt(s(f))}\blacksquare L}{\blacksquare Nt(s(f)) \Rightarrow \boxed{\exists aNa}}\exists R \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\boxed{Nt(s(m))} \Rightarrow Nt(s(m))}}{\boxed{\blacksquare Nt(s(m))} \Rightarrow Nt(s(m))}\blacksquare L}{\blacksquare Nt(s(m)) \Rightarrow \boxed{\exists gNt(s(g))}}\exists R}{[\blacksquare Nt(s(m))] \Rightarrow \boxed{\langle\rangle\exists gNt(s(g))}}\langle\rangle R \quad \dfrac{}{\boxed{Sf} \Rightarrow Sf}}{[\blacksquare Nt(s(m))], \boxed{\langle\rangle\exists gNt(s(g))\backslash Sf} \Rightarrow Sf}\backslash L}{[\blacksquare Nt(s(m))], \boxed{(\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa}, \blacksquare Nt(s(f)) \Rightarrow Sf}/L}{[\blacksquare Nt(s(m))], \boxed{\square((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa)}, \blacksquare Nt(s(f)) \Rightarrow Sf}\square L$$

Reading upwards from the endsequent, the first inference removes the intensionality modality from the transitive verb, and then over left selects the object to analyse as the argument of the transitive verb; this is done by existential right instantiating the agreement feature to third person singular feminine, followed by (semantically inactive) intensionality modality left. The right hand branch is the same as for example (7) after the first inference. All this delivers semantics:

(23) $(\mathit{Pres}\ ((\check{}\mathit{love}\ m)\ j))$

The next example has a subordinate clause:

(24) $[\mathbf{john}]+\mathbf{thinks}+[\mathbf{mary}]+\mathbf{walks} : Sf$

Lexical lookup yields the following; note that the propositional attitude verb is polymorphic with respect to a complementised or uncomplementised sentential argument, expressed with a semantically inactive additive disjunction:

(25) $[\blacksquare Nt(s(m)) : j], \Box((\langle\rangle\exists gNt(s(g))\backslash Sf)/(CPthat\sqcup\Box Sf)) :$
$\hat{}\lambda A\lambda B(Pres\ ((\check{}think\ A)\ B)), [\blacksquare Nt(s(f)) :$
$m], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf) : \hat{}\lambda C(Pres\ (\check{}walk\ C)) \ \Rightarrow\ Sf$

This has the derivation:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\boxed{Nt(s(f))} \Rightarrow Nt(s(f))}}{\boxed{\blacksquare Nt(s(f))} \Rightarrow Nt(s(f))}\blacksquare L}{\blacksquare Nt(s(f)) \Rightarrow \boxed{\exists gNt(s(g))}}\exists R}{[\blacksquare Nt(s(f))] \Rightarrow \boxed{\langle\rangle\exists gNt(s(g))}}\langle\rangle R \qquad \dfrac{}{\boxed{Sf} \Rightarrow Sf}}{[\blacksquare Nt(s(f))], \boxed{\langle\rangle\exists gNt(s(g))\backslash Sf} \Rightarrow Sf}\backslash L}{[\blacksquare Nt(s(f))], \boxed{\Box(\langle\rangle\exists gNt(s(g))\backslash Sf)} \Rightarrow Sf}\Box L}{[\blacksquare Nt(s(f))], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf) \Rightarrow \Box Sf}\Box R}{[\blacksquare Nt(s(f))], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf) \Rightarrow \boxed{CPthat\sqcup\Box Sf}}\sqcup R \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\boxed{Nt(s(m))} \Rightarrow Nt(s(m))}}{\boxed{\blacksquare Nt(s(m))} \Rightarrow Nt(s(m))}\blacksquare L}{\blacksquare Nt(s(m)) \Rightarrow \boxed{\exists gNt(s(g))}}\exists R}{[\blacksquare Nt(s(m))] \Rightarrow \boxed{\langle\rangle\exists gNt(s(g))}}\langle\rangle R \qquad \dfrac{}{\boxed{Sf} \Rightarrow Sf}}{[\blacksquare Nt(s(m))], \boxed{\langle\rangle\exists gNt(s(g))\backslash Sf} \Rightarrow Sf}\backslash L}{[\blacksquare Nt(s(m))], \boxed{(\langle\rangle\exists gNt(s(g))\backslash Sf)/(CPthat\sqcup\Box Sf)}, [\blacksquare Nt(s(f))], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf) \Rightarrow Sf}/L}{[\blacksquare Nt(s(m))], \boxed{\Box((\langle\rangle\exists gNt(s(g))\backslash Sf)/(CPthat\sqcup\Box Sf))}, [\blacksquare Nt(s(f))], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf) \Rightarrow Sf}\Box L$$

The derivation delivers semantics:

(26) $(\mathit{Pres}\ ((\check{}\mathit{think}\ \hat{}(\mathit{Pres}\ (\check{}\mathit{walk}\ m)))\ j))$

The following example involves a ditransitive verb:

(27) $[\mathbf{mary}]{+}\mathbf{buys}{+}\mathbf{john}{+}\mathbf{coffee} : Sf$

Lexical lookup is as follows; note the use of product (multiplicative conjunction) for the ditransitive verb, and the use of additive conjunction for the polymorphism of the mass noun *coffee* which can appear either as a bare nominal or with an article:

(28) $$\begin{array}{l} [\blacksquare Nt(s(f)) : m], \Box((\langle\rangle\exists gNt(s(g))\backslash Sf)/(\exists aNa\bullet\exists aNa)) : \\ \hat{}\lambda A\lambda B(Pres\ (((\check{}buy\ \pi_1 A)\ \pi_2 A)\ B)), \blacksquare Nt(s(m)) : \\ j, \Box(Nt(s(n))\&CNs(n)) : \hat{}((gen\ \check{}coffee), \check{}coffee) \ \Rightarrow \ Sf \end{array}$$

There is the derivation:

$$
\begin{prooftree}
\AxiomC{}
\UnaryInfC{$\boxed{Nt(s(m))} \Rightarrow Nt(s(m))$}
\RightLabel{$\blacksquare L$}
\UnaryInfC{$\boxed{\blacksquare Nt(s(m))} \Rightarrow Nt(s(m))$}
\RightLabel{$\exists R$}
\UnaryInfC{$\blacksquare Nt(s(m)) \Rightarrow \boxed{\exists aNa}$}
\AxiomC{}
\UnaryInfC{$\boxed{Nt(s(n))} \Rightarrow Nt(s(n))$}
\RightLabel{$\& L$}
\UnaryInfC{$\boxed{Nt(s(n))\& CNs(n)} \Rightarrow Nt(s(n))$}
\RightLabel{$\square L$}
\UnaryInfC{$\boxed{\square(Nt(s(n))\& CNs(n))} \Rightarrow Nt(s(n))$}
\RightLabel{$\exists R$}
\UnaryInfC{$\square(Nt(s(n))\& CNs(n)) \Rightarrow \boxed{\exists aNa}$}
\RightLabel{$\bullet R$}
\BinaryInfC{$\blacksquare Nt(s(m)), \square(Nt(s(n))\& CNs(n)) \Rightarrow \boxed{\exists aNa \bullet \exists aNa}$}
\AxiomC{}
\UnaryInfC{$\boxed{Nt(s(f))} \Rightarrow Nt(s(f))$}
\RightLabel{$\blacksquare L$}
\UnaryInfC{$\boxed{\blacksquare Nt(s(f))} \Rightarrow Nt(s(f))$}
\RightLabel{$\exists R$}
\UnaryInfC{$\blacksquare Nt(s(f)) \Rightarrow \boxed{\exists gNt(s(g))}$}
\RightLabel{$\langle\rangle R$}
\UnaryInfC{$[\blacksquare Nt(s(f))] \Rightarrow \boxed{\langle\rangle\exists gNt(s(g))}$}
\AxiomC{}
\UnaryInfC{$\boxed{Sf} \Rightarrow Sf$}
\RightLabel{$\backslash L$}
\BinaryInfC{$[\blacksquare Nt(s(f))], \boxed{\langle\rangle\exists gNt(s(g))\backslash Sf} \Rightarrow Sf$}
\RightLabel{$/L$}
\BinaryInfC{$[\blacksquare Nt(s(f))], \boxed{(\langle\rangle\exists gNt(s(g))\backslash Sf)/(\exists aNa\bullet\exists aNa)}, \blacksquare Nt(s(m)), \square(Nt(s(n))\& CNs(n)) \Rightarrow Sf$}
\RightLabel{$\square L$}
\UnaryInfC{$[\blacksquare Nt(s(f))], \boxed{\square((\langle\rangle\exists gNt(s(g))\backslash Sf)/(\exists aNa\bullet\exists aNa))}, \blacksquare Nt(s(m)), \square(Nt(s(n))\& CNs(n)) \Rightarrow Sf$}
\end{prooftree}
$$

After removal of the outer modality of the ditransitive verb, the partitioning of over left selects the two objects as the verb's product argument, partitioned in turn by continuous product right. The indirect object *John* is analysed by existential right and inactive modality left inferences; the direct object *coffee* is analysed by existential right and (active) modality left inferences followed by selection of the bare noun type by additive conjunction left. The rightmost subtree is as usual for an intransitive sentence. This delivers semantics as follows in which a 'generic' operator applies to *coffee*:

(29) $(\mathit{Pres}\ (((\check{}\mathit{buy}\ j)\ (\mathit{gen}\ \check{}\mathit{coffee}))\ m))$

The next example includes a definite article:

(30) [**the**$+$**man**]$+$**walks** : Sf

We treat the definite article simply as an iota operator which returns the unique individual in the context of discourse satisfying its common noun argument (Carpenter 1997[2]); this unicity is presupposed by the use of the definite. Lexical lookup yields the semantically labelled sequent:

(31) $$[\blacksquare\forall n(Nt(n)/CNn) : \iota, \Box CNs(m) : \\ man], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf) : \hat{}\lambda A(Pres\ (\check{}walk\ A)) \ \Rightarrow\ Sf$$

There is the derivation:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\boxed{CNs(m)} \Rightarrow CNs(m)}}{\boxed{\Box CNs(m)} \Rightarrow CNs(m)}\Box L \qquad \dfrac{}{\boxed{Nt(s(m))} \Rightarrow Nt(s(m))}}{\boxed{Nt(s(m))/CNs(m)}, \Box CNs(m) \Rightarrow Nt(s(m))}/L}{\boxed{\forall n(Nt(n)/CNn)}, \Box CNs(m) \Rightarrow Nt(s(m))}\forall L}{\boxed{\blacksquare\forall n(Nt(n)/CNn)}, \Box CNs(m) \Rightarrow Nt(s(m))}\blacksquare L}{\blacksquare\forall n(Nt(n)/CNn), \Box CNs(m) \Rightarrow \boxed{\exists g Nt(s(g))}}\exists R}{[\blacksquare\forall n(Nt(n)/CNn), \Box CNs(m)] \Rightarrow \boxed{\langle\rangle\exists g Nt(s(g))}}\langle\rangle R \qquad \dfrac{}{\boxed{Sf} \Rightarrow Sf}}{[\blacksquare\forall n(Nt(n)/CNn), \Box CNs(m)], \boxed{\langle\rangle\exists g Nt(s(g))\backslash Sf} \Rightarrow Sf}\backslash L}{[\blacksquare\forall n(Nt(n)/CNn), \Box CNs(m)], \boxed{\Box(\langle\rangle\exists g Nt(s(g))\backslash Sf)} \Rightarrow Sf}\Box L$$

The derivation delivers semantics:

(32) $(Pres\ (\check{}walk\ (\iota\ \check{}man)))$

The next two examples have adverbial and adnominal prepositional modification respectively. We consider the adverbial case first:

(33) [**john**]$+$**walks**$+$**from**$+$**edinburgh** : *Sf*

Lexical lookup inserts a single value-polymorphic prepositional type, which uses semantically active additive conjunction:

(34) $[\blacksquare Nt(s(m)) : j], \Box(\langle\rangle\exists g Nt(s(g))\backslash Sf) :$
$\hat{}\lambda A(Pres\ (\check{}walk\ A)), \Box((\forall a\forall f((\langle\rangle Na\backslash Sf)\backslash(\langle\rangle Na\backslash Sf))\&\forall n(CNn\backslash CNn))/\exists bNb) :$
$\hat{}\lambda B((\check{}fromadv\ B), (\check{}fromadn\ B)), \blacksquare Nt(s(n)) : e \ \Rightarrow\ Sf$

There is the derivation:

$$
\begin{prooftree}
\AxiomC{$\boxed{Nt(s(n))} \Rightarrow Nt(s(n))$}
\RightLabel{$\blacksquare L$}
\UnaryInfC{$\boxed{\blacksquare Nt(s(n))} \Rightarrow Nt(s(n))$}
\RightLabel{$\exists R$}
\UnaryInfC{$\blacksquare Nt(s(n)) \Rightarrow \boxed{\exists bNb}$}
\AxiomC{$Nt(s(m)) \Rightarrow Nt(s(m))$}
\RightLabel{$\exists R$}
\UnaryInfC{$Nt(s(m)) \Rightarrow \boxed{\exists gNt(s(g))}$}
\RightLabel{$\langle\rangle R$}
\UnaryInfC{$[Nt(s(m))] \Rightarrow \boxed{\langle\rangle\exists gNt(s(g))}$}
\AxiomC{$\boxed{Sf} \Rightarrow Sf$}
\RightLabel{$\backslash L$}
\BinaryInfC{$[Nt(s(m))], \boxed{\langle\rangle\exists gNt(s(g))\backslash Sf} \Rightarrow Sf$}
\RightLabel{$\Box L$}
\UnaryInfC{$[Nt(s(m))], \boxed{\Box(\langle\rangle\exists gNt(s(g))\backslash Sf)} \Rightarrow Sf$}
\RightLabel{$\langle\rangle L$}
\UnaryInfC{$\langle\rangle Nt(s(m)), \Box(\langle\rangle\exists gNt(s(g))\backslash Sf) \Rightarrow Sf$}
\RightLabel{$\backslash R$}
\UnaryInfC{$\Box(\langle\rangle\exists gNt(s(g))\backslash Sf) \Rightarrow \langle\rangle Nt(s(m))\backslash Sf$}
\AxiomC{$\boxed{Nt(s(m))} \Rightarrow Nt(s(m))$}
\RightLabel{$\blacksquare L$}
\UnaryInfC{$\boxed{\blacksquare Nt(s(m))} \Rightarrow Nt(s(m))$}
\RightLabel{$\langle\rangle R$}
\UnaryInfC{$[\blacksquare Nt(s(m))] \Rightarrow \boxed{\langle\rangle Nt(s(m))}$}
\AxiomC{$\boxed{Sf} \Rightarrow Sf$}
\BinaryInfC{$[\blacksquare Nt(s(m))], \boxed{\langle\rangle Nt(s(m))\backslash Sf} \Rightarrow Sf$}
\RightLabel{$\backslash L$}
\BinaryInfC{$[\blacksquare Nt(s(m))], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf), \boxed{(\langle\rangle Nt(s(m))\backslash Sf)\backslash(\langle\rangle Nt(s(m))\backslash Sf)} \Rightarrow Sf$}
\RightLabel{$\forall L$}
\UnaryInfC{$[\blacksquare Nt(s(m))], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf), \boxed{\forall f((\langle\rangle Nt(s(m))\backslash Sf)\backslash(\langle\rangle Nt(s(m))\backslash Sf))} \Rightarrow Sf$}
\RightLabel{$\forall L$}
\UnaryInfC{$[\blacksquare Nt(s(m))], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf), \boxed{\forall a\forall f((\langle\rangle Na\backslash Sf)\backslash(\langle\rangle Na\backslash Sf))} \Rightarrow Sf$}
\RightLabel{$\& L$}
\UnaryInfC{$[\blacksquare Nt(s(m))], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf), \boxed{\forall a\forall f((\langle\rangle Na\backslash Sf)\backslash(\langle\rangle Na\backslash Sf))\&\forall n(CNn\backslash CNn)} \Rightarrow Sf$}
\RightLabel{$/L$}
\BinaryInfC{$[\blacksquare Nt(s(m))], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf), \boxed{(\forall a\forall f((\langle\rangle Na\backslash Sf)\backslash(\langle\rangle Na\backslash Sf))\&\forall n(CNn\backslash CNn))/\exists bNb}, \blacksquare Nt(s(n)) \Rightarrow Sf$}
\RightLabel{$\Box L$}
\UnaryInfC{$[\blacksquare Nt(s(m))], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf), \boxed{\Box((\forall a\forall f((\langle\rangle Na\backslash Sf)\backslash(\langle\rangle Na\backslash Sf))\&\forall n(CNn\backslash CNn))/\exists bNb)}, \blacksquare Nt(s(n)) \Rightarrow Sf$}
\end{prooftree}
$$

After elimination of the outer modality of the preposition, over left selects as the prepositional argument the prepositional object, which is analysed in the leftmost subtree. In the sister subtree additive conjunction left selects the adverbial type for the prepositional phrase and for all left instantiates the subject agreement and verb form features to third person singular masculine, and finite. Following under left, in the middle subtree *walks* is analysed as the intransitive verb second argument of the adverbial preposition; note the analysis of the higher-order type by the under right rule, which lowers the conclusion succedent hypothetical subtype into the premise antecedent. The rightmost subtree is an intransitive sentence case again. All this delivers the semantics:

(35) $(((\check{}\mathit{fromadv}\ e)\ \lambda B(\mathit{Pres}\ (\check{}\mathit{walk}\ B)))\ j)$

The adnominal case is:

(36) [**the**$+$**man**$+$**from**$+$**edinburgh**]$+$**walks** : *Sf*

Lexical lookup yields:

(37) $[\blacksquare\forall n(Nt(n)/CNn) : \iota, \Box CNs(m) :$
$man, \Box((\forall a\forall f((\langle\rangle Na\backslash Sf)\backslash(\langle\rangle Na\backslash Sf))\&\forall n(CNn\backslash CNn))/\exists bNb) :$
$\hat{}\lambda A((\check{}fromadv\ A), (\check{}fromadn\ A)), \blacksquare Nt(s(n)) : e], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf) :$
$\hat{}\lambda B(Pres\ (\check{}walk\ B)) \ \Rightarrow \ Sf$

The semantics delivered is:

(38) $(\mathit{Pres}\ ({}^{\vee}\mathit{walk}\ (\iota\ (({}^{\vee}\mathit{fromadn}\ e)\ {}^{\vee}\mathit{man}))))$

The last two initial examples involve the copula with nominal and (intersective) adjectival complementation respectively. We consider first the nominal case:

(39) $[\textbf{tully}]{+}\textbf{is}{+}\textbf{cicero} : Sf$

Lexical lookup inserts a single argument-polymorphic copula type, which uses both semantically active and semantically inactive additive disjunction:

(40) $[\blacksquare Nt(s(m)) : t], \blacksquare((\langle\rangle \exists g Nt(s(g)) \backslash Sf)/(\exists a Na \oplus (\exists g((CNg/CNg) \sqcup (CNg \backslash CNg)) - I))) : \lambda A \lambda B(Pres\ (A \rightarrow C.[B = C]; D.((D\ \lambda E[E = B])\ B))), \blacksquare \forall g Nt(s(g)) : c \ \Rightarrow\ Sf$

There is the derivation:

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\boxed{Nt(s(m))} \Rightarrow Nt(s(m))}}{\boxed{\blacksquare Nt(s(m))} \Rightarrow Nt(s(m))}\,\blacksquare L}{\blacksquare Nt(s(m)) \Rightarrow \boxed{\exists aNa}}\,\exists R}{\blacksquare Nt(s(m)) \Rightarrow \boxed{\exists aNa{\oplus}(\exists g((CNg/CNg){\sqcup}(CNg\backslash CNg)){-}I)}}\,\oplus R \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\boxed{Nt(s(m))} \Rightarrow Nt(s(m))}}{\boxed{\blacksquare Nt(s(m))} \Rightarrow Nt(s(m))}\,\blacksquare L}{\blacksquare Nt(s(m)) \Rightarrow \boxed{\exists gNt(s(g))}}\,\exists R}{[\blacksquare Nt(s(m))] \Rightarrow \boxed{\langle\rangle\exists gNt(s(g))}}\,\langle\rangle R \qquad \dfrac{}{\boxed{Sf} \Rightarrow Sf}}{[\blacksquare Nt(s(m))], \boxed{\langle\rangle\exists gNt(s(g))\backslash Sf} \Rightarrow Sf}\,\backslash L}{[\blacksquare Nt(s(m))], \boxed{(\langle\rangle\exists gNt(s(g))\backslash Sf)/(\exists aNa{\oplus}(\exists g((CNg/CNg){\sqcup}(CNg\backslash CNg)){-}I))}, \blacksquare Nt(s(m)) \Rightarrow Sf}\,/L}{[\blacksquare Nt(s(m))], \boxed{\blacksquare((\langle\rangle\exists gNt(s(g))\backslash Sf)/(\exists aNa{\oplus}(\exists g((CNg/CNg){\sqcup}(CNg\backslash CNg)){-}I)))}, \blacksquare Nt(s(m)) \Rightarrow Sf}\,\blacksquare L$$

After elimination of the outer copula modality the copula is applied to its nominal complement. Additive disjunction right selects the first, nominal, disjunct. The derivation delivers semantics:

(41) $(Pres\ [t = c])$

The (intersective) adjectival case is:

(42) [**tully**]$+$**is**$+$**humanist** : Sf

Lexical lookup yields:

(43) $[\blacksquare Nt(s(m)) : t], \blacksquare((\langle\rangle\exists g Nt(s(g))\backslash Sf)/(\exists a Na \oplus (\exists g((CNg/CNg) \sqcup (CNg\backslash CNg)) - I))) : \lambda A \lambda B(Pres\ (A \to C.[B = C]; D.((D\ \lambda E[E = B])\ B))), \Box\forall n(CNn/CNn) : \hat{}\lambda F \lambda G[(F\ G) \wedge (\check{}humanist\ G)]\ \Rightarrow\ Sf$

There is the derivation:

$$\dfrac{\dfrac{}{CNA \Rightarrow CNA} \qquad \dfrac{}{\boxed{CNA} \Rightarrow CNA}}{\boxed{CNA/CNA}, CNA \Rightarrow CNA}\,/L$$

$$\dfrac{\boxed{CNA/CNA}, CNA \Rightarrow CNA}{\boxed{\forall n(CNn/CNn)}, CNA \Rightarrow CNA}\,\forall L$$

$$\dfrac{\boxed{\forall n(CNn/CNn)}, CNA \Rightarrow CNA}{\boxed{\Box\forall n(CNn/CNn)}, CNA \Rightarrow CNA}\,\Box L$$

$$\dfrac{\boxed{\Box\forall n(CNn/CNn)}, CNA \Rightarrow CNA}{\Box\forall n(CNn/CNn) \Rightarrow CNA/CNA}\,/R$$

$$\dfrac{\Box\forall n(CNn/CNn) \Rightarrow CNA/CNA}{\Box\forall n(CNn/CNn) \Rightarrow \boxed{(CNA/CNA)\sqcup(CNA\backslash CNA)}}\,\sqcup R$$

$$\dfrac{\Box\forall n(CNn/CNn) \Rightarrow \boxed{(CNA/CNA)\sqcup(CNA\backslash CNA)}}{\Box\forall n(CNn/CNn) \Rightarrow \boxed{\exists g((CNg/CNg)\sqcup(CNg\backslash CNg))}}\,\exists R$$

$$\dfrac{\Box\forall n(CNn/CNn) \Rightarrow \boxed{\exists g((CNg/CNg)\sqcup(CNg\backslash CNg))}}{\Box\forall n(CNn/CNn) \Rightarrow \boxed{\exists g((CNg/CNg)\sqcup(CNg\backslash CNg)){-}I}}\,{-}R$$

$$\dfrac{\Box\forall n(CNn/CNn) \Rightarrow \boxed{\exists g((CNg/CNg)\sqcup(CNg\backslash CNg)){-}I}}{\Box\forall n(CNn/CNn) \Rightarrow \boxed{\exists aNa\oplus(\exists g((CNg/CNg)\sqcup(CNg\backslash CNg)){-}I)}}\,\oplus R$$

$$\dfrac{\dfrac{}{\boxed{Nt(s(m))} \Rightarrow Nt(s(m))}}{\boxed{\blacksquare Nt(s(m))} \Rightarrow Nt(s(m))}\,\blacksquare L$$

$$\dfrac{\boxed{\blacksquare Nt(s(m))} \Rightarrow Nt(s(m))}{\blacksquare Nt(s(m)) \Rightarrow \boxed{\exists gNt(s(g))}}\,\exists R$$

$$\dfrac{\blacksquare Nt(s(m)) \Rightarrow \boxed{\exists gNt(s(g))}}{[\blacksquare Nt(s(m))] \Rightarrow \boxed{\langle\rangle\exists gNt(s(g))}}\,\langle\rangle R$$

$$\dfrac{[\blacksquare Nt(s(m))] \Rightarrow \boxed{\langle\rangle\exists gNt(s(g))} \qquad \dfrac{}{\boxed{Sf} \Rightarrow Sf}}{[\blacksquare Nt(s(m))], \boxed{\langle\rangle\exists gNt(s(g))\backslash Sf} \Rightarrow Sf}\,\backslash L$$

$$\dfrac{\Box\forall n(CNn/CNn) \Rightarrow \boxed{\exists aNa\oplus(\exists g((CNg/CNg)\sqcup(CNg\backslash CNg)){-}I)} \qquad [\blacksquare Nt(s(m))], \boxed{\langle\rangle\exists gNt(s(g))\backslash Sf} \Rightarrow Sf}{[\blacksquare Nt(s(m))], \boxed{(\langle\rangle\exists gNt(s(g))\backslash Sf)/(\exists aNa\oplus(\exists g((CNg/CNg)\sqcup(CNg\backslash CNg)){-}I))}, \Box\forall n(CNn/CNn) \Rightarrow Sf}\,/L$$

$$\dfrac{[\blacksquare Nt(s(m))], \boxed{(\langle\rangle\exists gNt(s(g))\backslash Sf)/(\exists aNa\oplus(\exists g((CNg/CNg)\sqcup(CNg\backslash CNg)){-}I))}, \Box\forall n(CNn/CNn) \Rightarrow Sf}{[\blacksquare Nt(s(m))], \boxed{\blacksquare((\langle\rangle\exists gNt(s(g))\backslash Sf)/(\exists aNa\oplus(\exists g((CNg/CNg)\sqcup(CNg\backslash CNg)){-}I)))}, \Box\forall n(CNn/CNn) \Rightarrow Sf}\,\blacksquare L$$

After elimination of its outer modality, the copula is applied to its adjectival complement. Semantically active additive disjunction right selects the second disjunct. The difference right rule checks that the antecedent is not empty, but this is not displayed. Exists right substitutes the existentially quantified variable for a metavariable *A* and semantically inactive additive disjunction right then selects the adjectival disjunct. The following semantics is delivered:

(44) $(\mathit{Pres}\ (\check{\ }\mathit{humanist}\ t))$

Relativisation

Our account of relativisation rests on the lexical projection of islands by argument bracketing ($\langle\rangle$) and value antibracketing ($[\,]^{-1}$), and a single relative pronoun type of overall shape $R/((\langle\rangle N\sqcap !N)\backslash S)$ for both subject and object relativisation.

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In analysis of the body of relative clauses the higher order succedent argument essentially of form $\langle\rangle N \sqcap !N$ is lowered into the antecedent according to the deduction theorem;
in subject relativisation $\langle\rangle N$ is selected by conjunction left, and satisfies the (bracketed) subject valency.
In object relativisation $!N$ is selected. When the $!L$ rule is applied to $!N$, the hypothetical subtype N moves into the stoup, from where it can move by $!P$ to any (nonisland) position in its zone, realising nonparasitic extraction.

However, in addition it can be copied by !*C* to the stoup of a newly created weak island domain, realising parasitic extraction. The *N* in the outer stoop can be copied by !*C* repeatedly, capturing that there may be parasitic gaps in any number of local weak islands; at the end of this process it moves by !*P* to a host position in its zone. The *N* in an inner stoup can also be copied by !*C* to the stoup of any number of newly created weak subislands, and so on recursively, capturing that parasitic gaps can also be hosts to further parasitic gaps; finally the stoup contents are copied by !*P* to an extraction site in their zone.

The first example is a minimal subject relativisation; note that the relative clause is doubly bracketed, corresponding to the fact that relative clauses are strong islands (relative clauses themselves, being doubly bracketed, will not allow parasitic gaps):

(45) $\mathbf{man}+[[\mathbf{that}+\mathbf{walks}]] : \mathit{CNs(m)}$

Lexical lookup yields the following, where there is semantically inactive additive conjunction of the hypothetical subtypes $\langle\rangle N$ for subject relativisation and $!\blacksquare N$ for object relativisation; the (semantically inactive) modality on the object gap subtype is to permit object relativisation from embedded modal/intensional domains:

(46) $\Box \mathit{CNs(m)} : \mathit{man}, [[\blacksquare\forall n([]^{-1}[]^{-1}(\mathit{CN}n\backslash \mathit{CN}n)/\blacksquare((\langle\rangle \mathit{Nt}(n)\sqcap !\blacksquare \mathit{Nt}(n))\backslash \mathit{Sf})) :$
$\lambda A\lambda B\lambda C[(B\ C) \wedge (A\ C)], \Box(\langle\rangle\exists g\mathit{Nt}(s(g))\backslash \mathit{Sf}) :$
$\hat{}\lambda D(\mathit{Pres}\ (\check{}\mathit{walk}\ D))]] \ \Rightarrow\ \mathit{CNs(m)}$

There is the following derivation:

$$
\begin{prooftree}
\AxiomC{}
\UnaryInfC{$Nt(s(m)) \Rightarrow Nt(s(m))$}
\RightLabel{$\exists R$}
\UnaryInfC{$Nt(s(m)) \Rightarrow \boxed{\exists gNt(s(g))}$}
\RightLabel{$\langle\rangle R$}
\UnaryInfC{$[Nt(s(m))] \Rightarrow \boxed{\langle\rangle\exists gNt(s(g))}$}
\AxiomC{}
\UnaryInfC{$\boxed{Sf} \Rightarrow Sf$}
\RightLabel{$\backslash L$}
\BinaryInfC{$[Nt(s(m))], \boxed{\langle\rangle\exists gNt(s(g))\backslash Sf} \Rightarrow Sf$}
\RightLabel{$\square L$}
\UnaryInfC{$[Nt(s(m))], \boxed{\square(\langle\rangle\exists gNt(s(g))\backslash Sf)} \Rightarrow Sf$}
\RightLabel{$\langle\rangle L$}
\UnaryInfC{$\langle\rangle Nt(s(m)), \square(\langle\rangle\exists gNt(s(g))\backslash Sf) \Rightarrow Sf$}
\RightLabel{$\sqcap L$}
\UnaryInfC{$\boxed{\langle\rangle Nt(s(m))\sqcap !\blacksquare Nt(s(m))}, \square(\langle\rangle\exists gNt(s(g))\backslash Sf) \Rightarrow Sf$}
\RightLabel{$\backslash R$}
\UnaryInfC{$\square(\langle\rangle\exists gNt(s(g))\backslash Sf) \Rightarrow (\langle\rangle Nt(s(m))\sqcap !\blacksquare Nt(s(m)))\backslash Sf$}
\RightLabel{$\blacksquare R$}
\UnaryInfC{$\square(\langle\rangle\exists gNt(s(g))\backslash Sf) \Rightarrow \blacksquare((\langle\rangle Nt(s(m))\sqcap !\blacksquare Nt(s(m)))\backslash Sf)$}
\AxiomC{}
\UnaryInfC{$\boxed{CNs(m)} \Rightarrow CNs(m)$}
\RightLabel{$\square L$}
\UnaryInfC{$\boxed{\square CNs(m)} \Rightarrow CNs(m)$}
\AxiomC{}
\UnaryInfC{$\boxed{CNs(m)} \Rightarrow CNs(m)$}
\RightLabel{$\backslash L$}
\BinaryInfC{$\square CNs(m), \boxed{CNs(m)\backslash CNs(m)} \Rightarrow CNs(m)$}
\RightLabel{$[]^{-1}L$}
\UnaryInfC{$\square CNs(m), [\boxed{[]^{-1}(CNs(m)\backslash CNs(m))}] \Rightarrow CNs(m)$}
\RightLabel{$[]^{-1}L$}
\UnaryInfC{$\square CNs(m), [[\boxed{[]^{-1}[]^{-1}(CNs(m)\backslash CNs(m))}]] \Rightarrow CNs(m)$}
\RightLabel{$/L$}
\BinaryInfC{$\square CNs(m), [[\boxed{[]^{-1}[]^{-1}(CNs(m)\backslash CNs(m))/\blacksquare((\langle\rangle Nt(s(m))\sqcap !\blacksquare Nt(s(m)))\backslash Sf)}, \square(\langle\rangle\exists gNt(s(g))\backslash Sf)]] \Rightarrow CNs(m)$}
\RightLabel{$\forall L$}
\UnaryInfC{$\square CNs(m), [[\boxed{\forall n([]^{-1}[]^{-1}(CNn\backslash CNn)/\blacksquare((\langle\rangle Nt(n)\sqcap !\blacksquare Nt(n))\backslash Sf))}, \square(\langle\rangle\exists gNt(s(g))\backslash Sf)]] \Rightarrow CNs(m)$}
\RightLabel{$\blacksquare L$}
\UnaryInfC{$\square CNs(m), [[\boxed{\blacksquare\forall n([]^{-1}[]^{-1}(CNn\backslash CNn)/\blacksquare((\langle\rangle Nt(n)\sqcap !\blacksquare Nt(n))\backslash Sf))}, \square(\langle\rangle\exists gNt(s(g))\backslash Sf)]] \Rightarrow CNs(m)$}
\end{prooftree}
$$

This delivers the required semantics:

(47) $\lambda C[(\check{}\,man\ C) \wedge (Pres\ (\check{}\,walk\ C))]$

The next sentence contains a minimal example of object relativisation:

(48) $[\mathbf{the}+\mathbf{man}+[[\mathbf{that}+[\mathbf{mary}]+\mathbf{loves}]]]+\mathbf{walks} : Sf$

Lexical lookup yields:

(49) $[\blacksquare\forall n(Nt(n)/CNn) : \iota, \Box CNs(m) :$
$man, [[\blacksquare\forall n([]^{-1}[]^{-1}(CNn\backslash CNn)/\blacksquare(((\langle\rangle Nt(n)\sqcap !\blacksquare Nt(n))\backslash Sf)) :$
$\lambda A\lambda B\lambda C[(B\ C) \wedge (A\ C)], [\blacksquare Nt(s(f)) :$
$m], \Box((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa) :$
$\hat{}\lambda D\lambda E(Pres\ ((\check{}love\ D)\ E))]]], \Box(\langle\rangle\exists gNt(s(g))\backslash Sf) :$
$\hat{}\lambda F(Pres\ (\check{}walk\ F)) \ \Rightarrow\ Sf$

There is the following derivation:

$$\frac{}{\boxed{Nt(s(m))} \Rightarrow Nt(s(m))}$$

$$\frac{\boxed{Nt(s(m))} \Rightarrow Nt(s(m))}{\boxed{\blacksquare Nt(s(m))} \Rightarrow Nt(s(m))}\ \blacksquare L$$

$$\frac{\boxed{\blacksquare Nt(s(m))} \Rightarrow Nt(s(m))}{\boxed{\blacksquare Nt(s(m))}; \Rightarrow Nt(s(m))}\ !P$$

$$\frac{\boxed{\blacksquare Nt(s(m))}; \Rightarrow Nt(s(m))}{\blacksquare Nt(s(m)); \Rightarrow \boxed{\exists aNa}}\ \exists R$$

$$\frac{}{\boxed{Nt(s(f))} \Rightarrow Nt(s(f))}$$

$$\frac{\boxed{Nt(s(f))} \Rightarrow Nt(s(f))}{\boxed{\blacksquare Nt(s(f))} \Rightarrow Nt(s(f))}\ \blacksquare L$$

$$\frac{\boxed{\blacksquare Nt(s(f))} \Rightarrow Nt(s(f))}{\blacksquare Nt(s(f)) \Rightarrow \boxed{\exists gNt(s(g))}}\ \exists R$$

$$\frac{\blacksquare Nt(s(f)) \Rightarrow \boxed{\exists gNt(s(g))}}{[\blacksquare Nt(s(f))] \Rightarrow \boxed{\langle\rangle\exists gNt(s(g))}}\ \langle\rangle R$$

$$\frac{}{\boxed{Sf} \Rightarrow Sf}$$

$$\frac{[\blacksquare Nt(s(f))] \Rightarrow \boxed{\langle\rangle\exists gNt(s(g))} \qquad \boxed{Sf} \Rightarrow Sf}{[\blacksquare Nt(s(f))], \boxed{\langle\rangle\exists gNt(s(g))\backslash Sf} \Rightarrow Sf}\ \backslash L$$

$$\frac{\blacksquare Nt(s(m)); \Rightarrow \boxed{\exists aNa} \qquad [\blacksquare Nt(s(f))], \boxed{\langle\rangle\exists gNt(s(g))\backslash Sf} \Rightarrow Sf}{\blacksquare Nt(s(m));\ [\blacksquare Nt(s(f))], \boxed{(\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa} \Rightarrow Sf}\ /L$$

$$\frac{\blacksquare Nt(s(m));\ [\blacksquare Nt(s(f))], \boxed{(\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa} \Rightarrow Sf}{\blacksquare Nt(s(m));\ [\blacksquare Nt(s(f))], \boxed{\square((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa)} \Rightarrow Sf}\ \square L$$

$$\frac{\blacksquare Nt(s(m));\ [\blacksquare Nt(s(f))], \boxed{\square((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa)} \Rightarrow Sf}{!\blacksquare Nt(s(m)), [\blacksquare Nt(s(f))], \square((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa) \Rightarrow Sf}\ !L$$

$$\frac{!\blacksquare Nt(s(m)), [\blacksquare Nt(s(f))], \square((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa) \Rightarrow Sf}{\boxed{\langle\rangle Nt(s(m))\sqcap !\blacksquare Nt(s(m))}, [\blacksquare Nt(s(f))], \square((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa) \Rightarrow Sf}\ \sqcap L$$

$$\frac{\boxed{\langle\rangle Nt(s(m))\sqcap !\blacksquare Nt(s(m))}, [\blacksquare Nt(s(f))], \square((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa) \Rightarrow Sf}{[\blacksquare Nt(s(f))], \square((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa) \Rightarrow (\langle\rangle Nt(s(m))\sqcap !\blacksquare Nt(s(m)))\backslash Sf}\ \backslash R$$

$$\frac{[\blacksquare Nt(s(f))], \square((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa) \Rightarrow (\langle\rangle Nt(s(m))\sqcap !\blacksquare Nt(s(m)))\backslash Sf}{[\blacksquare Nt(s(f))], \square((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa) \Rightarrow \blacksquare((\langle\rangle Nt(s(m))\sqcap !\blacksquare Nt(s(m)))\backslash Sf)}\ \blacksquare R$$

$$\frac{}{\boxed{CNs(m)} \Rightarrow CNs(m)}$$

$$\frac{\boxed{CNs(m)} \Rightarrow CNs(m)}{\boxed{\square CNs(m)} \Rightarrow CNs(m)}\ \square L \qquad \boxed{CN$$

$$\square CNs(m), \boxed{CNs(m)\backslash CNs(m)}$$

$$\square CNs(m), [\boxed{[]^{-1}(CNs(m)\backslash CNs(m)$$

$$\square CNs(m), [[\boxed{[]^{-1}[]^{-1}(CNs(m)\backslash CNs($$

$$\square CNs(m), [[\boxed{[]^{-1}[]^{-1}(CNs(m)\backslash CNs(m))/\blacksquare((\langle\rangle Nt(s(m))\sqcap !\blacksquare Nt(s(m)))\backslash Sf)}, [\blacksquare Nt(s(f))], \square((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa)]]$$

$$\square CNs(m), [[\boxed{\forall n([]^{-1}[]^{-1}(CNn\backslash CNn)/\blacksquare((\langle\rangle Nt(n)\sqcap !\blacksquare Nt(n))\backslash Sf))}, [\blacksquare Nt(s(f))], \square((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa)]] \Rightarrow C$$

$$\square CNs(m), [[\boxed{\blacksquare\forall n([]^{-1}[]^{-1}(CNn\backslash CNn)/\blacksquare((\langle\rangle Nt(n)\sqcap !\blacksquare Nt(n))\backslash Sf))}, [\blacksquare Nt(s(f))], \square((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa)]] \Rightarrow$$

This delivers the required semantics:

(50) $(\mathit{Pres}\ (\check{}\,\mathit{walk}\ (\iota\ \lambda D[(\check{}\,\mathit{man}\ D) \wedge (\mathit{Pres}\ ((\check{}\,\mathit{love}\ D)\ m))])))$

An example with longer-distance object relativisation, in the context of an entire sentence, is:

(51)

$$[\textbf{the}+\textbf{man}+[[\textbf{that}+[\textbf{john}]+\textbf{thinks}+[\textbf{mary}]+\textbf{loves}]]]+\textbf{walks} : Sf$$

Lexical lookup yields the following; note how the propositional attitude verb is polymorphic between a complementised and an uncomplementised sentential argument, expressed with a semantically inactive additive disjunction:

(52)

$$\begin{array}{l}[\blacksquare\forall n(Nt(n)/CNn) : \iota, \square CNs(m) : \\ man, [[\blacksquare\forall n([]^{-1}[]^{-1}(CNn\backslash CNn)/\blacksquare(((\langle\rangle Nt(n)\sqcap !\blacksquare Nt(n))\backslash Sf)) : \\ \lambda A\lambda B\lambda C[(B\ C)\wedge(A\ C)], [\blacksquare Nt(s(m)) : \\ j], \square((\langle\rangle\exists gNt(s(g))\backslash Sf)/(CPthat\sqcup\square Sf)) : \\ {}^\wedge\lambda D\lambda E(Pres\ (({}^\vee think\ D)\ E)), [\blacksquare Nt(s(f)) : \\ m], \square((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa) : \\ {}^\wedge\lambda F\lambda G(Pres\ (({}^\vee love\ F)\ G))]]], \square(\langle\rangle\exists gNt(s(g))\backslash Sf) : \\ {}^\wedge\lambda H(Pres\ ({}^\vee walk\ H)) \ \Rightarrow\ Sf\end{array}$$

Derivation delivers the correct semantics:

(53) $(\mathit{Pres}\ ({}^{\vee}\mathit{walk}\ (\iota\ \lambda D[({}^{\vee}\mathit{man}\ D) \wedge$
$(\mathit{Pres}\ (({}^{\vee}\mathit{think}\ {}^{\wedge}(\mathit{Pres}\ (({}^{\vee}\mathit{love}\ D)\ m)))\ j))])))))$

There follows an example of medial object relativisation (the gap is in a non-peripheral position left of the adverb):

(54) $\mathbf{man}+[[\mathbf{that}+[\mathbf{mary}]+\mathbf{likes}+\mathbf{today}]] : \mathit{CNs(m)}$

Appropriate lexical lookup yields:

(55) $\Box \mathit{CNs(m)}$:
$man, [[\blacksquare\forall n([]^{-1}[]^{-1}(CNn\backslash CNn)/\blacksquare((\langle\rangle Nt(n)\sqcap !\blacksquare Nt(n))\backslash Sf))$:
$\lambda A\lambda B\lambda C[(B\ C) \wedge (A\ C)], [\blacksquare Nt(s(f))$:
$m], \Box((\langle\rangle\exists g Nt(s(g))\backslash Sf)/\exists a Na)$:
$\hat{}\lambda D\lambda E(Pres\ ((\check{}like\ D)\ E)), \Box\forall a\forall f((\langle\rangle Na\backslash Sf)\backslash(\langle\rangle Na\backslash Sf))$:
$\hat{}\lambda F\lambda G(\check{}today\ (F\ G))]] \;\Rightarrow\; \mathit{CNs(m)}$

The semantics delivered is:

(56) $\lambda C[(\check{}man\ C) \wedge (\check{}today\ (Pres\ ((\check{}like\ C)\ m)))]$

As we remarked subjects are weak islands (the Subject Condition of Chomsky 1973[3]); accordingly in our CatLog2 fragment there is no derivation of simple relativisation from a subject such as:

(57) **man**+[[**that**+[**the**+**friends**+**of**]+**walk**]] : *CNs(m)*

This is because *walk* projects brackets around its subject, but the permutation of the ! hypothetical gap subtype issued by the relative pronoun is limited to its zone and cannot penetrate a bracketed subzone. Roughly, the derivation blocks at * in:

(58)
$$\dfrac{\dfrac{\dfrac{[N/CN, CN/PP, PP/N, N], N\backslash S \Rightarrow S}{N; [N/CN, CN/PP, PP/N], N\backslash S \Rightarrow S}\,*!P}{!N, [N/CN, CN/PP, PP/N], N\backslash S \Rightarrow S}\,!L}{[N/CN, CN/PP, PP/N], N\backslash S \Rightarrow\, !N\backslash S}\,\backslash R$$

However, a weak island ‘parasitic’ gap can be licensed by a host gap:

(59) **man**+[[**that**+**the**+**friends**+**of**+**admire**]] : *CNs(m)*

Lexical lookup yields:

(60) $\square CNs(m)$:
$man, [[\blacksquare\forall n([]^{-1}[]^{-1}(CNn\backslash CNn)/\blacksquare(((\langle\rangle Nt(n)\sqcap !\blacksquare Nt(n))\backslash Sf))$:
$\lambda A\lambda B\lambda C[(B\ C)\wedge(A\ C)], \blacksquare\forall n(Nt(n)/CNn)$:
$\iota, \square(CNp/PPof)$:
$friends, \square((\forall n(CNn\backslash CNn)/\blacksquare\exists bNb)\&(PPof/\exists aNa))$:
$\hat{}(\check{}of, \lambda DD), \square(((\langle\rangle(\exists aNa - \exists gNt(s(g))))\backslash Sf)/\exists aNa)$:
$\hat{}\lambda E\lambda F(Pres\ ((\check{}admire\ E)\ F))]] \Rightarrow CNs(m)$

The following semantics is delivered in which the gap variable is multiply bound:

(61) $\lambda C[(\check{\ }\mathit{man}\ C) \wedge (\mathit{Pres}\ ((\check{\ }\mathit{admire}\ C)\ (\iota\ (\check{\ }\mathit{friends}\ C))))]$

Parasitic extraction from strong islands such as coordinate structures is not acceptable:

(62) *that$_i$ Mary showed [[John and the friends of t_i]] to t_i

This is successfully blocked because strong islands are doubly bracketed. Although contraction could apply twice to introduce two bracketings, a copy of the hypothetical gap subtype would remain trapped in the stoup at the intermediate level of bracketing, blocking overall derivation. Likewise, parasitic extraction is not possible from relative clauses themselves, for the same reason: a superfluous gap subtype would remain in between the double brackets required for the strong island.

A parasitic gap can also appear in an adverbial weak island:

(63) **paper**$+$[[**that**$+$[**john**]$+$**filed**$+$**without**$+$**reading**]] : *CNs(n)*

Lexical lookup for this example yields:

(64) $\Box CNs(n)$:

$paper, [[\blacksquare\forall n([\,]^{-1}[\,]^{-1}(CNn\backslash CNn)/\blacksquare((\langle\rangle Nt(n)\sqcap !\blacksquare Nt(n))\backslash Sf))$:
$\lambda A\lambda B\lambda C[(B\ C) \wedge (A\ C)], [\blacksquare Nt(s(m))$:
$j], \Box((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa)$:
$\hat{}\lambda D\lambda E(Past\,((\check{}file\ D)\ E)), \blacksquare\forall a\forall f([\,]^{-1}(((\langle\rangle Na\backslash Sf)\backslash(\langle\rangle Na\backslash Sf))/(\langle\rangle N$
$\lambda F\lambda G\lambda H[(G\ H) \wedge \neg(F\ H)], \Box((\langle\rangle\exists aNa\backslash Spsp)/\exists aNa)$:
$\hat{}\lambda I\lambda J((\check{}read\ I)\ J)]] \;\Rightarrow\; CNs(n)$

There is delivers semantics:

(65) $\lambda C[(\check{}\mathit{paper}\ C) \wedge [(\mathit{Past}\ ((\check{}\mathit{file}\ C)\ j)) \wedge \neg((\check{}\mathit{read}\ C)\ j)]]$

In our final relativisation example the host gap licences two parasitic gaps, in the subject noun phrase and in an adverbial phrase:

(66)

paper$+$[[**that**$+$**the**$+$**editor**$+$**of**$+$**filed**$+$**without**$+$**reading**]] : $CNs(n)$

Lexical lookup yields:

(67) $\Box CNs(n)$:
$paper$, $[[\blacksquare\forall n([]^{-1}[]^{-1}(CNn\backslash CNn)/\blacksquare((\langle\rangle Nt(n)\sqcap!\blacksquare Nt(n))\backslash Sf))$:
$\lambda A\lambda B\lambda C[(B\ C) \wedge (A\ C)]$, $\blacksquare\forall n(Nt(n)/CNn)$:
ι, $\Box(\forall gCNs(g)/PPof)$:
$editor$, $\Box((\forall n(CNn\backslash CNn)/\blacksquare\exists bNb)\&(PPof/\exists aNa))$:
$\hat{}(\check{}of, \lambda DD)$, $\Box((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa)$:
$\hat{}\lambda E\lambda F(Past((\check{}file\ E)\ F))$, $\blacksquare\forall a\forall f([]^{-1}((\langle\rangle Na\backslash Sf)\backslash(\langle\rangle Na\backslash Sf))/(\langle\rangle N$
$\lambda G\lambda H\lambda I[(H\ I) \wedge \neg(G\ I)]$, $\Box((\langle\rangle\exists aNa\backslash Spsp)/\exists aNa)$:
$\hat{}\lambda J\lambda K((\check{}read\ J)\ K)]]$ $\Rightarrow$ $CNs(n)$

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}{[\blacksquare\forall n(Nt(n)/CNn), \Box(\forall gCNs(g)/PPof), \Box((\forall n(CNn\backslash CNn)/\blacksquare\exists bNb)\&(PPof/\exists aNa)), \blacksquare Nt(s(n))], \boxed{\langle\rangle Nt(s(A))\backslash Sf} \Rightarrow}
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$$[\blacksquare\forall n(Nt(n)/CNn), \Box(\forall gCNs(g)/PPof), \Box((\forall n(CNn\backslash CNn)/\blacksquare\exists bNb)\&(PPof/\exists aNa)),$$

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$$[\blacksquare\forall n(Nt(n)/CNn), \Box(\forall gCNs(g)/PPof), \Box((\forall n(CNn\backslash CNn)/\blacksquare\exists bNb)\&(PPof/\exists aNa)), \blacksquare Nt$$

$$[\blacksquare\forall n(Nt(n)/CNn), \Box(\forall gCNs(g)/PPof), \Box((\forall n(CNn\backslash CNn)/\blacksquare\exists bNb)\&(PPof/\exists aNa)), \blacksquare Nt(s(n))], \Box((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa), \blacksquare Nt(s(n))$$

$$[\blacksquare\forall n(Nt(n)/CNn), \Box(\forall gCNs(g)/PPof), \Box((\forall n(CNn\backslash CNn)/\blacksquare\exists bNb)\&(PPof/\exists aNa)), \blacksquare Nt(s(n))], \Box((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa), \blacksquare Nt(s(n)), [$$

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$$\blacksquare\forall n(Nt(n)/CNn), \Box(\forall gCNs(g)/PPof), \Box((\forall n(CNn\backslash CNn)/\blacksquare\exists bNb)\&(PPof/\exists aNa)), \Box((\langle\rangle\exists gNt(s(g))\backslash Sf)/\exists aNa), \blacksquare\forall a\forall f([]^{-1}$$

This delivers the correct semantics:

(68) $\lambda C[(\check{}\mathit{paper}\ C) \wedge [(\mathit{Past}\ ((\check{}\mathit{file}\ C)\ (\iota\ (\check{}\mathit{editor}\ C)))) \wedge$
$\neg((\check{}\mathit{read}\ C)\ (\iota\ (\check{}\mathit{editor}\ C)))]]$

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