Algorithmic Game Theory
Fall 2018
Exercises 3

23. Consider a keyword auction in the model and social welfare described at class. Provide a way to compute the payments of the VCG mechanism and illustrate it with some examples.

24. Consider a GSP auction for n players.

    - Show that any bid $b_i > v_i$ is dominated by bid $b_i = v_i$.
    - Show that every envy-free equilibrium is efficient.
    - Show that the PoS of the GSP mechanism is 1.

25. Consider a GSP auction on n bidders in which all the valuation factors are 1. We say that a bid profile b is *up-Nash* for player i if he can't increase his utility by taking some slot above the one in the corresponding allocation π. A bid profile is *up-Nash* if it is up-Nash for all players.

    Show that if a bid profile b is a Nash equilibrium, then the bid profile $b'_i = b_{\pi(i)}$ is up-Nash.

26. Consider a GSP auction on n bidders in which all the valuation factors are 1 ($\gamma_i = 1$) and the corresponding VCG mechanism.

    - Show that GSP can generate more revenue than VCG and viceversa.
    - Show that the GSP revenue in a Nash equilibrium is always at least half of the revenue of the VCG mechanism on the auction in which the participants are all the bidders except the one with the highest valuation. (Hint. Use the construction on the previous problem)

27. Consider a GSP auction for n players. Recall that in such an auction each bid profile b defines an allocation π mapping slots to players. We say that an allocation is *reasonable* if for each pair i, j of slots

    $$\frac{\alpha_j}{\alpha_i} + \frac{\gamma_{\pi(i)} v_{\pi(i)}}{\gamma_{\pi(j)} v_{\pi(j)}} \geq 1.$$

    - Prove that when b is a NE, the corresponding allocation π is reasonable.
    - Use the previous fact to show that the price of anarchy, on pure strategies, of the GSP auction is at most 2.
    - Can the bound on the price of anarchy be improved, in the case in which there are only 2 slots?