

Algorithmic Game Theory
Fall 2017
Exercises 3

19. The owner of an item is planning to increase his benefit by being one of the bidders in the selling auction. The auction will take the format of an English auction. If the owner wins, he will not pay anything and the item remains his property. Assume that there are n bidders (other than the owner) and that the value of the item for each bidder is drawn uniformly at random from $[0, d]$. Also the value of the good for the owner is v . The auction bids increases by a value e which is vanishingly small compare to d . Can the owner get a higher benefit by participating? If so, what is the owners optimal bidding strategy?
20. Consider a combinatorial auction in which we have n bidders and a set \mathcal{O} of m objects for sale. Each bidder has a valuation $v_i(S)$, for $S \subseteq \mathcal{O}$. The aim of the seller is to maximize revenue.
- Assuming that $\mathcal{O} = \{a, b, c\}$ and that there are three players $\{A, B, C\}$. The valuations each player has for each subset of the items is

| | a | b | c | ab | bc | ac | abc |
|-----|-----|-----|-----|------|------|------|-------|
| A | 1 | 2 | 1 | 6 | 4 | 5 | 11 |
| B | 3 | 5 | 3 | 5 | 5 | 3 | 5 |
| C | 4 | 5 | 0 | 7 | 5 | 7 | 9 |

What is the allocation produced by the VCG mechanism? What are the VCG payments?

- Assuming that the bids are truthful give an integer program formulation to compute the VCG allocation.
21. Consider an auction in which the seller announces a reserve price of r before running the auction. With a reserve price, the item is sold to the highest bidder if the highest bid is above r ; otherwise, the item is not sold. In a first-price auction with a reserve price, the winning bidder (if there is one) still pays her bid. In a second-price auction with a reserve price, the winning bidder (if there is one) pays the maximum of the second-place bid and the reserve price r .
- Is truthful bidding a dominant strategy on such auctions?
 - Assuming that the seller assigns value u to the object and chooses the reserve price r in order to maximize its expected revenue in a SP auction. Show that r might be higher than u . (Hint. Provide a value for the simple case in which the item is worth $u = 0$ to the seller and a second-price auction with a single bidder, whose value is uniformly distributed on $[0, 1]$ is run.)
22. Consider a keyword auction in the model described at class. Provide a way to compute the payments of the VCG mechanism and illustrate it with some examples.

23. Consider a GSP auction for n players.

- Show that any bid $b_i > v_i$ is dominated by bid $b_i = v_i$.
- Show that every envy-free equilibrium is efficient.
- Show that the PoS of the GSP mechanism is 1.

24. Consider a GSP auction on n bidders in which all the valuation factors are 1. We say that a bid profile b is *up-Nash* for player i if he can't increase his utility by taking some slot above the one in the corresponding allocation π . A bid profile is *up-Nash* if it is up-Nash for all players.

Show that if a bid profile b is a Nash equilibrium, then the bid profile $b'_i = b_{\pi(i)}$ is up-Nash.

25. Consider a GSP auction on n bidders in which all the valuation factors are 1 ($\gamma_i = 1$) and the corresponding VCG mechanism.

- Show that GSP can generate more revenue than VCG and viceversa.
- Show that the GSP revenue in a Nash equilibrium is always at least half of the revenue of the VCG mechanism on the auction in which the participants are all the bidders except the one with the highest valuation. (Hint. Use the construction on the previous problem)

26. Consider a GSP auction for n players. Recall that in such an auction each bid profile b defines an allocation π mapping slots to players. We say that an allocation is *reasonable* if for each pair i, j of slots

$$\frac{\alpha_j}{\alpha_i} + \frac{\gamma_{\pi(i)} v_{\pi(i)}}{\gamma_{\pi(j)} v_{\pi(j)}} \geq 1.$$

- Prove that when b is a NE, the corresponding allocation π is reasonable.
- Use the previous fact to show that the price of anarchy, on pure strategies, of the GSP auction is at most 2.
- Can the bound on the price of anarchy be improved, in the case in which there are only 2 slots?